Optimization of geometrical parameter for V-type alpha Stirling engine based on dimensionless analysis

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Abstract. This paper aims to study the effect of mechanism and geometrical parameters on the shaft work of alpha-type Stirling engine with V-cylinder arrangement and find the appropriate value of geometrical parameters to get the maximum value of shaft work output. The equation of motions of both expansion and compression pistons and the variations of volumes and pressures inside the cylinders are expressed in dimensionless forms. Schmidt analysis and Senft's shaft work theory are used to build dimensionless thermodynamic models in order to derive dimensionless shaft work and optimal geometrical parameters for achieving the maximum efficiency

1 Introduction

Stirling engine is a heat engine that works as a cycle of expansion and compression of air or other working gas by the effects of temperature differences. From these processes, the conversion of energy from heat to mechanical energy in addition, the Stirling engine is a closed-loop cycle engine that has a constant working gas is sealed without transferring to the environment. In practice, the Stirling engine are classified into three types, based on the configurations and varying displacements of piston and displacer, which are alpha, beta, and gamma. The alpha Stirling engine the simplest pattern form of the Stirling engine.

Karabulut H. [1] built and tested performance of the alpha Stirling engine. The engine provided the maximum power under the heater temperature of 1100 °C and 2.5 bar of engine's pressurization. Batmaz I, Ustun S [2] employed double heaters for solar power heating V-type Stirling engine which the unpressurized engine gave the best performance at 950 °C heater surface. A.A. El-Ehwany et al. [3] tested and tried to use elbow-bend as heater and cooler part of alpha Stirling engine. The result of the engine power delivers about 13% more power per cc per ΔT than the literature from this previous study. The optimization of the alpha Stirling engine were proposed by Homutescu and Balanescu [4]. The results show that the diameter of the expansion piston must be greater than the compression piston diameter. Lately, Scollo et al. [5]

proposed the constructive and redesign of the prototype of the twin cylinder alpha Stirling engine. The engine performance was greatly increased. Recently, Martaj N., Rochelle P. [6] presented a 1D modeling of an alpha double-acting Stirling engine engines and carried out the optimal values of the engine geometry.

According to optimization of the Stirling engine, Senft [7] introduced the concept of effectiveness of mechanism and the relationship between force work and the shaft work of an engine. Later, the same author [8] the Schmidt theory was used to combined and find the optimal values for the phase angle and the swept volume ratio for gamma type Stirling engine. Cheng and Yang [9]. also used this method to evaluate and compare the power of Stirling engine between the alpha, beta, and gamma. The result shows that a beta type has the highest power, followed by the alpha and gamma. But the gamma Stirling engine is the most suitable for low temperature different applications. As the same author [10] the rhombic mechanism variable of the beta Stirling engine is also considered to be integrated with the method, in order to find the optimal mechanism variable and other variable. So that in this paper, propose the optimization by using the slider crank mechanism variable of an alpha Stirling engine to cooperate with this method. All variables considered were phase angle, area ratio between expansion side and compression side, temperature ratio, dead volume ratio and the ratio between connecting rod and crank radius at expansion side and compression side.

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2. Dimensionless analytical models

2.1 Displacement of expansion and compression pistons

Since the changing of the displacement of expansion, $\bar{y}_e$ and compression, $\bar{y}_c$ pistons which is driven by slider crank mechanisms affect to the variation of volume inside the engine. The equations of displacement of slider crank mechanisms can be presented in dimensionless form as

$$\bar{y}_e(\theta) = \cos \theta + \sqrt{e_d^2 + (\sin \theta)^2} \quad (1a)$$

$$\bar{y}_c(\theta) = \cos(\theta - \alpha) + \sqrt{e_p^2 + (\sin(\theta - \alpha))^2} \quad (1b)$$

Where $e_d = l_e/r_e$ and $e_p = l_c/r_c$. Note that in practical, the variable e_p and e_d must be more than one. If it isn't, it cannot be a slider crank mechanism.

2.2 The variations of the volumes inside the cylinders

The volumes of the expansion side and compression side are changing because of movement of these two pistons in this case are assumed that no dead volume from clearance displacement between cylinder and piston. Let V_{se} is the swept volume of the expansion side ($= A_e r_e \bar{S}_e$) so the equation of volume variation of expansion side, $\bar{V}_e$ and compression side, $\bar{V}_c$ can be written in dimensionless form as

$$\bar{V}_e(\theta) = [\bar{y}_{e,max} - \bar{y}_e(\theta)]/\bar{S}_e \quad (2a)$$

$$\bar{V}_c(\theta) = [\bar{y}_{c,max} - \bar{y}_c(\theta)]/a\bar{S}_c \quad (2b)$$

Where a is an area ratio that is $a = A_e/A_c$, $\bar{V}_e(\theta) = V_e/V_{se}$, $\bar{V}_c(\theta) = V_c/aV_{se}$, when $r_e = r_c$ so that the equation of total working gas volume variation can be written as

$$\bar{V}_t(\theta) = \bar{V}_h + \bar{V}_r + \bar{V}_k + \bar{V}_e(\theta) + \bar{V}_c(\theta) \quad (3)$$

At the first three terms are dimensionless volumes of heater $\bar{V}_h$, regenerator $\bar{V}_r$, and cooler $\bar{V}_k$, respectively. The first three terms can be written as $\chi = \bar{V}_h + \bar{V}_r + \bar{V}_k$.

2.3 Pressure and back pressure

From the Schemidt theory, this theory is used to determine the pressure inside the working space. The temperature at total dead space is estimated as an average of T_e and T_c . The Schemidt theory can be rearranged and presented in dimensionless form as

$$\begin{aligned} \bar{p}(\theta, \alpha, \tau, \chi, e_p, e_d) &= p(\theta) / \frac{mRT_e}{V_{se}} \\ &= \tau / (2\tau\chi / (1 + \tau) + \tau\bar{V}_e(\theta) + \bar{V}_c(\theta)) \end{aligned} \quad (4)$$

Where $\tau = T_c/T_e$ is the temperature ratio, and $\bar{p}$ is dimensionless pressure

As discussed by Senft [7], the optimal value of the back pressure is the average cyclic pressure, which is

$$\bar{p}_b = \sqrt{\bar{p}_{max}\bar{p}_{min}} \quad (5)$$

2.4 Force, Indicated, and shaft works

The area under $\bar{p} - \bar{V}_t$ diagram is the dimensionless indicated work can be evaluated from $\oint \bar{p} d\bar{V}_t$.

Force work is work loss in the mechanisms. For the alpha Stirling engine force work will act at the expansion and compression pistons, the result of the pressure different on both sides of this two pistons. Based on the definition given by Senft [7]. Dimensionless force work can be calculated by

$$\bar{W}_f = \oint [(\bar{p} - \bar{p}_b) d\bar{V}_t]^- \quad (6)$$

Where the symbol $[]^-$ means

$$[n]^- = \begin{cases} -n, & n \leq 0 \\ 0, & n > 0 \end{cases} \quad (7)$$

From equation (7) as it is described by Senft [7] can be determine dimensionless shaft work by

$$\bar{W}_s = E\bar{W}_i - (1/E - E)\bar{W}_f \quad (8)$$

Where E is the effectiveness of mechanism, depending on the mechanisms design, friction, speed revolution, mechanism parts manufacturing, lubrication and etc. In this study, value of E is defined to 0.8.

3. Numerical Results

The study effects of mechanism size and parameters were analyzed by theoretical analysis with search method. The results of the parameters are as follows: Fig. 1 shows the effect of temperature ratios on $\bar{W}_i$, $\bar{W}_s$ and $\bar{W}_f$ at $E = 0.8$, $\chi = 0.1$, $\alpha = 90^\circ$, $a = 1$ and $e_p = e_d = 5$, then the values of $\bar{W}_i$ and $\bar{W}_s$ are continuously decreasing, but the value of $\bar{W}_f$ increases with the value of τ . From τ at 0.759, the value of $\bar{W}_s$ is zero. This indicates that the engine cannot work and it implies this engine type is not suitable for use at low temperature differences. The value of $\bar{W}_i$ is zero when the value of $\tau = 1$, where this point means there is no difference in temperature between hot and cold side.

Fig. 1. Effects of temperature ratio on dimensionless works

Fig. 2. Effect of total dead volume ratio on dimensionless works

The effect of dead volume ratio χ on $\bar{W}_i$, $\bar{W}_s$ and $\bar{W}_f$ is shown in Fig. 2 at $E = 0.8$, $\tau = 0.3$, $\alpha = 90^\circ$, $a = 1$ and $e_p = e_d = 5$. The results clearly show that the increase in the dead volume ratio will decrease the value of the output. It is observed that the values of $\bar{W}_i$, $\bar{W}_s$ and $\bar{W}_f$ in the range χ are between 0 and 0.8, the value of the output decreases rapidly. But after this, the value starts to drop slowly. In theory, the value of the dead volume ratio should be as low as possible, but in practices, the design and construction of the engine, the heater, regenerator and the cooler part also have dead volume inside. So that this problem cannot be avoided. That why the study of the effect of the dead volume ratio is important.

Fig. 3 show the effect of e_p and e_d on dimensionless work at $E = 0.8$, $\tau = 0.3$, $\chi = 0.1$, $\alpha = 90^\circ$ and $a = 1$. Let's $e_p = e_d$. The results of e_p and e_d are between 1.1 and 5, the value of the dimensionless work is high rate variable. And sooner or later, the work will increase slowly and little by little. As a result, the greater e_d and e_p will be better transmit work.

Fig. 3. Effect of e_p and e_d on dimensionless works

The effect of the phase angle on $\bar{W}_i$, $\bar{W}_s$ and $\bar{W}_f$ is shown in Fig. 4 at $E = 0.8$, $\tau = 0.3$, $\chi = 0.1$, $a = 1$ and $e_p = e_d = 5$. The result is that in the initial phase, the values of $\bar{W}_i$ and $\bar{W}_s$ are increased to the inverted curve by the maximum $\bar{W}_s$ is $\alpha = 77^\circ$, and then it decreases.

Fig. 5 shows the effect of the area ratio on the expansion on the compression side in case $E = 0.8$, $\tau = 0.3$, $\chi = 0.1$, $\alpha = 90^\circ$ and $e_p = e_d = 5$. From Fig. 5, it is notice that a must more than 0.22 so the engine can operate. In the range of $a = 0.1$ to 1. It is found that the values of $\bar{W}_i$ and $\bar{W}_s$ are increasing rapidly, while the value of $\bar{W}_f$ decreases rapidly. And then $\bar{W}_i$ and $\bar{W}_s$ will increase slowly and very little. $\bar{W}_s$ have the maximum value of $a = 1.79$, and then $\bar{W}_s$ will gradually decrease but very little. The result is that the value of the area ratio must be greater than 1, because if the area ratio is less than 1, then the area of the expansion side smaller than the compression area will be a mechanical disadvantage, resulting in the output power will be decrease or in practices, the engine may not be run.

Fig. 4. Effect of phase angle on dimensionless works

Fig. 5. Effect of area ratio on dimensionless works

The maximum value of the shaft work can be found from two variables that are area ratio and phase angle. So we use these two variables primarily to find the work from the maximum shaft work. Fig. 6 shows contour diagram as the parametric dimensionless shaft work in the function of the area ratio and the phase angle for the alpha V-type Stirling engines at $E = 0.8$, $\tau = 0.3$, $\chi = 0.1$ and $e_p = e_d = 5$. We discover that dimensionless shaft work optimize value are 0.4292 at $a = 1.73$ and $\alpha = 76^\circ$. However the output from variable of area ratio and the phase angle are also depends on the value of effectiveness of mechanism, temperature ratio, dead volume ratio and e_p , e_d . If one value changes, the optimize value will change.

Fig. 6. Contour diagram of dimensionless shaft work as a function of phase angle and area ratio

4. Conclusion and Remarks

The result of the assumption at $E = 0.8$, $\chi = 0.1$, $\alpha = 90^\circ$, $a = 1$ and $e_p = e_d = 5$ indicates that when the temperature ratio increases to 0.759, the value of dimensionless shaft work $\bar{W}_s$ is zero. It's implies that this engine is not suitable for use in low temperature different

applications. In addition, the dead volume ratio too much affective to the work of the engine. It is notice that $\bar{W}_i$, $\bar{W}_s$ and $\bar{W}_f$ are rapidly decreasing as the volume ratio increases. Especially in the range $\chi = 0$ to $\chi = 0.8$. For the area ratio, it is found that should greater than 0.22 so the engine can operate. At $a = 0.1$ to $a = 1$ where $\bar{W}_i$ and $\bar{W}_s$ increase rapidly. It's mean the value of the area ratio must be greater than 1 and the expansion cylinder area must be greater than the compression side.

In terms of the variable of size of the slider crank mechanism, we discover that $e_d > 1$ and $e_p > 1$, but the results of these two variables don't have an optimize values. When these two values are higher, the mechanisms are greater transmission power and reduce the effect of side forces on the cylinder wall. Therefore, it is concluded that the variables e_p and e_d should be designed according to the appropriate proportions. In the other way, when the variable are tried to optimize at $E = 0.8$, $\tau = 0.3$, $\chi = 0.1$ and $e_p = e_d = 5$. The maximum value of dimensionless shaft work is 0.4292 at $a = 1.73$ and $\alpha = 76^\circ$

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