A trajectory design method for RLV via artificial-memory-principle optimization

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Abstract. A trajectory optimization method for RLV based on artificial memory principles is proposed. Firstly the optimization problem is modelled in Euclidean space. Then in order to solve the complicated optimization problem of RLV in entry phase, Artificial-memory-principle optimization (AMPO) is introduced. AMPO is inspired by memory principles, in which a memory cell consists the whole information of an alternative solution. The information includes solution state and memory state. The former is an evolutionary alternative solution, the latter indicates the state type of memory cell: temporary, short- and long-term. In the evolution of optimization, AMPO makes a various search (stimulus) to ensure adaptability, if the stimulus is good, memory state will turn temporary to short-term, even long-term, otherwise it not. Finally, simulation of different methods is carried out respectively. Results show that the method based on AMPO has better performance and high convergence speed when solving complicated optimization problems of RLV.

1 Introduction

Since the retirement of space shuttle, many countries start to develop the 2nd generation reusable launch vehicles (RLVs), which are more autonomous and reliable. After fulfilling its orbital tasks, RLV has to reentry the atmosphere. However, the reentry environment is very tough and complicated, a well-designed trajectory can bring about soft and tractable guidance commands, and make control easier to realize it. Accordingly, it is very important for RLV to seek a trajectory that satisfies all path constraints, such as heat rate, structural loads and dynamic pressure.

Generally, aerospace engineers use optimization algorithms to design different types of trajectories. Traditional optimization algorithms can find global optimum solutions in many cases [1], in which the objective functions and constraints are simple. Engineers expect the algorithms to be universal, nevertheless, it is not reality. When confronted with complicated optimizations, researchers realized that the emerging nature-inspired algorithms (NIAs), are more suitable with the help of high performance computers [2]. In order to achieve a much wider adaptability, these algorithms abandon mathematical or physical searching ways, and turn to imitate the behaviors of creatures or phenomenon in nature. Their searching process seems to be fuzzy or random, however researches showed NIAs have a global convergence.

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These algorithms have a large family, including artificial neural network (ANN) [3-4], ant colony algorithm (ACA) [5], genetic algorithm (GA) [6], differential evolution (DE) [7] and particle swarm optimization (PSO) [8], and so on.

Biologists agreed that human is a kind of cleverest animal in nature, and studies point out that human's wisdom is closely related with memory. According to [9], if a NIA has the abilities of synergy, information exchange, diversity and evolution, and then it can solve a quite complicated problem. In the past decades, researches indicate that human memory principles can be transferred into a NIA [10-12]. They simplified the memory mechanism, and applied in different fields.

Based on the artificial memory principles, this paper proposed a NIA, which is called AMPO. There are three types of memory: temporary memory (TM), short-term memory (STM), long-term memory (LTM). A large number of memory cells build up the artificial memory system (AMS). In the process of optimization, a new alternative solution is firstly set to be TM, if the searching direction is "right", or the stimulus makes objective function lower, then the alternative solution turn to be STM. In the next search, if the same thing happens, then the alternative solution turn to be LTM, which has potential to be optimum solution, otherwise it turn back to be TM. As the search moves on, AMS will forget most alternative solutions or memory cells, and retain a small piece of the cells in LTM. The goal of trajectory design for RLV via AMPO is realized. Finally the convergence and validity of AMPO are verified through simulation.

This paper is organized as follows. Section 2 presents the optimization model of RLV. Section 3 develops the design of AMPO. Section 4 shows the simulation results. Section 5 contains conclusions.

2 Problem Statement and Preliminaries

2.1 Problem statement of optimization

In this paper, we consider the following optimization problem:

$$\begin{cases} \min f(\mathbf{X}) \\ s.t. \\ g_i(\mathbf{X}) \geq 0 \quad i = 1, 2, \dots, I \\ \mathbf{X} \in S \in R^n \end{cases} \quad (1)$$

where R^n is a n-dimensional Euclidean space; $\mathbf{X} = (x_1, x_2, \dots, x_n)$ is a control vector; S is a search space; $f(\mathbf{X})$ is the original objective function; $g_i(\mathbf{X})$ is the i th inequality constraint. To make AMPO more adaptive, here the objective function can be rewritten as:

$$F(\mathbf{X}) = \begin{cases} f(\mathbf{X}), & g_i(\mathbf{X}) \geq 0 \\ F_{\max}, & \text{others} \end{cases} \quad (2)$$

where $F_{\max}$ is a man-made large number which is a punishment for an alternative solution that can not satisfy all constraints.

2.2 Preliminaries of AMS

According to theory of Ebbinghaus [13], the principles of memory is abstracted and viewed as a process of how brains (creatures) interact with the stimulus from the objective world.

Fig. 1. The simplified model of memory.

3 Construction of AMPO

3.1 Memory cell of AMS

First of all, define the memory cell M_i as below, which consists of an alternative solution $\mathbf{X}_i$, and its characteristics, namely

$$M_i = (\mathbf{X}_i, m_i, t_i, s_i, f_i), i = 1, 2, \cdots, N \tag{3}$$

where m_i is the memory residual value; t_i is the iteration running time; s_i is the type of memory state, $s_i = T, S, L$ respectively corresponding to TM, SM and LM; if $f_i = 0$, it means memory cell M_i is not forgotten, if $f_i = 1$, forgotten.

AMS includes N alternative solutions $\{\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_N\}$. In each iteration, it will practice all memory cells, in other words, it scans each alternative solution and calculates $F(\mathbf{X}'_i)$, which represents the object function value that corresponds to the scanning (stimulus). There are two types of stimuli:

- (1) If $F(\mathbf{X}'_i) < F(\mathbf{X}^{t-1}_i)$, it means the stimulus are good for finding the optimum solution, and memory cell M_i will strengthen memory, which makes m_i bigger than before. The stimulus is so-called good stimulus.
- (2) If $F(\mathbf{X}'_i) \geq F(\mathbf{X}^{t-1}_i)$, it means the stimulus are useless, and M_i will weaken memory, which makes m_i smaller. The stimulus is so-called bad stimulus.

AMS scans all memory cells for a large number of iterations. Finally, this enables some memory cells with better characteristics of optimum to be strengthen-memorized, and some memory cells with worse characteristics to be forgotten.

3.2 Evolution strategies of alternative solutions

The position of $\mathbf{X}_i$ has some relationship with others while iteration going. Remark the position of all alternatives as $\mathbf{X}^{t-1}_1, \mathbf{X}^{t-1}_2, \cdots, \mathbf{X}^{t-1}_N$ at time $t - 1$. Since there are some memory cells in the LTM library, to accelerate the convergence speed, randomly select L cells from

them, that is $A^{t-1} = \{\mathbf{X}_{i1}^{t-1}, \mathbf{X}_{i2}^{t-1}, \dots, \mathbf{X}_{iL}^{t-1}\}$. To assure the randomness, here we proposed 3 evolution strategies.

3.2.1 Average evolution

At time t , the state of $\mathbf{X}_i$ is transferred to

$$x_{ij}^t = \begin{cases} \frac{1}{|A^{t-1}|} \sum_{k \in A^{t-1}} x_{ij}^{t-1}, j \in B \\ x_{ij}^{t-1}, j \in \{1, 2, \dots, n\} - B \end{cases} \quad (4)$$

where $\mathbf{X}_i^t = (x_{i1}^t, x_{i2}^t, \dots, x_{in}^t)$, $\mathbf{X}_i^{t-1} = (x_{i1}^{t-1}, x_{i2}^{t-1}, \dots, x_{in}^{t-1})$, $B = \{1, 2, \dots, n\}$, and $W = \max\{2, (0.1\% \sim 5.0\%)n\}$.

3.2.2 Compound evolution

To avoid the local minimum and make the evolution more flexible, here we select two alternative solutions from set A , the new compound state value is

$$x_{ij}^t = \begin{cases} \sum_{k=1}^{m_I} \alpha_k x_{i_{2(k+1)+1}j}^{t-1} - \sum_{k=1}^{m_E} \gamma_k x_{i_{2kj}}^{t-1}, j \in B \\ x_{ij}^{t-1}, j \in \{1, 2, \dots, n\} - B \end{cases} \quad (5)$$

where m_I and m_E are number of cells, $m_I \geq 2$, $m_E \geq 1$, $m_I > m_E$, $L = m_I + m_E$, $\forall k, s \in \{i_1, i_2, \dots, i_L\}$, $k \neq s \neq i$, $0 < \alpha_k, \gamma_k < 1$.

3.2.3 Copy evolution

Obviously there are more than one better alternative solution (or memory cell M_k), $F(\mathbf{X}_k^{t-1}) < F(\mathbf{X}_i^{t-1})$, at time t , memory cell M_i can simply copy the state of M_k , then the state of $\mathbf{X}_i$ is transferred to

$$x_{ij}^t = \begin{cases} x_{kj}^{t-1}, j \in B \\ x_{ij}^{t-1}, j \in \{1, 2, \dots, n\} - B \end{cases} \quad (6)$$

3.3 The mathematical model for updating and forgetting memory

At time t , assume the memory residual value of memory cell M_i is $m_i(t)$, then

$$s_i = \begin{cases} I, & m_i(t) \leq M_S \\ S, & M_S < m_i(t) \leq M_L \\ L, & m_i(t) > M_L \end{cases} \quad (7)$$

where M_S and M_L are critical constant to judge memory state, TM, SM or LM.

3.3.1 Model for forgetting memory

Here we utilize the Ebbinghaus's forgetting curve [13] to model the proceed of memory forgetting, then the memory decaying function of memory cell M_i is as follows:

$$m_i(t + \Delta t) = m_i(t)e^{-\alpha\Delta t},$$
$$\alpha = \begin{cases} \alpha_I, s_i = I \\ \alpha_S, s_i = S \\ \alpha_L, s_i = L \end{cases} \quad (8)$$

where constant Δt is the time increment, memory starts at time $t = 0$; coefficient α is used to adjust to memory decaying; For TM, SM and LM, the memory decaying rate $\alpha_I, \alpha_S, \alpha_L$ are different, and as normal, $\alpha_I > \alpha_S > \alpha_L$.

This model is very close to the actual situation of memory decaying, because

$$\lim_{\Delta t \rightarrow 0} m_i(t)e^{-\alpha\Delta t} = m_i(t)$$
$$\lim_{\Delta t \rightarrow \infty} m_i(t)e^{-\alpha\Delta t} = 0 \quad (9)$$

3.3.2 Model for updating memory

At time t , memory cell M_i accepts a good stimulus and its memory increase $\Delta m_i(t)$, the better the stimulus is, the bigger the objective function increment $\Delta F(\mathbf{X}_i^t)$ will be, so it is with $\Delta m_i(t)$. Here $\Delta m_i(t)$ is as follows:

$$\Delta m_i(t) = h |\Delta F(\mathbf{X}_i^t)| \quad (10)$$

where coefficient h indicates the magnitude of stimulus.

For memory cell M_i , after the stimulus, its memory will increase to

$$m_i(t + \Delta t) = m_i(t) + \Delta m_i(t) \quad (11)$$

Then the real memory becomes to

$$m_i(t + \Delta t) = m_i(t)e^{-\alpha\Delta t} + \lambda m_i(t)f_m(\Delta t) \quad (12)$$

where λ is the decaying adjustment coefficient, decaying function $f_m()$ is used to control memory decaying rate, which suits the Ebbinghaus's forgetting curve.

$$\lim_{\Delta t \rightarrow \infty} f_m(\Delta t) = 0 \quad (13)$$

Here assume $f_m(x) = e^{-bx}$ (b is a constant, $b > \alpha_L$), then formula (12) above can be written as:

$$m_i(t + \Delta t) = m_i(t)[e^{-\alpha\Delta t} + \lambda e^{-b\Delta t}] \quad (14)$$

Then after memory updating, forgetting and stimulus, the memory updating model of memory cell M_i is defined as

$$m_i(t + \Delta t) = m_i(t)[e^{-\alpha\Delta t} + \lambda e^{-b\Delta t}] + \Delta m_i(t) \quad (15)$$

Let $\beta = e^{-\alpha \Delta t} + \lambda e^{-b \Delta t}$, after $\alpha, b, \Delta t$ are given, β is constant and $0 < \beta < 1$, finally memory function can be rewritten as

$$m_i(t + \Delta t) = m_i(t)\beta + \Delta m_i(t), \beta = \begin{cases} \beta_I, & s_i = I \\ \beta_S, & s_i = S \\ \beta_L, & s_i = L \end{cases}, 0 < \beta_L < \beta_S < \beta_I < 1 \tag{16}$$

4 Simulation results

The computer used to test the performance of AMPO is a Lenovo Notebook computer, its CPU is Intel Core I5, N3450 @ 1.10 GHz, its memory size is 4 GB, its OS is Windows 10.

The setting of parameters used in AMPO are $G = 1000, n = 15, N = 50$,
 $\beta_I = 0.55, \beta_S = 0.35, \beta_L = 0.15$, $h = 1.2$,
 $M_S = 50h, M_L = 3M_S, F_I = 0.7, F_S = 0.3, F_L = 0.05$, $L = 3, W = \max\{2, 0.01n\}$,
 $m_I = 2, m_E = 1$, $\alpha_1 = 1, \alpha_2 = 0.5$, $P_A = 1/3, P_B = 2/3, P_C = 1, \gamma_1 = 0.5, \Delta t = 0.1$.

We choose two popular optimization algorithms to make comparison with AMPO, they are DE and GA, then choose objective function as $f(\mathbf{X}) = t_f$, namely to design a trajectory while RLV takes a shortest time to re-entry. The initial states are $h_0 = 120 \text{ km}$, $V_0 = 7200 \text{ m/s}$, $\theta_0 = -1.1^\circ$, flight is over when height reaches 28 km. If iteration times reach the max times limits, or objective function value changes less than 0.1%, then we assume that AMPO find the global optima and terminate the iteration.

The algorithms are run independently 100 iterations or 200s, the best objective function value, time of convergence, iteration times, are reported in Table 1 and Table 2.

Table.1 Results of same iterations.

Optimization algorithm	Best value	Convergence time	Iterations
AMPO	992.5	200.0	100
DE	994.1	256.9	100
GA	1074.6	360.2	100

Table 2. Results of same convergence time.

Optimization algorithm	Best value	Convergence time	Iterations
AMPO	992.5	200.0	100
DE	1022.7	200.0	78
GA	1294.1	200.0	35

From Table 1 and Table 2 we can see that AMPO successfully converge, it terminate iteration by reaching the maximum of iterations and convergence time limit simultaneously, while DE and GA don’t converge. For the same iterations, AMPO’s convergence takes less time than DE, and DE takes less time than GA. For the same convergence time, AMPO has less iterations than DE, and DE has less iterations than GA. The results show that AMPO has the ability of high convergence speed, which is very critical for RLV for trajectory design.

Further research shows that if AMPO has a pre-stored LTM library, AMPO will converge faster (less than 10 seconds) and makes it possible for online trajectory design.

The results for the trajectory generated by AMPO are shown below.

Fig. 2. Trajectory and entry corridor.

Fig. 3. Velocity history of attack angle.

Fig. 4. Velocity history of bank angle.

Fig. 5. Time history of dynamic pressur.

Fig. 6. Time history of load.

Figure 2 gives the entry trajectory in the velocity-altitude space, including the initial decent and phase of Quasi-equilibrium, plus dynamic pressure and load (normal acceleration) limits. The trajectory is quite smooth and reliable. Figure 2 shows the angle-of-attack (or alpha) profile in the alpha-velocity space. Figure 3 shows the bank angle profile in the bank-velocity space, obviously bank angle never exceeds the limits. From Figure 5 and Figure 6 we can find that the maximum dynamic pressure is 7.8 kPa (less than the limit of 8 kPa), the maximum load is 1.96 g (less than the limit of 2.0 g).

5 Conclusion

This paper proposed a AMPO algorithm for RLV to design the re-entry trajectory, and make a comparison with DE and GA. Results show that AMPO has a big advantage in speed of convergence, which is quite essential for complicated optimization problems.

The potential drawback of AMPO is that the LTM system requires off-line design of LTM, which works as the initial value for optimization. And AMPO has to improve its speed and flexibility. The concurrent and future work involves the development of evolution strategies that makes AMPO faster, and applicable for online trajectory design.

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