Joints dynamic identification and modeling based on FRFs data

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Abstract. The problem of joints dynamic identification and modeling is discussed in this paper. The theoretical dynamic model of joints is established by FRFs (frequency response functions) data, and formulas for identifying the joints dynamic properties is deduced. The equivalent value of dynamic stiffness is extracted by solving the inconsistent equation using the least square method. The experimental example is provided to validate the feasibility and accuracy of the proposed method, the predicted result showing good fitting with experimental results.

1 Introduction

Mechanical structures are generally composed of substructures which are integrated by joints (such as bolts, welds and glue). When the mechanical system vibrates, the energy is absorbed, stored and released by joints, which reflect the stiffness and damping of joints dynamic characteristic. Due to the complexity and variability of joints' dynamics properties, purely theoretical analysis or finite element modeling could not establish the dynamical model of joints [1, 2]. The way to extract the joints' properties from measured data becomes one of the research hotspots in the related field.

The most suitable approach from the practical point of view is using frequency response function (FRF) data to identify the dynamic characteristic of joints, which can be measured directly. The first FRF method for the joint identification was probably mentioned by Okubo and Miyazaki [3], and developed by Tsai and Chou [4], Ren and Beards [5]. The model of joints was considered as a set of linear spring-damper elements in translational degree of freedom in above researches. Thereafter, the impact of rotational degree of freedom was investigated by Liu [6]. Instead of just a set of translational and rotational springs, the model of joints was considered as a coupled stiffness matrix in [7]. The identification formulas were deduced by Čelič, and the influence of the coordinate reduction on the identification of the joint dynamic properties was investigated in [8]. In above researches, there is a common problem: in the identification process, FRF data can't be measured completely, to solve this problem, the FRFs data of joints coordinates system is estimated by the measurable FRF or ignored. No matter in which way, few errors are mixed into the following identification formulas, and the influence of the errors is not ignorable at times.

The purpose of this paper is to discuss the problem of the joints dynamic identification

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and modeling based on FRFs data. The theoretical dynamic model of joints is established by FRFs (frequency response functions) data, and formulas for identifying the joints dynamic properties are deduced. The equivalent value of dynamic stiffness is extracted by solving the inconsistent equation using the least square method. The experimental example is provided to validate the feasibility and the accuracy of the proposed method, showing good fitting with experimental results.

2 Theoretical model

In this paper, the mechanical structure is divided into three systems: the first system is the substructure system consisting of substructures, the second system is the joint system comprised of joints, and the third system is the assembly system assembled by the substructure system and joint systems. Schematic diagram of theoretical model is shown in figure 1.

Fig. 1. Schematic diagram of theoretical model.

The input–output relationship of an assembled structure can be expressed as

$$\begin{Bmatrix} x_i \\ x_j \end{Bmatrix} = \begin{bmatrix} H_{ii} & H_{ij} \\ H_{ji} & H_{jj} \end{bmatrix} \begin{Bmatrix} f_i \\ f_j \end{Bmatrix} \quad (1)$$

where H is the FRF, superscripts “1” and “2” denote the substructure in the assembled structure, x_i and x_j are external node and joint node displacement vectors of the substructure, respectively, f_i and f_j are force vectors applied at the external nodes of substructure and joint nodes of the assembled structure, respectively.

Similarly, the input–output relationship for substructures can be written as

$$\begin{Bmatrix} x_a \\ x_b \end{Bmatrix} = \begin{bmatrix} H_{aa} & H_{ab} \\ H_{ba} & H_{bb} \end{bmatrix} \begin{Bmatrix} f_a \\ f_b \end{Bmatrix} \quad (2)$$

The dynamic characteristics of a joint system can be expressed as

$$f_c = D_j x_c \quad (3)$$

From the mechanical equilibrium condition and displacement compatibility condition, it is obvious that

$$f_b + f_c = f_j \quad (4)$$

$$x_b = x_c = x_j \quad (5)$$

Rearranging Eq. (2) leads to

$$x_b = x_c = H_{ba} f_a + H_{bb} f_b \quad (6)$$

Substituting Eq. (3) into Eq. (6) yields

$$f_j - f_b = D_j x_c = D_j H_{ba} f_a + D_j H_{bb} f_b \quad (7)$$

From Eq. (3) ~ Eq. (7), it is obvious that

$$\begin{Bmatrix} x_i \\ x_j \end{Bmatrix} = \begin{bmatrix} H_{aa} \\ H_{ba} \end{bmatrix} - \begin{bmatrix} H_{ab} \\ H_{bb} \end{bmatrix} (I + D_j H_{bb})^{-1} D_j H_{ba} \begin{Bmatrix} f_i \\ f_j \end{Bmatrix} + \begin{bmatrix} H_{ab} \\ H_{bb} \end{bmatrix} (I + D_j H_{bb})^{-1} \begin{Bmatrix} f_i \\ f_j \end{Bmatrix} \quad (8)$$

Comparing Eq. (8) with Eq. (1) yields

$$\begin{bmatrix} H_{ii} \\ H_{ji} \end{bmatrix} = \begin{bmatrix} H_{aa} \\ H_{ba} \end{bmatrix} - \begin{bmatrix} H_{ij} \\ H_{jj} \end{bmatrix} D_j H_{ba} \quad (9)$$

$$\begin{bmatrix} H_{ij} \\ H_{jj} \end{bmatrix} = \begin{bmatrix} H_{ab} \\ H_{bb} \end{bmatrix} (I + D_j H_{bb})^{-1} \quad (10)$$

Rearranging Eq. (9) and Eq. (10) leads to

$$\begin{aligned} H_{ij} D_j H_{ba} &= H_{aa} - H_{ii} \\ H_{jj} D_j H_{ba} &= H_{ba} - H_{ji} \\ H_{ij} D_j H_{bb} &= H_{ab} - H_{ij} \\ H_{jj} D_j H_{bb} &= H_{bb} - H_{jj} \end{aligned} \quad (11)$$

The dynamic stiffness can be obtained by using generalized inverse matrix based on Eq. (11):

$$Y \cdot T f(\omega) = H_j^{-1}(\omega) \quad (12)$$

$$Y = [K \quad C \quad M] \quad (13)$$

$$T_f(\omega) = [I \quad i\omega I \quad -\omega^2 I]^T \quad (14)$$

where $Y(\omega)$ is the coefficient matrix structured by stiffness, damper, and mass matrices,

$T_f(\omega)$ is transformation matrix used to uncouple $Y(\omega)$.

Extending Eq. (12) leads to

$$\tilde{Y} \cdot [T_f(\omega_1) \quad \dots \quad T_f(\omega_n)] = [H_j^{-1}(\omega_1) \quad \dots \quad H_j^{-1}(\omega_n)] \quad (15)$$

$$\tilde{Y} = [H_j^{-1}(\omega_1) \quad \dots \quad H_j^{-1}(\omega_n)] \cdot [T_f(\omega_1) \quad \dots \quad T_f(\omega_n)]^+ \quad (16)$$

$$\tilde{Y} = [\tilde{K} \quad \tilde{C} \quad \tilde{M}] \quad (17)$$

where $\tilde{Y}$ is the result matrix structured by the equivalent stiffness, damper, and mass factor ($\tilde{K}$, $\tilde{C}$ and $\tilde{M}$).

3 Examples

To validate the feasibility and accuracy of the proposed method, an experimental examples is presented here for dynamic modeling of single stiffness to prove the correctness of the method proposed in this paper. The test assembly is used in the experimental example, which is consisted of one steel beam and one cast iron beam, with a same rectangular cross section (30*10 mm²) and a length of 350 mm. Both the beams are suspended by two flexible springs during testing. The responses of each structure are obtained by applying an impact force on the input point with a hammer whose response being obtained at the output points with an accelerometer. All the data are processed by LMS test lab, and transmitted to computer for further calculation. A bolted joint with a torque of 60Nm applied on is used to assemble the substructures. The experimental set-up and distribution of test points are shown in figure 2, the picture of field test is shown in figure 3, and the material data of beams is shown in table 1.

Fig. 2. (a) Experimental set-up for impact hammer test; (b) Distribution of test point.

Fig. 3. Picture of field test.

Table 1. Material data of beams.

	Substructure 1	Substructure 2
Material	Steel	Cast iron
Young’s modulus(N/m²)	2.10E11	1.3E11
Poisson ratio	0.3	0.3
Density(kg/m³)	7850	7350

The measured FRF data of substructure and assembled structure is shown in figure 4.

Fig. 4. (a) Part of FRF data of substructure; (b) FRF data of assembly structure.

The FEA model is built, which is shown in figure 5. The comparison between the measured value and the predicted value is shown in figure 6.

Fig. 5. Finite element model of assembly system.

Fig. 6. Comparison between the measured value and the predicted value.

4 Conclusion

In this paper, the formulas for identifying the joints dynamic properties are deduced based on the mechanical equilibrium condition and the displacement compatibility condition. The theoretical model is built based on FRFs, and the equivalent dynamic stiffness is obtained by using least square method. One experimental example is provided to validate the feasibility and the accuracy of the proposed method, showing good fitting with experimental results.

5 Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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