Shear stresses in rectangular panels of ship structures in the calculations according to Reissner theory

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Abstract. This study calculates and analyzes torsion moments of a rectangular panel with clamped edges as an element of ship structures under the action of uniform pressure with allowance for transverse shear deformation and examines the contribution of the corresponding shear stresses to the general stress state. In order to solve this problem, the method of infinite superposition of corrective functions of bending and stresses is applied. It involves an iterative process of mutually correcting the discrepancies from the said functions while meeting all boundary conditions. A particular solution for the bending function in the form of a quadratic polynomial is chosen as the initial approximation. It is established that torsion moment series diverge at the corner points of the plate going into infinity, which yields infinite values for the shear stresses at these points as well. Results of torsion moment calculation for square plates with different width ratios are provided. A 3D distribution diagram of moments is obtained. The computational experiment confirms the correctness of theoretical conclusions about infinite torsion moments at the corner points of the plate. Comparison with bending moments shows that torsion moments cannot be ignored during the assessment of the stress-strain state. The behavior of torsion moments near the corner points is qualitatively different from the simplified Kirchhoff theory, where they turn into zero.

1 Introduction

Modern ship structures widely use metallic and non-metallic materials with high interlaminar shear strength. Panels (plates) that serve as the basic elements of plating, deck sheathing, bulkheads, etc. are often made of such materials. This requires using more accurate methods of analysis during calculation that would reduce the weight of the structure without compromising its strength.

The classical (simplified) theory of thin plates based on the hypotheses of Kirchhoff and Love yields a significant error near the edges of the plate, in areas of steep changes in boundary conditions and points of application of concentrated forces, and during the calculation of medium thick plates.

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The linear plate-bending theory, which qualitatively specifies the classical theory, was first introduced by Reissner [1], who obtained new differential equations of plate bending and the corresponding boundary conditions with allowance for the effects of the transverse shear. The system of two differential equations consists of a fourth-order equation with respect to the bending function and an additional second-order equation with respect to the stress function. This makes it possible to meet three boundary conditions (instead of two in the classical theory) on each edge of the plate.

The above theory and the similar shear theories that followed are called Reissner – Mindlin [2] – Timoshenko [3] theories due to their similarity. For example, the difference between the Reissner and Mindlin theories virtually comes down to transverse shear ratio values, which are not that different.

Also worth mentioning are the variants of the shear theory from recent works [4-12].

There are few known results on the problem of bending of a rectangular panel with clamped edges using shear theories due to the complexity of the corresponding boundary problem. Known works [12-19] mainly contain numerical results for bending or, less commonly, calculate bending moments and shear forces at specific points. However, these works lack calculations of torsion moments, which can significantly affect shear stresses as elements of the general stress state.

It should be noted that during the calculation of plates according to the simplified Kirchhoff theory researchers rarely calculate torsion moments because they consider the contribution of the corresponding shear stresses to the general stress state to be insignificant. However, there may occur a redistribution of shear stresses at the corner points of the contour, which is ignored by the Kirchhoff theory. This can change the qualitative picture of the stress state near these points.

The main aim of this study is to obtain reliable numerical results on the torsion moments of a rectangular panel with clamped edges with allowance for transverse shear deformation according to the Reissner theory and to assess the impact of the corresponding shear stresses on the general stress state.

2 Methods and Materials

The system of differential equations in dimensionless form is written as follows [3].

$$\begin{aligned}\nabla^2 \nabla^2 w(x, y) &= -1, \\ \psi(x, y) - \alpha \nabla^2 \psi(x, y) &= 0.\end{aligned}\tag{1}$$

where ∇^2 is the Laplace bivariate operator; $w(x, y) = W/(qb^4/D)$ is the relative bending of the points of the middle surface of the plate; $x = X/b$, $y = Y/b$ are the relative coordinates; W is the absolute bending of the corresponding points; q is the transverse distributed load; b is the plate width; $D = EH^3/[12(1-\nu^2)]$ is the bending rigidity of the plate; E is the Young modulus; H is the plate thickness; ν is the Poisson ratio; $\psi(x, y) = \Psi(X, Y)/qb^2$ is the dimensionless stress function; $\Psi(X, Y)$ is the stress function; $\alpha = h^2/10$ is the shear ratio; $h = H/b$ is the relative plate thickness.

A plate with clamped edges often serves as the computational model for rectangular elements of ship structures. Boundary conditions consist in the absence of deflections and angles of clamped edges $x = \pm \gamma/2$, $y = \pm 1/2$

$$w = 0, \quad \varphi_x = 0, \quad \varphi_y = 0.\tag{2}$$

where $\gamma = a/b$ is the aspect ratio of the plate; φ_x, φ_y are the section angles $x = const$, $y = const$:

$$\begin{aligned} \varphi_x &= \frac{\partial}{\partial x} (w + \alpha_1 \nabla^2 w) - \alpha_1 \frac{\partial \psi}{\partial y}; & \varphi_y &= \frac{\partial}{\partial y} (w + \alpha_1 \nabla^2 w) + \alpha_1 \frac{\partial \psi}{\partial x}; \\ & \left(\alpha_1 = \frac{2\alpha}{1-\nu} \right). \end{aligned} \quad (3)$$

According to [3], torsion moments are related to the bending and stress functions through the following formula:

$$M_{xy} = (1-\nu) \frac{\partial^2 w}{\partial x \partial y} + 2\alpha \frac{\partial^2}{\partial x \partial y} \nabla^2 w - \alpha \left(\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right). \quad (4)$$

Here, the torsion moment is assigned to value qb^2 .

It is necessary to find an analytical expression for torsion moments at each point in order to determine the corresponding shear stresses. According to (4), this involves solving the basic problem of finding unknown deflection and stress functions.

According to the Reissner shear model, stresses along the thickness of the plate are determined using the following formulas:

$$\begin{aligned} \sigma_x &= \frac{12M_x z}{h^3}, \quad \sigma_y = \frac{12M_y z}{h^3}, \quad \tau_{xy} = \frac{12M_{xy} z}{h^3}, \\ \tau_{yz} &= \frac{3Q_y}{2h} \left[1 - \left(\frac{2z}{h} \right)^2 \right], \quad \tau_{zx} = \frac{3Q_x}{2h} \left[1 - \left(\frac{2z}{h} \right)^2 \right], \end{aligned} \quad (5)$$

where $-h/2 \leq z \leq h/2$ is the applicate of a random point of the plate. Formulas (5) feature linear bending moment M_x , M_y , torsion moment M_{xy} , shear forces Q_x , Q_y . The maximum values of normal stresses σ_x , σ_y , and shear stresses τ_{xy} will be valid with $z = h/2$

$$\sigma_{x\max} = \frac{6M_x}{h^2}, \quad \sigma_{y\max} = \frac{6M_y}{h^2}, \quad \tau_{xy\max} = \frac{6M_{xy}}{h^2},$$

while the maximum values of shear stresses τ_{yz} , τ_{zx} will be valid with $z = 0$:

$$\tau_{yz\max} = \frac{3Q_y}{2h}, \quad \tau_{zx\max} = \frac{3Q_x}{2h}.$$

The problem of determining bending functions $w(x, y)$ and stress functions $\psi(x, y)$ are solved in works [20] using superposition of corrective functions. The required bending and stress functions are represented by the following expressions:

$$w(x, y) = w_0(x, y) + \sum_{n=1}^{\infty} w_{1n}(x, y) + w_{2n}(x, y), \quad (6)$$

$$\psi(x, y) = \sum_{n=1}^{\infty} \psi_{1n}(x, y) + \psi_{2n}(x, y), \quad (7)$$

where

$$w_0(x, y) = -\frac{1}{8} \left(x^2 - \frac{\gamma^2}{4} \right) \left(y^2 - \frac{1}{4} \right) \quad (8)$$

is the initial deflection, while corrective functions are selected in the form of hyperbolic-trigonometric series.

$$w_{1n}(x, y) = \sum_{k=1,3,\dots}^{\infty} (-1)^{k*} \frac{A_{kn}}{\cosh \tilde{\lambda}_k} \left(x \sinh \lambda_k x - \frac{\gamma}{2} \tanh \tilde{\lambda}_k \cosh \lambda_k x \right) \cos \lambda_k y, \quad (9)$$

$$w_{2n}(x, y) = \sum_{s=1,3,\dots}^{\infty} (-1)^s \frac{B_{sn}}{\cosh \tilde{\mu}_s} \left(y \sinh \mu_s y - \frac{1}{2} \tanh \tilde{\mu}_s \cosh \mu_s y \right) \cos \mu_s x, \quad (10)$$

$$\psi_{1n}(x, y) = \sum_{k=1,3,\dots}^{\infty} (-1)^{k*} C_{kn} \sinh \beta_k x \sin \lambda_k y, \quad (11)$$

$$\psi_{2n}(x, y) = \sum_{s=1,3,\dots}^{\infty} (-1)^s D_{sn} \sinh \xi_s y \sin \mu_s x. \quad (12)$$

Here, A_{kn} , B_{sn} , C_{kn} , D_{sn} are the undetermined ratios that are found in pairs when boundary conditions on parallel edges are met,

$$\lambda_k = k\pi, \quad \mu_s = \frac{s\pi}{\gamma}, \quad k^* = \frac{k+1}{2}, \quad s^* = \frac{s+1}{2}.$$

$$\tilde{\lambda}_k = \frac{\lambda_k \gamma}{2}, \quad \tilde{\mu}_s = \frac{\mu_s}{2}, \quad \beta_k = \sqrt{\lambda_k^2 + \frac{1}{\alpha}}, \quad \xi_s = \sqrt{\mu_s^2 + \frac{1}{\alpha}}. \quad (13)$$

The essence of the method is that all boundary conditions of the problem are met as a result of an infinite iterative process of eliminating shear deformations (discrepancies) from initial function (8) and from each pair of corrective functions (9), (11) and (10), (12), which result in discrepancies on the other two edges as they eliminate discrepancies on two parallel edges through undetermined ratios.

Initial bending (8) is a particular solution of the first differential equation (1) that "automatically" ensures zero bending of clamped edges. The corrective bending functions are biharmonic, i.e. they satisfy the corresponding homogeneous equation for the first equation (1). They also ensure zero bending at the plate contour. The corrective stress function satisfy the second equation (1).

Work [20] calculates deflections, bending moments, and shear forces for different relative thicknesses of a square uniformly loaded plate and provides detailed tables. Torsion moments are not calculated as it is assumed that the corresponding shear stresses are minor compared with normal stresses. However, the subsequent analysis of the expression for torsion moments shows specific aspects of their behavior and effect on the stress state of the plate, particularly near the corner points. The numerical experiment confirms theoretical conclusions and provides a complete picture of torsion moment distribution in the plate. Below we provide the main results of the study.

By substituting the found bending functions (6) and (7) into formula (4) for torsion moments we obtain the final expression used in calculations:

$$M_{xy} = -\frac{1-\nu}{2} xy - \sum_{k=1,3,\dots}^{\infty} (-1)^{k*} \left\{ \left[(1-\nu) \left(1 - \tilde{\lambda}_k \tanh \tilde{\lambda}_k \right) + 4\alpha \lambda_k^2 \right] \sinh \lambda_k x + (1-\nu) \lambda_k x \cosh \lambda_k x \right. \\ \left. - 2\alpha \frac{\lambda_k \cosh \tilde{\lambda}_k}{\beta_k \cosh \tilde{\beta}_k} \left(\lambda_k^2 + \beta_k^2 \right) \sinh \beta_k x \right\} \frac{\lambda_k A_{k\Sigma}}{\cosh \tilde{\lambda}_k} + 2\alpha \frac{\lambda_k^2 + \beta_k^2}{\lambda_k^2 \beta_k \cosh \tilde{\beta}_k} \sinh \beta_k x \Big\} \sin \lambda_k y$$

$$- \sum_{s=1,3,\dots}^{\infty} (-1)^s \left\{ \left[(1-\nu)(1-\tilde{\mu}_s \tanh \tilde{\mu}_s) + 4\alpha\mu_s^2 \right] \sinh \mu_s y + (1-\nu)\mu_s y \cosh \mu_s y \right. \\ \left. - 2\alpha \frac{\mu_s \cosh \tilde{\mu}_s}{\xi_s \cosh \tilde{\xi}_s} (\mu_s^2 + \xi_s^2) \sinh \xi_s y \right\} \frac{\mu_s B_{s\Sigma}}{\cosh \tilde{\mu}_s} + \frac{2\alpha}{\gamma} \frac{\mu_s^2 + \xi_s^2}{\mu_s^2 \xi_s \cosh \tilde{\xi}_s} \sinh \xi_s y \Big\} \sin \mu_s x \quad (14)$$

where $A_{k\Sigma} = A_{k1} + A_{k2} + \dots + A_{kn}; \dots; B_{s\Sigma} = B_{s1} + B_{s2} + \dots + B_{sn}$ are the cumulative ratios for all iterations.

It should be noted that ratios C_{kn}, D_{sn} , which are parts of stress functions, are expressed through "basic" ratios A_{kn}, B_{sn} providing that boundary conditions are met.

The expression of torsion moments (14) is an odd function of both variables. Therefore, it is enough to obtain numerical results for the first quarter of the plate.

It is easy to see that in the center of the plate torsion moments turn into zero. Let us find their expression at the corner points, particularly at point (0.5γ, 0.5). After simplification, we get

$$M_{xy} \Big|_{x=\gamma/2, y=1/2} = -\frac{1-\nu}{2} \frac{\gamma}{4} + \sum_{k=1,3,\dots}^{\infty} \left\{ \left[(1-\nu) \left(1 + \frac{\tilde{\lambda}_k}{\cosh \tilde{\lambda}_k} \right) + 4\alpha\lambda_k^2 \right] \tanh \tilde{\lambda}_k \right. \\ \left. - 2\alpha \frac{\lambda_k}{\beta_k} (\lambda_k^2 + \beta_k^2) \tanh \tilde{\beta}_k \right\} \lambda_k A_{k\Sigma} + 2\alpha \frac{\lambda_k^2 + \beta_k^2}{\lambda_k^2 \beta_k} \tanh \tilde{\beta}_k \Big\} \\ + \sum_{s=1,3,\dots}^{\infty} \left\{ \left[(1-\nu) \left(1 + \frac{\tilde{\mu}_s}{\cosh \tilde{\mu}_s} \right) + 4\alpha\mu_s^2 \right] \tanh \tilde{\mu}_s \right. \\ \left. - 2\alpha \frac{\mu_s}{\xi_s} (\mu_s^2 + \xi_s^2) \tanh \tilde{\xi}_s \right\} \mu_s B_{s\Sigma} + \frac{2\alpha}{\gamma} \frac{\mu_s^2 + \xi_s^2}{\mu_s^2 \xi_s} \tanh \tilde{\xi}_s \Big\}. \quad (15)$$

Using formulas (13) it is possible to find the limit of the following components of this expression:

$$\lim_{k \rightarrow \infty} \left[4\alpha\lambda_k^2 \tanh \tilde{\lambda}_k - 2\alpha \frac{\lambda_k}{\beta_k} (\lambda_k^2 + \beta_k^2) \tanh \tilde{\beta}_k \right] \\ = \lim_{k \rightarrow \infty} \left[4\alpha\lambda_k^2 \tanh \tilde{\lambda}_k \left(1 - \frac{\lambda_k}{\beta_k} \frac{\tanh \tilde{\beta}_k}{\tanh \tilde{\lambda}_k} \right) - 2 \frac{\lambda_k}{\beta_k} \tanh \tilde{\beta}_k \right] = \frac{2}{1-\nu} - 2 = \frac{2\nu}{1-\nu}.$$

Ratios A_k and B_s have estimates obtained in [20]: $A_{kn} = O(1/k^2)$, $B_{sn} = O(1/s^2)$ for any iteration. Then the general first-order term in (15) has k -index exponent $O(1/k)$. Similarly, the second-order term in (15) has exponent $O(1/s)$.

This means that series in (15) diverge at the corner point and yield infinite torsion moments and, consequently, shear stresses.

3 Results

Numerical results for torsion moments are obtained for a rectangular plate ($\gamma = 1$) with fully clamped edges (CCCC plate according to the international nomenclature) under the effect of uniform pressure according to the refined Reissner theory. Calculations are made using a computer program compiled in the Maple system. The Poission number is taken equal to 0.3. The number of terms held in series (14) is taken equal to 299, the number of iterations is 10. After the 10th iteration, discrepancies in the fulfillment of boundary conditions, which are printed after each iteration, are practically zero. The relative thickness is taken equal to $h = 0.01, 0.05, 0.1, 0.2$, and 0.3 . Tables 1–5 show torsion moment values at different points of the plate for the given relative thicknesses. Computational points are condensed near the contour in order to examine the effect of the plate edge. Fig. 1–5 show the corresponding torsion moment diagrams.

Table 1. Torsion moment values in relation to value $-qb^2\times10^{-6}$ for a square Reissner plate with relative thickness $h = 0.01$ (CCCC plate, $q = const$).

y	x									
	0	0.1	0.2	0.3	0.4	0.42	0.44	0.46	0.48	0.5
0	0	0	0	0	0	0	0	0	0	0
0.1	0	1721	3109	3730	2965	2576	2091	1504	809	195
0.2	0	3109	5652	6860	5565	4862	3972	2877	1560	380
0.3	0	3730	6860	8483	7095	6255	5166	3790	2085	516
0.4	0	2965	5565	7095	6109	5417	4508	3346	1877	478
0.42	0	2576	4862	6255	5417	4797	3980	2943	1650	420
0.44	0	2091	3972	5166	4508	3980	3280	2400	1323	335
0.46	0	1504	2877	3790	3346	2943	2400	1710	900	214
0.48	0	809	1560	2085	1877	1650	1323	900	412	65
0.5	0	195	380	516	478	420	335	214	65	-21

Table 2. Torsion moment values in relation to value $-qb^2\times10^{-6}$ for a square Reissner plate with relative thickness $h = 0.05$ (CCCC plate, $q = const$).

y	x									
	0	0.1	0.2	0.3	0.4	0.42	0.44	0.46	0.48	0.5
0	0	0	0	0	0	0	0	0	0	0
0.1	0	1684	3042	3652	2911	2535	2076	1555	1063	962
0.2	0	3042	5524	6699	5439	4762	3922	2958	2043	1850
0.3	0	3652	6700	8257	6883	6073	5048	3848	2689	2452
0.4	0	2911	5440	6883	5885	5218	4364	3349	2359	2158
0.42	0	2535	4762	6073	5218	4625	3861	2957	2075	1895
0.44	0	2076	3922	5049	4364	3862	3213	2445	1701	1546
0.46	0	1555	2958	3848	3350	2957	2445	1836	1248	1125
0.48	0	1063	2043	2689	2359	2076	1701	1248	785	668
0.5	0	962	1851	2452	2159	1895	1547	1125	668	-111

Table 3. Torsion moment values in relation to value $-qb^2\times10^{-6}$ for a square Reissner plate with relative thickness $h = 0.1$ (CCCC plate, $q = const$).

y	x									
	0	0.1	0.2	0.3	0.4	0.42	0.44	0.46	0.48	0.5
0	0	0	0	0	0	0	0	0	0	0
0.1	0	1584	2859	3430	2799	2507	2185	1880	1685	1782
0.2	0	2859	5182	6270	5186	4661	4078	3521	3163	3348
0.3	0	3430	6270	7689	6490	5862	5155	4470	4026	4268
0.4	0	2799	5186	6490	5591	5063	4458	3866	3483	3704
0.42	0	2507	4661	5862	5063	4578	4018	3470	3118	3324
0.44	0	2185	4078	5155	4458	4018	3505	2988	2656	2846
0.46	0	1880	3521	4470	3866	3470	2988	2476	2110	2268
0.48	0	1685	3163	4026	3483	3118	2656	2110	1562	1563
0.5	0	1782	3348	4268	3704	3324	2846	2268	1563	-712

Table 4. Torsion moment values in relation to value $-qb^2\times10^{-6}$ for a square Reissner plate with relative thickness $h = 0.2$ (CCCC plate, $q = const$).

y	x									
	0	0.1	0.2	0.3	0.4	0.42	0.44	0.46	0.48	0.5
0	0	0	0	0	0	0	0	0	0	0
0.1	0	1331	2400	2917	2709	2612	2528	2483	2517	2687
0.2	0	2400	4344	5313	4967	4796	4644	4564	4628	4945
0.3	0	2917	5313	6558	6177	5970	5788	5694	5784	6196
0.4	0	2709	4967	6177	5704	5448	5217	5093	5186	5630
0.42	0	2612	4796	5970	5448	5155	4873	4696	4758	5204
0.44	0	2528	4644	5788	5217	4873	4507	4221	4197	4639
0.46	0	2483	4564	5694	5093	4696	4221	3742	3491	3888
0.48	0	2517	4628	5784	5186	4758	4197	3491	2732	2824
0.5	0	2687	4945	6196	5630	5204	4639	3888	2824	-3983

Table 5. Torsion moment values in relation to value $-qb^2\times10^{-6}$ for a square Reissner plate with relative thickness $h = 0.3$ (CCCC plate, $q = const$).

y	x									
	0	0.1	0.2	0.3	0.4	0.42	0.44	0.46	0.48	0.5
0	0	0	0	0	0	0	0	0	0	0
0.1	0	1125	2047	2578	2691	2696	2714	2759	2849	3011
0.2	0	2047	3729	4709	4928	4940	4977	5064	5238	5543
0.3	0	2578	4709	5941	6170	6182	6231	6354	6595	7014
0.4	0	2691	4928	6170	6000	5871	5794	5840	6089	6602
0.42	0	2696	4940	6182	5871	5663	5488	5441	5640	6173
0.44	0	2714	4977	6231	5794	5488	5171	4953	5027	5570
0.46	0	2759	5064	6354	5840	5441	4953	4459	4226	4727
0.48	0	2849	5238	6595	6089	5640	5027	4226	3342	3468
0.5	0	3011	5543	7014	6602	6173	5570	4727	3468	-9968

Fig. 1. Distribution diagram of torsion moments M_{xy} for a clamped square Reissner plate with relative thickness $h = 0.01$.

Fig. 2. Distribution diagram of torsion moments M_{xy} for a clamped square Reissner plate with relative thickness $h = 0.05$.

Fig. 3. Distribution diagram of torsion moments M_{xy} for a clamped square Reissner plate with relative thickness $h = 0.1$.

Fig. 4. Distribution diagram of torsion moments M_{xy} for a clamped square Reissner plate with relative thickness $h = 0.2$.

Fig. 5. Distribution diagram of torsion moments M_{xy} for a clamped square Reissner plate with relative thickness $h = 0.3$.

4 Discussion

For square plates with relative thickness $h = 0.01, 0.05, 0.1, 0.2$, minimum values of torsion moment M_{xy} are achieved at point $(0.3, 0.3)$ and are $-0.008483, -0.008257, -0.007689, -0.006558$ respectively, i.e. absolute extremum values decrease with the increase of relative thickness. Maximum values are achieved at corner point $(0.5, 0.5)$ and are $0.000021, 0.000111, 0.000712, 0.003983$ respectively, i.e. they grow with the increase of thickness. That said, minimum values change gradually, whereas values at the corner point increase dramatically, producing peak values. This can be seen in the corresponding diagrams and substantiates theoretical conclusions about the infinity of torsion moments at the corner points. A Reissner plate with relative thickness $h = 0.01$ is similar to the Kirchhoff plate ($h \approx 0$) in that torsion moments at the corner points are practically zero. This is where the main feature of the refined Reissner theory becomes apparent.

With thickness $h = 0.3$, minimum values are achieved at two points with coordinates $(0.3, 0.5)$ and $(0.5, 0.3)$ and are -0.007014 . Maximum values 0.009968 are achieved at corner point $(0.5, 0.5)$.

The behavior pattern of the distribution diagrams of torsion moments shows that the corner points of the Reissner plate are special points, at which torsion moments are infinite. Consequently, the corresponding shear stresses τ_{xy} are also infinite.

Now let us compare normal and shear stresses at the critical points of the plate. For this purpose, let us use the results of work [20], which calculates bending moments M_x for the similar Reissner plates.

Bending moment M_x at point (0.3, 0.3) with thickness $h = 0.05$ equals [20] -0.00185, achieving maximum value equal to 0.05067 at point (0.5, 0). Comparison with $M_{xy\max}(0.3, 0.3) = -0.008257$ shows that at this point the torsion moment is approximately 4.5 times higher than the bending moment but 6.1 times lower than the maximum bending moment.

Bending moment M_x at point (0.3, 0.3) with thickness $h = 0.1$ equals [20] -0.00197, achieving maximum value equal to 0.04894 at point (0.5, 0). Comparison with $M_{xy\max}(0.3, 0.3) = -0.007689$ shows that at this point the torsion moment is approximately 3.9 times higher than the bending moment but 6.4 times lower than the maximum bending moment.

Bending moment M_x at point (0.3, 0.3) with thickness $h = 0.2$ equals [20] -0.002823, achieving maximum value equal to 0.04458 at point (0.5, 0). Comparison with $M_{xy\max}(0.3, 0.3) = -0.006558$ shows that at this point the torsion moment is approximately 2.3 times higher than the bending moment but 6.8 times lower than the maximum bending moment.

Bending moment M_x at point (0.5, 0.3) with thickness $h = 0.3$ equals [20] -0.002572, achieving maximum value equal to 0.04049 at point (0.5, 0). Comparison with $M_{xy\max}(0.5, 0.3) = -0.007014$ shows that at this point the torsion moment is approximately 2.7 times higher than the bending moment but 5.8 times lower than the maximum bending moment.

Stresses relate to each other the same way as they are calculated by multiplying moments by $6/h^2$.

5 Conclusion

1. An analytical expression for torsion moments M_{xy} for a rectangular Reissner plate with clamped edges is obtained and examined.
2. Torsion moments along the horizontal of a square plate are calculated, and the corresponding 3D distribution diagrams are built for different relative thicknesses of the plate.
3. Changes in torsion moments depending on plate thickness are analyzed.
4. It is analytically substantiated and numerically confirmed that the corner points of the Reissner plate are special points, at which torsion moments are infinite, whereas in Kirchhoff plates they are zero.
5. Comparative analysis of normal and shear stress values is performed.

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References

1. E. Reissner, J. Appl. Mech. **12**, 69-77 (1945)
2. R.D. Mindlin, J. Appl. Mech. **18**, 31-38 (1951)
3. S. Timoshenko, S. Woinowsky-Krieger, *Theory of Plates and Shells* (McGraw-Hill Book Company, New York, Toronto, London, 1959)
4. S.A. Ambartsumyan, *Theory of Anisotropic Plates* (New York, Technomic Publishing Co., 1970)

5. K. Vijayakumar, Arch. Appl. Mech. **81**, 1717-1724 (2011) DOI: 10.1007/s00419-011-0513-4
6. P.E. Tovstik, T.P. Tovstik, Doklady Physics. **8**, 389-392 (2014) DOI: 10.7868/S0869565214240128
7. J.M. Martínez Valle, P. Martínez-Jiménez, Lat. Am. J. Solids Struct. **12**, 295-316 (2015) DOI: 10.1590/1679-78251275
8. K. Nareen, R.P. Shimpi, Proc. Inst. Mech. Eng. **203-210**, 1989-1996 (2015) DOI: 10.1177 / 0954406214563739
9. I. Senjanović, N. Vladimir, M. Tomić, Mech. Res. Commun. **73**, 31-38 (2016) DOI.org/10.1016/j.mechrescom.2016.02.005
10. H.T. Thai, T.-K. Nguyen, T.P. Vo, T. Ngo, Compos. Struct. **171**, 277-285 (2017)
11. N.F. Morozov, P.E. Tovstik, T.P. Tovstik, Mech. Solids. **51**, 527-537 (2016) DOI: 10.3103/S0025654416050034
12. K. Hassan, S. Guirguis, H. El-Hamouly, Afr. J. Eng. Res. **5**, 7-17 (2017)
13. K. Shirakawa, Mech. Res. Commun. **10**, 205-211 (1983)
14. Z. Xu, Acta Mech. Sinica-PRC. **2**, 185-192 (1986)
15. Z. Xu, Int. J. Num. Meth. Eng. **33**, 963-973 (1992)
16. O.C. Zienkiewicz, Z. Xu, L.F. Zeng, A. Samuelsson, N-E. Wiberg, Int. J. Num. Meth. Eng. **36**, 3043-3056 (1993)
17. R. Ayad, G. Dhatt, J.L. Batoz, Int. J. Num. Meth. Eng. **42**, 1149-1179 (1998)
18. S. Weiming, Y. Guangsong, Appl. Math. Mech. (Engl. Edit.) **20**, 193-199 (1999)
19. H.R. Dhananjaya, Electr. J. Struct. Eng. **10**, 1-10 (2010)
20. M.V. Sukhoterlin, S.O. Baryshnikov, T.P. Knysh, Magaz. Civ. Eng. **76**, 225-240 (2017) DOI: 10.18720/MCE.76.20