Investigation of the effect of the angle between drive shafts on vibration responses of main reducer

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Abstract. The drive shaft arrangement has a considerable influence on the vibration responses of the shaft-final drive system. In this research, a coupled vibration model is developed based on force analysis of hypoid gear and lumped mass method. The effect of time-varying mesh stiffness, gear backlash and transfer error are included to investigate the effect of the angle between drive shafts on vibration responses of main reducer. The vibration responses of main reducer are acquired using this model. The results show that the vibration amplitude of the gears of main reducer increase with the angle between drive shafts. This paper presents an analytical method to determine the value of the angle between drive shafts, so as to control the vibration of main reducer.

1 Introduction

As the requirement for comfort of automobile from customers is increasing, the requirement for vehicle vibration and noise control is becoming more demanding. The main reducer is a prominent source of vibration for automobile products and it has been investigated intensively. In the early stage, Singh [1] developed a mathematical model of single degree of freedom gear vibration with the backlash considered. Kahraman et al. [2, 3] developed a nonlinear vibration model of a spur gear system with rolling bearings, which took into account the bearing clearance and backlash. In addition, time-varying stiffness was introduced into the model, in which the single degree of freedom model was extended to a 3 degree of freedom coupled torsional vibration model with nonlinear backlash. A few years later, Kahraman and Blankenship [4] analysed vibration characteristics of nonlinear systems using periodic time-varying parameters and gear backlash under external excitation. Teodossiades et al. [5, 6] developed a model with time-varying meshing stiffness and nonlinear backlash to analyse the vibration characteristics of the gear.

To sum up, most of the studies are based on the dynamic performance analysis of the gear system. For automobile transmission system, the vibration analysis should not only consider the dynamic performance of gear system, but also the influence of the vibration generated by the drive shaft's excitation on main reducer. However, the effects of the drive

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shaft arrangement and responses of the gear transmission system have not been investigated simultaneously in details.

In this research, a model about the coupled vibration of shaft-final drive system is developed, based on the models developed previously [7-10], which includes the inhomogeneity of multiple universal joint transfer and the nonlinear time-varying factors of gear system. Using this model, the vibration responses of gears at different values of angle between drive shafts are simulated.

2 Methodologies

In this part, the dynamic model of the shaft-final drive system is introduced, which is based on force analysis of gear system and lumped mass method, including coupled vibration model of gear pair and torsional vibration model of drive shaft.

2.1 Hypoid gear model

In this model, the uniformly distributed load along the contact line is simplified to a concentrated force acting on the midpoint of the tooth width. The force analysis of pinion as illustrated in Fig. 1.

Fig. 1. Force analysis of pinion.

Fig. 2. The coupled vibration model of hypoid gears.

According to [10], the normal dynamic meshing force of pinion and its components can be expressed as,

$$F_n = k_m(t)f(x_n) + c_m\dot{x}_n \tag{1}$$

$$\begin{cases} F_x = -F_n \cos \alpha_n \sin \beta_1 \cos \delta_1 + F_n \sin \alpha_n \sin \delta_1 \\ F_y = -F_n \cos \alpha_n \sin \beta_1 \cos \delta_1 - F_n \sin \alpha_n \cos \delta_1 \\ F_z = -F_n \cos \alpha_n \cos \beta_1 \end{cases} \tag{2}$$

where c_m is the average damping of hypoid gear. $\dot{x}_n$ is the relative velocity between two gears at meshing point in the normal direction of meshing point. $k_m(t)$ is gear pair meshing stiffness function. δ_1 is the pitch cone angle of the pinion. β_1 is mean spiral angle. α_n is average pressure angle. Their values are listed in Table 1.

Table 1. Parameters of pinion and gear.

	Pinion	Gear
The number of teeth	8	41
Pitch cone angle	12.75°	76.50°
Average pressure angle	21.25°	21.25°
Mean spiral angle	47.97°	28.27°
Radius of nodal circle	18.8mm	71.7mm

Using the lumped mass method as illustrated in Fig. 2, a coupled vibration model of a hypoid gear pair is developed. c_{ij} is the equivalent damping in x, y and z direction (where $i = p, g; j = x, y, z$). K_{ij} is the equivalent stiffness in x, y and z direction (where $i = p, g; j = x, y, z$). Their values are listed in Table 2.

Table 2. The equivalent stiffness and damping of pinion and gear.

Stiffness (N/m)				Damping (N·s/m)			
K_{px}	864250000	K_{gx}	2820700000	c_{px}	1050	c_{gx}	1000
K_{py}	1635600000	K_{gy}	692000000	c_{py}	1050	c_{gy}	1000
K_{pz}	1635600000	K_{gz}	2820700000	c_{pz}	1050	c_{gz}	1000

2.2 Nonlinear dynamic model of shaft-final drive system

In this paper, the lumped mass method is used to develop the mechanical model of the shaft-final drive system. Using this method, universal joint, gears, differential and hub are simplified as lumped rotary inertia. Rotary inertia of the drive shaft is assigned to universal joint that is connected to it. Gears and differential are simplified as lumped mass; drive shaft is replaced by a torsion spring and a damper; bearing is taken as a massless elastomer, replaced by a spring and a damper.

The torsional vibration model of the shaft-final drive system is illustrated in Fig. 3, where K_i is the equivalent stiffness of drive shafts and half shafts (where $i = 1, 2, 3, 4$). c_i is the damping of the drive shafts and half shafts (where $i = 1, 2, 3, 4$). J_i is the equivalent rotary inertia of transmission components (where $i = 1, \dots, 8$), and their values are listed in Table 3.

Table 3. The rotary inertia, stiffness and damping of drive shaft.

Rotary inertia ($\text{kg} \cdot \text{mm}^2$)				Stiffness ($\text{N} \cdot \text{m}/\text{rad}$)		Damping ($\text{N} \cdot \text{s} \cdot \text{m}/\text{rad}$)	
J_1	1488.122	J_5	939.644	K_1	51945.046	c_1	0.5203
J_2	1798.169	J_6	19862.712	K_2	30020.573	c_2	0.4351
J_3	1970.879	J_7	11398.368	K_3	10506.904	c_3	0.8374
J_4	1969.705	J_8	122659.200	K_4	10506.904	c_4	0.8374

Fig. 3. Torsional vibration model of the shaft-final drive system.

With the reference to the relationship between displacement of shaft and angle between drive shafts [8], the theoretical model for the coupled vibration of the shaft-final drive system can be acquired,

$$\left\{ \begin{array}{l} J_1 \ddot{\theta}_1 + c_1(\dot{\theta}_1 - \dot{\theta}_2) + K_1(\theta_1 - \theta_2) - T_1 = 0 \\ J_2 M_2 \ddot{\theta}_2 + J_3 \left(\frac{\ddot{\theta}_2}{A-B \cos 2\theta_2} - \frac{2B \sin 2\theta_2}{(A-B \cos 2\theta_2)^2} \dot{\theta}_2^2 \right) + c_1 M_2(\dot{\theta}_2 - \dot{\theta}_1) + \\ c_2 \left(\frac{\dot{\theta}_2}{A-B \cos \theta_2} - \frac{\dot{\theta}_5}{A-B \cos \theta_5} \right) + K_1 M_2(\theta_2 - \theta_1) + \\ K_2 \left(\arctan \frac{\tan \theta_2}{\cos \alpha} - \arctan \frac{\tan \theta_5}{\cos \alpha} \right) = 0 \\ J_5 M_5 \ddot{\theta}_5 + J_4 \left(\frac{\ddot{\theta}_5}{A-B \cos 2\theta_5} - \frac{2B \sin 2\theta_5}{(A-B \cos 2\theta_5)^2} \dot{\theta}_5^2 \right) + \\ c_2 \left(\frac{\dot{\theta}_5}{A-B \cos \theta_5} - \frac{\dot{\theta}_2}{A-B \cos \theta_2} \right) + \\ K_2 \left(\arctan \frac{\tan \theta_5}{\cos \alpha} - \arctan \frac{\tan \theta_2}{\cos \alpha} \right) - F_z R_5 M_5 = 0 \\ 2J_7 \ddot{\theta}_6 + J_6 \ddot{\theta}_6 + 2c_3(\dot{\theta}_6 - \dot{\theta}_8) + 2K_3(\theta_6 - \theta_8) + F_z R_6 = 0 \\ J_8 \ddot{\theta}_8 + c_3(\dot{\theta}_8 - \dot{\theta}_6) + K_3(\theta_8 - \theta_6) = -T_8 \\ m_{px} \ddot{x}_p + c_{px} \dot{x}_p + K_{px} x_p = F_x \\ m_{py} \ddot{y}_p + c_{py} \dot{y}_p + K_{py} y_p = F_y \\ m_{pz} \ddot{z}_p + c_{pz} \dot{z}_p + K_{pz} z_p = F_z \\ m_{gx} \ddot{x}_g + c_{gx} \dot{x}_g + K_{gx} x_g = -F_x \\ m_{gy} \ddot{y}_g + c_{gy} \dot{y}_g + K_{gy} y_g = -F_y \\ m_{gz} \ddot{z}_g + c_{gz} \dot{z}_g + K_{gz} z_g = -F_z \end{array} \right. \quad (3)$$

where m_{ij} is the equivalent mass of pinion and gear (where $i = p, g; j = x, y, z$), and $m_{px} = m_{py} = m_{pz} = 1.04\text{kg}$, $m_{gx} = m_{gy} = m_{gz} = 6.16\text{kg}$. $M_2 = (A - B \cos \theta_2)$, $M_5 = (A - B \cos \theta_5)$. $A = \frac{1 + (\cos \alpha)^2}{2 \cos \alpha}$, $B = \frac{1 - (\cos \alpha)^2}{2 \cos \alpha}$.

3 Results and discussion

The output torque of the automobile engine is transmitted to main reducer through drive shaft, so the parameters of drive shaft affect the output torque of the engine, which is transmitted to pinion and gear of main reducer. Using the model developed in this paper, the vibration displacement curves of pinion and gear in x , y and z direction at different values of angle between drive shafts, such as 4° , 6° , 8° , 10° and 12° are simulated. The effects of the angle between drive shafts on the vibration amplitude are illustrated in Fig. 4(a). It can be seen from Fig. 4(a) that the angle between drive shafts can affect the displacement and vibration amplitude of pinion and gear in x , y and z direction. With the increase of the angle between drive shafts, the vibration amplitudes of pinion and gear in all directions increase. At large angle beyond 6° , the vibration amplitude will increase with the increase of the angle evidently; while at small angle, increasing the angle has little effect on the vibration amplitude. In particular, at small angles such as 4° and 6° , the vibration responses are similar and the vibration amplitudes are quite small and steady. To acquire the optimum angle, the vibration amplitude of pinion and gear in each direction is simulated with an increment of 0.2° from 4° to 6° , and the results are illustrated in Fig. 4(b).

Fig. 4. Vibration amplitude of pinion and gear in x , y and z direction (a) from 4° to 12° , (b) from 4° to 6° .

It can be seen that the vibration amplitudes of pinion and gear in three directions decrease first and then become larger. In particular, the vibration amplitude of pinion and gear in each direction has a minimum value when the angle between drive shafts is 4.2° . In particular, the vibration responses in all directions keep a small value at the range of angle between 4.2° and 4.8° .

4 Conclusions

In this paper, a coupled vibration model of the shaft-final drive system is developed. In the modelling and simulation of this research, the nondimensionalization which is common in previous research is avoided to acquire a clear physical meaning. Using this model, the vibration responses of main reducer at different angles between drive shafts are simulated. In general, the results show that the vibration amplitudes of gears in all three directions

increase with the angle between drive shafts, especially when it is larger than 6° . In particular, the vibration of the gears at the angle range between 4.2° and 4.8° is quite small and stable. Therefore, the model developed in this paper provides an insight for determining the proper angel between drive shafts.

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