

Sensitivity of the inverse problem solution related to detection of rebars buried in concrete by using GPR scanning

Zoubaida Mechbal^{1,a} and Abdellatif Khamlichi²

¹ *Department of Physics, Faculty of Sciences at Tetouan, University Abdelmalek Essaadi, Tetouan 93030, Morocco*

² *Department STIC, National School of Applied Sciences at Tetouan, University Abdelmalek Essaadi, Tetouan 93030, Morocco*

Abstract. Ground Penetrating Radar is a device which is nowadays largely used in civil engineering applications. Considering a rebar buried in a concrete medium, this work addresses sensitivity of the inverse problem solution associated to identification of the object radius and its depth from B-scan data acquired by the GPR. The approach uses a closed form parameterisation of the hyperbola trace emerging in the radargram as function of the hyperbola apex coordinate along the direction of B-scan, the cover depth, the radius of the object and the relative permittivity of the medium. Estimation of the wave velocity, the hyperbola apex coordinates and the rebar radius was performed through solution of an appropriate nonlinear least mean squares problem. Perturbation analysis was then conducted by assuming that the hyperbola points coordinates, extracted from raw data of radargram, are randomly distributed according to Gaussian densities of probabilities. The effect of the amount of data was also analyzed. The method was implemented in Matlab environment. The obtained results have shown that identification process is extremely sensitive to noise affecting the B-scan raw data, but not to the number of points used in identification.

1 Introduction

Ground Penetrating Radar (GPR) is commonly used in civil engineering for the localization of supply lines, land mines, pipes and many other buried objects in soils. Nowadays, its use encompasses diagnosis and detection of rebars in reinforced concrete structures. One of the changing issues in this field is the localization and the identification of radius of buried steel bars.

Detection of a reinforcement steel rebar can be made from the raw data recorded as reflection hyperbola trace in the GPR radargram. The hyperbola has typical shape that varies according to the depth and material of the rebar as well as the surrounding medium dielectric properties. One of the most important problems to deal with in the field of GPR based detection techniques is to retrieve from the actual radargram the hyperbola characteristics by post-processing of acquired B-scan data and inverse problem solution.

The hyperbola characteristics enable to identify the centre position and the radius of the buried cylindrical object. Several methods have been developed in recent years in order to automate this task [1-3]. The various authors who dealt with this matter have used essentially three different approaches in order to perform image processing techniques of the obtained radargrams, Ristic et al. [4]. These include Hough transform based techniques [5-6], interactive visual overlapping of hyperbola [7] and direct fitting of the raw data to a general hyperbola equation [8-9].

It was recognized that the two first approaches suffer from critical drawbacks as the error affecting the radius estimation of buried cylinder radius may be in some cases as large as 50%, because of numerical bad conditioning that impedes correct localisation of hyperbola centre [8]. However, the third approach was judged to be more interesting as it allowed reducing the computational cost with higher accuracy.

In the following the third approach according to [4] is considered. This method is based on mathematical representation of the raw data of radargram by means of a parameterized hyperbola which is strained then to fit the observed trace hyperbola as formed by reflected amplitudes from rebar obstacle. The first work in this context was performed by Shihab et al. [8]. The authors have given an estimation of the radius by assuming prior knowledge of the velocity from measurement. They proposed a hyperbola fitting technique using weighted least squares with respect to the mathematical error expectation of two-way travel time.

Dolgiy et al. [10] have presented several techniques for optimal radius estimation of subsurface metal pipes. These included the classical weighted least squares, the recursive Kalman filter, the maximum likelihood, the direct least-square fitting of hyperbola and the Nelder-Mead optimization. Ristic et al. [4] have proposed a multi-step nonlinear least squares fitting procedure in order to simultaneously estimate velocity and radius of buried cylinders. The method was implemented in a Matlab environment and validated by means of

^a Corresponding author: mechbal.zoubaida@gmail.com

experimental results. The new method was found to be significantly more accurate and robust with regard to both noise and the amount of raw data.

In this work, a closed form parameterisation of the hyperbola trace that is acquired in the B-scan radargram is considered as function of the hyperbola apex coordinate along the direction of B-scan, the cover depth, the radius of the object and the wave velocity in the medium. Use is made of an appropriate least squares based method in order to assess more effectively sensitivity of the inverse problem solution associated to identification of rebars buried in homogeneous concrete. This approach to identification problem is recognized to be fast enabling to perform large parametric studies to deal with the effects of the amount of data used and uncertainties affecting the trace hyperbola points.

To take into account uncertainties, probabilistic analysis is conducted. The coordinates of hyperbola points, extracted from raw data of radargram, are assumed to be randomly distributed according to Gaussian densities of probabilities. Then, sensitivity of the obtained estimated parameters to the number of points used in identification and inherent uncertainties in the raw data is analyzed.

2 Materials and method

2.1 Inverse problem solution

The hyperbolic signatures in GPR data are created by reflections that occur on the target boundary due to changing distance between the antenna and the target.

Let's assume a B-scan for the target having the form of a disk of radius R and centre with the coordinates, $(x_0, t_0 + 2R/v)$ where v is the wave velocity in the host medium, x_0 is the location of the antenna position at the nearest distance from the scanning surface and t the two-way travel time, see Figure 1. If the antenna is located at the abscissa x , then the following geometrical relation holds [12]

$$\left(\frac{v}{2}t + R\right)^2 = (x - x_0)^2 + \left(\frac{v}{2}t_0 + R\right)^2 \quad (1)$$

The hyperbola equation can be rewritten under the form

$$t = t(x_0, t_0, v, R, x) = -\frac{2R}{v} + \frac{2}{v} \sqrt{\left(\frac{v}{2}t_0 + R\right)^2 + (x - x_0)^2} \quad (2)$$

Denoting N the total number of points used and for a given set of data points $(t_i, x_i)_{i=1, \dots, N}$ located on the hyperbola which mathematical model is defined by equation (2), the unknowns (x_0, t_0, v, R) can be determined by a least squares based method as solution of the following minimisation problem

$$(x_0, t_0, v, R) = \underset{(\xi, \tau, \zeta, \rho)}{\operatorname{Argmin}} \sum_{i=1}^N [t(\xi, \tau, \zeta, \rho, x_i) - t_i]^{2\alpha} \quad (3)$$

where α is a parameter that fixes the cost function to be minimized.

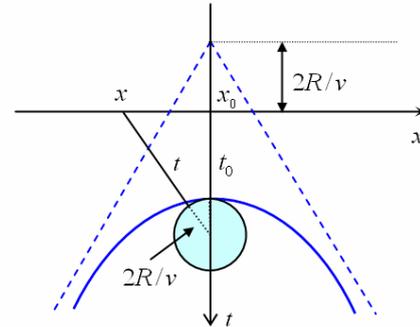


Figure 1. The GPR hyperbola trace signature showing the problem parameters (x_0, t_0, v, R) .

The previous problem can be readily solved by using suitable minimization algorithms. The algorithm based on pattern search method for linearly constrained problems proposed by Lewis and Torczon [2] proved to be very efficient. This algorithm can be applied through the command *patternsearch* available in Matlab. In order to stabilize the inversion problem, it was suggested [1] to add the following bound constraints associated to the domain where the solution is expected to be found:

$$\begin{bmatrix} 0 \\ 0 \\ c/10 \\ 0.004 \end{bmatrix} \leq \begin{bmatrix} x_0 \\ t_0 \\ v \\ R \end{bmatrix} \leq \begin{bmatrix} L \\ 20L/c \\ c \\ 0.04 \end{bmatrix} \quad (4)$$

where L is the maximum dimension of the concrete member being monitored and c the speed of wave in air. In equation (4), the lengths are expressed in m , time is in s and the velocities are in m/s .

The inverse problem solution depends like this, among other things, on the total number of points N which is fixed a priori at the start of the procedure.

2.2 Probabilistic analysis

If the raw data acquired by GPR during a B-scan is perfect, then the solution of the optimization problem defined by equations (3) and (4) will be only affected by the errors associated to the numerical procedure used. But, in practice the obtained radargram suffers from various sources of noise. The acquired data points $(t_i, x_i)_{i=1, \dots, N}$ are perturbed and their actual positions may be far from the theoretical ones. To take into account this uncertainty, it is assumed here that these points are random variables with their coordinates distributed according to Gaussian densities of probabilities with uniform standard deviations in space and time. In practice, the real distributions of probabilities can be identified from experimental testing followed by standard statistics performed on the results obtained from a large number of samples.

Probabilistic analysis is performed in this work by using simulations with raw data perturbed by generated

random numbers. The uncertainty affecting hyperbola points is assumed to be distributed according to Gaussian density of probabilities. The Matlab command *normrnd* which relies on the Matlab random number generator *rand* is used.

Fixing the standard deviations in time σ_t and in space σ_x , simulations enable to quantify evolution of the errors on the identified hyperbola parameters as modified by the perturbed raw data.

3 Results and discussion

Let's consider a reinforced concrete specimen containing a buried rebar of radius $R=4mm$, see Figure 2. The concrete section has the form of a rectangle having the edge dimensions $1m$ (horizontally) and $0.8m$ (vertically). These dimensions are taken to be large enough in order to avoid any reflection from the bottom edge of the block during the selected time interval.

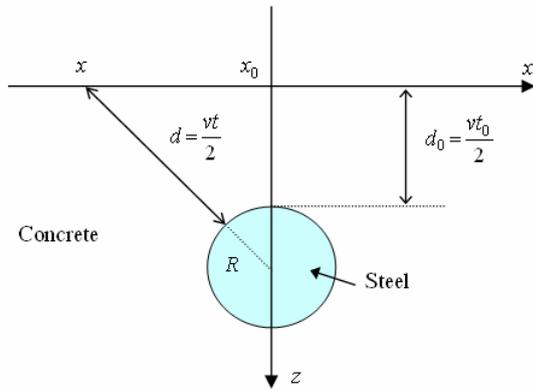


Figure 2. Geometrical characteristics of a buried steel rebar in concrete.

The dielectric properties chosen for the different materials are given in Table 1.

Table 1. Materials properties used for Reflexw simulation

Material	ϵ_r	μ_r	σ
Concrete	4	1	0.0255
Steel	1.45	1	9.931×10^6

The work frequency was fixed at $1.5GHz$, the cover depth d_0 was taken to be $0.52m$ and x_0 was fixed at $0.5m$. The wave speed is given by $c/\sqrt{\epsilon_r} = 1.5 \times 10^8 m/s$ and $t_0 = 6.9333 ns$.

For the direct problem simulations associated to the previous data have been performed by using Reflexw software package [11]. A B-scan which consisted in moving at constant speed the antenna along the x -line was performed. The simulation parameters under Reflexw were selected as follows: the wave form excitation was according to Kuepper pulse; the source type was exploding reflection; the boundary conditions were of linear absorbing range type; the output type was single line which corresponds to B-scan, the range of

time was fixed at $15ns$. The obtained radargram is depicted in Figure 3.

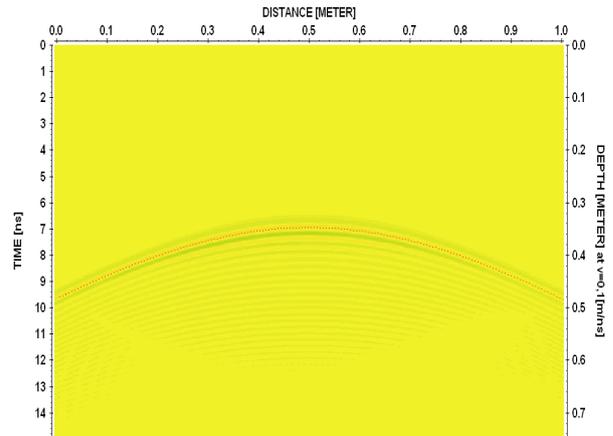


Figure 3. The obtained image radargrams corresponding to rebar have diameter $R=4mm$

Figure 3 shows the hyperbola trace on the obtained B-scan radargram. The raw data points located on the trace hyperbola are used to solve the optimisation problem as defined by equations (3) and (4) through using the command *patternsearch* in Matlab. Denoting X the vector which components are the unknowns (x_0, t_0, v, R) , the following instructions were used in Matlab:

```
LB=[0,0,c/10,4.e-3];
UB=[L,20*L/c,c,4.e-2];
xx0=(LB+UB)/2;
options = psoptimset('TolMesh',1.e-12,'MaxIter',1000,
'TolX',1.e-12,'TolFun',1.e-12,'MaxFunEvals',5000);
for k=1:10
[X,fval,exiflag]=patternsearch(@(X)Zt_object(X,xi,ti),
xx0,[],[],[],[],LB,UB,options);
xx0=X;
end
```

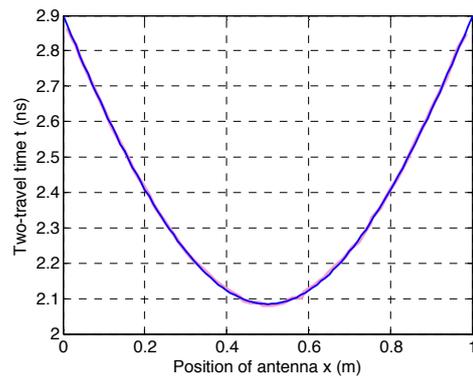


Figure 4. The identified hyperbola in blue and the input hyperbola corresponding to raw data in pink; $\alpha=2$ and $N=64$.

Figure 4 shows for $\alpha=2$ the identified hyperbola, by the procedure indicated above, superposed to the hyperbola resulting from the radargram raw data. Even if graphically the two curves are almost identical, the hyperbola centre which is also fixed by the hyperbola asymptotes is here inaccurate. Bad convergence in terms

of the hyperbola characteristics is in fact obtained when $\alpha=1$ or $\alpha=2$. The relative error for the rebar radius is too large $\Delta R/R \approx 800\%$, while the other errors remain small: $\Delta x_0/x_0 \approx 0.06\%$, $\Delta t_0/t_0 \approx 0.3\%$ and $\Delta v/v \approx 4\%$.

On the opposite, excellent convergence was obtained for integer values of the cost function parameter α that belong to the interval $[3,10]$. The obtained relative errors are in this case: $\Delta x_0/x_0 \approx 0.01\%$, $\Delta t_0/t_0 \approx 0.1\%$, $\Delta v/v \approx 1\%$ and $\Delta R/R \approx 0\%$.

Fixing $\alpha=10$, the number of points fixing the raw data hyperbola was reduced from $N=64$ to respectively $N=33$, $N=17$, $N=9$ and $N=5$.

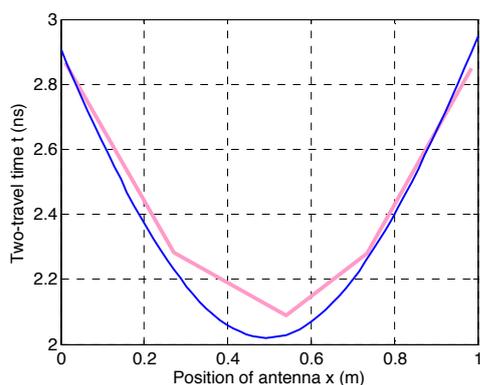


Figure 5. The identified hyperbola in blue and the input hyperbola in pink; $\alpha=10$ and $N=5$.

Figure 5 shows the obtained results for $\alpha=10$ and $N=5$. The obtained relative errors are: $\Delta x_0/x_0 \approx 1.3\%$, $\Delta t_0/t_0 \approx 2.9\%$, $\Delta v/v \approx 6\%$ and $\Delta R/R \approx 0\%$.

Fixing $\alpha=10$, perturbation of the raw data of the hyperbola trace given in Figure 3 was performed by sampling according to Gaussian density of probabilities. Use was made of the Matlab function *normrnd* which generates random numbers from the normal distribution with given mean parameter and standard deviation. Here the means are assumed to coincide with the initial input raw data, while the standard deviations are taken to be: $\sigma_t=0.0015$ and $\sigma_r=0.001$. This choice represents only small perturbations of the input raw data which is limited to a concentrated zone around the initial points. Performing a total number of 100 simulations, the error affecting rebar radius identification is shown in Figure 6.

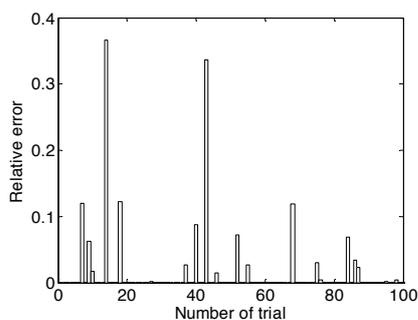


Figure 6. The error on the identified rebar radius for $\alpha=10$ and $N=64$, $\sigma_t=0.0015$ and $\sigma_r=0.001$.

The results regarding the influence of parameter α on convergence have shown that choosing large values of α enables to reduce the influence of accidental points and makes it possible to take into account more effectively contribution of normal points to the fitting hyperbola. Accuracy is then improved.

The results associated to the effect of the number of points taken on the raw data hyperbola have shown that these can be reduced favourably to a small number of points that are regularly spaced.

Figure 6 has shown that even small perturbation of the raw data points has considerable consequence on the identified rebar radius as the error was large.

4 Conclusions

In this work, sensitivity of the inverse problem solution associated to identification of rebar in concrete was analysed. The inverse problem was stated as an optimisation procedure enabling to search for the trace hyperbola characteristics that fits the raw data provided by a B-scan radargram. Solution of the inverse problem was fixed by appropriate selection of the cost function. Then random perturbation was applied to the raw data in order to guess robustness of the procedure. It was found that even small deviation of raw data points as modelled by a Gaussian density of probabilities yields significant false identified rebar radius. Reduction of the number of points was found to not affect largely the identification error if these are regularly spaced.

References

1. Z. Mechbal, A. Khamlichi, *NDT&E International*, **89** (2017)
2. R.M Lewis, V. Torczon, *SIAM Journal on Optimization*, 10 (2000)
3. A. Vaso Ristic, D. Petrovacki, M. Govedarica, *Computers & Geosciences* **35** (2009)
4. A.V. Ristic, D. Petrovacki, M. Govedarica, *Computers & Geosciences* **35** (2009)
5. C.G. Windsor, L. Capineri, *INSIGHT, Journal of the British Institute of Non Destructive Testing* **5** (1998)
6. G. Borgioli, L. Capineri, P.L. Falorni, S. Matucci, C.G. Windsor, *IEEE Transactions on Geoscience and Remote Sensing* **46**, 8 (2008)
7. G.R. Olhoeft, *Journal of Applied Geophysics* **43** (2000)
8. S. Shihab, W. Al-Nuaimy, A. Eriksen, *Proceedings 10th International Conference on Ground Penetrating Radar* (Delft, The Netherlands, 2004)
9. S. Shihab, W. Al-Nuaimy, *Subsurface Sensing Technologies and Applications* **6** (2005)
10. A. Dolgiy, A. Dolgiy, V. Zolotarev, *Proceedings of 11th International Conference on Ground Penetrating Radar* (Columbus Ohio, USA, 2006)
11. K.J. Sandmeier, *Reflexw version 6.0* (Karlsruhe, Germany, 1998)