The regularities of resistance and flow in pipes and wide channels

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Abstract. The data of hydraulic characteristics of flow are required to be more accurate to increase reliability and accident-free work of water conducting systems and hydraulic structures. One of the problems in hydraulic calculations is the determination of friction loss that is associated with the distribution of velocities over the cross section of the flow. The article presents a comparative analysis of the regularities of velocity distribution based on the logarithmic velocity profile and hydraulic resistance in pipes and open channels. It is revealed that the Karman parameter is associated with the second turbulence constant and depend on the hydraulic resistance coefficient. The research showed that the behavior of the second turbulence constant in the velocity profile is determined mainly by the Karman parameter. The attached illustrations picture the dependence of logarithmic velocity profile parameters based on different values of the hydraulic resistance coefficient. The results of the calculations were compared to the experimental-based Nikuradze formulas for smooth and rough pipes.

1 Introduction

Determination of the friction loss is one of the most important problems in hydraulic calculations for pipes and channels [1-15]. Nowadays the concept of identity of flow regularity and resistance in pipes and wide channels is widely used. In this concept half-empiric dependences for resistance and velocity distribution were established using the suggestion that the flow in pipes and channels are alike [3]. Till present days the logarithmic velocity profile distribution based on Prandtl theories is used in hydraulic calculations for pipes and channels, that differs for smooth type of resistance:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{u_* y}{\nu} + C_1, \quad (1)$$

and rough type of resistance:

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$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y}{k} + C_2, \quad (2)$$

This velocity distribution has two undefined parameters: Karman parameter κ , also often called as “the first turbulence constant”, and parameter C , that is called “the second turbulence constant”. Their values were found by Prandtl and Nikuradze based on the famous series of Nikuradze experiments. The κ value was approximate 0.4 and not depended on the resistance type. The C_1 value for smooth type of resistance was equal 5.5 and C_2 value for rough type of resistance was close to 8.48.

Further researches [2, 4, 6] showed, that the Karman parameter and C values don't stay constant for every case, so the defining them as constants is not quite correct. The factors that regulate κ and C values are still not revealed and the character of their variations are not defined yet. That slows down the opportunity of velocity profile connected hydraulic calculations' refinement.

Millikan discovered that the amount of C_1 parameter in the border layer changes depending on $\frac{z}{\delta}$ (where δ – stands for border layer depth):

$$C_1 = B + h_1 \left(\frac{z}{\delta} \right), \quad (3)$$

where B is a numerous constant picturing the result of experimental data averaging and equals 4.9; $h_1 \left(\frac{z}{\delta} \right)$ – correction function.

Rotta's theoretical researches showed that C_1 and κ parameters are connected with each other through ratio

$$C = \frac{1}{\kappa} (\ln 4\kappa - 1) + \frac{u_* \delta_v}{\nu}, \quad (4)$$

where $\frac{u_* \delta_v}{\nu}$ is a dimensionless depth of viscous sublayer.

However, the connection between velocity profile parameters and hydraulic flow characteristics (through Reynold's number and hydraulic resistance coefficient λ) is not defined. The attempts to determine this connection was taken by Klauser, Couls and other authors [7-13] yet they didn't give any defined results.

The goal of this work is to define the logarithmic velocity profile behavior pattern, their connection with hydraulic resistance coefficient for pipes and channels.

2 Materials and methods

Velocity profiles for smooth and rough pipes and wide channels were integrated considering κ and C constant.

The resulted formula of average flow velocity for smooth pipes is shown below:

$$\frac{U}{u_*} = \frac{1}{\pi r_0^2} \int_0^{r_0} \left(\frac{1}{\kappa} \ln \frac{u_* (r_0 - r)}{\nu} + C_1 \right) 2\pi r dr \quad (5)$$

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{u_* r}{\nu} - 1.5 \right) + C_1, \quad (6)$$

where U is the average flow velocity, u_* is dynamic velocity, r is the pipe radius, ν is kinematical viscosity coefficient.

Using Nikuradze resistance formula for smooth pipes we get:

$$\frac{1}{\sqrt{\lambda}} = 0.8 + 2 \lg \frac{u_*}{\nu} \quad (7)$$

where λ is hydraulic resistance coefficient. Considering $\frac{U}{u_*} = \sqrt{\frac{8}{\lambda}}$ the connection between velocity profile κ and C for smooth type of resistance is defined as:

$$C_1 = \sqrt{\frac{8}{\lambda}} - \frac{1}{\kappa} \left(\frac{1.15}{\sqrt{\lambda}} - 2.42 \right) \quad (8)$$

Considering stated by numerous researchers range of κ variations, the calculation of C variations with different λ values typical for smooth pipes was made.

The results of this analysis show that even with small possible variations of κ values, C parameter can vary greatly: from 2.4 to 6.1 (figure 1).

Fig. 1. Calculated dependence $\kappa = f(C_1)$ for hydraulically smooth pipes.

These results allow to conclusion that refined calculation of velocity profile distribution cannot be made without shown varieties of C parameter.

Same way while integrating velocity profiles for rough pipes we get the formula of average flow velocity for rough pipes. For integration the counting start is stated at 0.5 of rough grains diameter as it was in Nikuradze's experiments:

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{r}{\kappa} - 1.5 \right) + C_2 \quad (9)$$

Using Nikuradze resistance formula for rough pipes we get:

$$\frac{1}{\sqrt{\lambda}} = 2 \lg \frac{r}{k} + 1.74 \quad (10)$$

The connection between velocity profile κ and C_2 for rough type of resistance is defined as:

$$C_2 = \sqrt{\frac{8}{\lambda}} - \frac{1}{\kappa} \left(\frac{1.15}{\sqrt{\lambda}} - 3.5 \right) \quad (11)$$

The formulas (8) and (11) are similar and differ only in values of the term in brackets. The results of κ and C_2 calculations with different λ values typical for rough pipes are show on figure 2.

Fig. 2. Calculated dependence $\kappa = f(C_2)$ for hydraulically rough pipes.

The results' analysis shows that even with small possible variations of κ values require great variations of C_2 parameter to assure resistance regularity, securely stated by Nikuradze for rough pipes based on direct measures, just like for the smooth pipes. It should also be noted, that variations of hydraulic resistance coefficient λ for fixed values κ doesn't affect C_2 values much. Therefore, the character of C parameter in velocity is mostly defined by κ parameter behavior for both rough and smooth pipes.

For open wide channels it is correct to use their hydraulic radius $r = 2R = 2h$ for the logarithmic velocity profile distribution instead of pipe radius.

While defining the average flow velocity in wide channel ($R \approx h$) by integrating logarithmic profile on the depth, considering that Reynolds number for wide channels is

$$Re = \frac{Ud}{\nu} = \frac{4Uh}{\nu} \frac{u_*}{u_*} = \frac{4u_*h}{\nu} \frac{U}{u_*}, \quad (12)$$

the dependence of velocity profile distribution in wide channels can be shown as:

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{u_* h}{\nu} - 1 \right) + C_1 \tag{13}$$

The resistance formula for open channels with smooth type of resistance can be stated as

$$\frac{1}{\sqrt{\lambda}} = \frac{1}{\kappa} \left[0.406 \left(\frac{1}{\sqrt{\lambda}} + 0.8 \right) - 1.2 \right] + \frac{C_1}{\sqrt{8}} \tag{14}$$

using Nikuradze’s experimental dependence $\left(\frac{1}{\sqrt{\lambda}} = 2 \lg \operatorname{Re} \sqrt{\lambda} + 0.8 \right)$.

Formula (14) shows the dependence for smooth open channels through C_1 parameter:

$$C_1 = \sqrt{\frac{8}{\lambda}} - \frac{1}{\kappa} \left(\frac{1.15}{\sqrt{\lambda}} - 2.5 \right) \tag{15}$$

The formula for average velocity of the rough wide open channels following the analogy with pipe velocity regularities will appear as:

$$\frac{U}{u_*} = \frac{1}{\kappa} \left(\ln \frac{u_* h}{\nu} - 1 \right) + C_2 \tag{16}$$

The connection between velocity profiles parameters C and κ for open channels with rough type of resistance can be found by modifying Nikuradze resistance formula for open wide channels:

$$C_2 = \sqrt{\frac{8}{\lambda}} - \frac{1}{\kappa} \left(\frac{1.15}{\sqrt{\lambda}} - 3.69 \right) \tag{17}$$

3 Results

Results of the C_1 and κ values calculating with different λ are shown on the figures 3, 4, 5.

Fig. 3. Calculated dependence $\kappa = f(C_1)$ for hydraulically smooth and hydraulically rough open channels.

Fig. 4. The results of velocity profiles integrating for smooth type of resistance ($\lambda = 0,02$).

Fig. 5. The results of velocity profiles integrating for smooth type of resistance ($\lambda = 0,04$).

Calculation results, illustrated on figures 1, 2, 3, show that realization of flows with constant κ and C (correlated to key points on graphics) is possible. However, the analysis of Nikuradze’s experimental profiles done by numerous researchers [4, 6] shows, that the Karman parameter κ can differ from its main value (approximate 0.4). Therefore, the situation that is described with established connections (8), (11), (15), (17) between the parameters of the velocity distribution κ , C and hydraulic resistance λ is pictured as more general.

4 Discussion

Various formulas are known to describe the distribution of velocities in turbulent flow in pipes and channels, which, when compared, reveal significant differences between themselves, as well as divergence with experimental data. In engineering practice, the logarithmic profile of Prandtl–Nikuradze velocity containing two so - called "turbulence constants" - κ and C is very popular. In this article and other publications it is revealed that

the parameters of this profile of κ and C speed still remain not clearly defined, depend on each other and, generally speaking, are not constants. The article analyzes the kinematic and dynamic dependences-the dependence of these parameters on each other and on the hydraulic resistance coefficient. The results obtained complement the system of knowledge in the field of velocity distribution in turbulent flows, show patterns and differences in their behavior in pressure and open flows. It is known that the distribution of speeds is inextricably linked to hydraulic resistance, so the authors of the article made a check for compliance of speed profiles with changing parameters of κ and independently established laws of resistance.

The analysis showed that it is possible to implement flows with constant values of the pocket parameter and the second turbulence constant corresponding to the node points on the graphs. However, it is known that the parameter of the pocket can differ from the value accepted in engineering practice (0.4) in a fairly wide range. Therefore, the situation described by the established relations between the parameters of the logarithmic velocity profile and the hydraulic resistance coefficient has more General. The nature of the studied dependences for pipes and wide channels is similar, but there are some differences in the functional relations between the parameter of the Pocket, the second "constant" of turbulence and the coefficient of hydraulic resistance for pipes and channels. These differences should be taken into account when making accurate forecasts of hydraulic flow characteristics.

Further work in this area of research may be associated with fundamental research in the field of hydraulics in terms of the possibility of establishing a connection between the parameters of the velocity profile and the characteristics of turbulence.

5 Conclusion

The gained calculation data shows that minor Karman parameter changes lead to notable parameter C changes. The obtained calculated dependences are qualitatively and quantitatively consistent with the data of velocity distribution measurements in smooth and rough pipes. A special point with $\kappa \approx 0,4$ in which all calculated curves for rough pipes cross with all λ values, draws attention to itself. In this point κ and C values match the ones obtained by Nikuradze in his experiments.

The analysis of the regularities is analogous, however, there are some discrepancies in functional connections between κ , C and λ for pipes and channels, that should be taken into account to make more accurate predictions on the hydraulic flow characteristics. Thereby the calculations' results made using the experimental-based resistance formulas are required to be checked.

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