Hertz Model Based Contact Modeling for Joints with Clearance

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Abstract. This paper presents the techniques of contact modeling for revolute joints in flexible multibody systems, in which the dry clearance revolute joints have been coupled with the flexibility of connected bodies. The contact model for revolute joints takes into account the relative planar motion caused by the clearance between the outer and inner races. This model applies a penalty method to simulate the phenomenon of inner-penetration between contact/impact bodies. The relationship between the normal contact force and the inner-penetration is described by the nonlinear Hertz model with energy dissipation. Meanwhile, the friction force can be predicted from continuous Coulomb's law. Finally, an example of flexible multibody systems has been simulated by using the developed contact models.

1 Introduction

Engineering practices show that the clearances in the revolute joints are inevitable to allow for the assembly of the pair elements. Furthermore, manufacturing tolerances[1], wear[2] and thermal effects[3] also cause clearances. The problems of modeling simple mechanical systems with clearance joints were extensively investigated[4-8]. A typical problem is the slider-crank mechanism, in which only the radial clearance located in the planar revolute joint is considered[9]. Since rigid modeling of the links will affect the prediction of the system real dynamic behaviour, the modelling of mechanism flexibility is also required. A comprehensive review can be found in Ref.[10], where the commercial software MSC ADAMS was applied to simulate the crank-slider mechanism with rigid/flexible links and planar joint clearances. More recently, the combined influence of the axial and radial clearances in the revolute joint[11, 12] has been studied. Ref.[13] predicts the dynamic behaviour of slider-crank mechanism with single and two revolute clearance joints. It concludes that the dynamic response in a mechanism with two clearance joints is not a simple superposition of that in mechanism with one clearance joint, and then proposes that all the joints in a multibody system should be modeled as clearance joints. The coupling problem of joint clearances and link flexibilities of the complicated mechanism is still an open topic. The innovation of this paper is the implementation of a contact model for clearance joints with the purpose of providing a general solution to the coupling problem. The new contact model is also defined in the absolute nodal coordinate systems. It can be used to connect any rigid/flexible bodies with arbitrary topologies.

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When modeling joints with clearance, an appropriate contact/impact force model is important. Gilardi and Sharf[14] distinguish the associated force calculation approaches into two different categories, impulse-momentum and continuous. The first approach is only practical if there is only a single contact. A serious disadvantage is that it will stop the simulation when the contact/impact occurs. In second approach, the simulation is not stopped, and the contact force in each contact pair is predicted at any time during the simulation[15]. Based on a relationship between contact force and penetration, for example, by Flores etc.[16], this approach performs the contact calculation in three steps: contact detection, calculation of the contact normal force, and calculation of the friction force[17]. In this paper, an algorithm based on the continuous approach is developed to evaluate the contact/impact force based on nonlinear Hertz model of Hunt and Crossley[18], which is suitable for multiple impact pairs. When the bodies are in contact/impact, the friction force can be predicted from the continuous Coulomb's law. It is known the Coulomb's friction law[19] describes the transitions from sticking to sliding and vice-versa, but these discrete transitions can cause difficulties for their numerical simulations. Many continuous friction laws[20, 21] have been advised to approximate the discrete transitions. This paper regularizes the friction coefficient of Coulomb's law to obtain a continuous friction law, which can describe both sliding and rolling/sticking behaviour of revolute joints.

In summary, this paper focuses on studying the coupling effect of joint clearances and link flexibilities of the complicated mechanism, and designs a detailed contact model for revolute joint with clearance. The nonlinear Hertz model with energy dissipation describes the relationship between the normal contact force and the inner-penetration. The continuous Coulomb's law are implemented for the simulation of friction phenomena.

2 Contact models for revolute joints with clearance

With the unilateral contact condition[19], a contact model for revolute joints is developed in this section to capture the phenomena of contact/impact due to clearance.

2.1 Contact kinematics of revolute joints with clearance

For the modeling of clearance, the scheme of contact detection should be designed at first. A straightforward strategy is predicting the relative distance between the contact point-pair candidates with the unilateral contact condition and then make a judgment of weather the contact is happened or not.

Figure 1. Configuration of revolute joint with clearance.

Observation of the geometric configuration of revolute joint with clearance, as depicted in Fig.1, shows that the contact point-pair candidates could be identified along the normal direction $\mathbf{n}$ defined as

$$\mathbf{n} = \mathbf{u} / \|\mathbf{u}\| \quad (1)$$

where $\|\cdot\|$ presents the 2-norm of vector. The positions of contact point-pair candidates on inner and outer races k and l , respectively, are written as

$$\mathbf{z}^k = \mathbf{u}_0^k + \mathbf{u}^k + \rho^k \mathbf{n} \quad \text{and} \quad \mathbf{z}^l = \mathbf{u}_0^l + \mathbf{u}^l + \rho^l \mathbf{n} \quad (2)$$

where ρ^k and ρ^l are the radii of inner and outer races. The relative distance between the contact candidates predicts from

$$q = \mathbf{n}^T (\mathbf{z}^k - \mathbf{z}^l) \quad (3)$$

in details,

$$q = \mathbf{n}^T \mathbf{u} + \rho^k - \rho^l \quad (4)$$

During contact/impact, the inner and outer bodies will exert tension on one another and the inner-penetration will be happened. Obviously, if $q < 0$, the contact/impact occurs and the inter-penetration between the contacting bodies is denoted by a negative value of q as $a = -q$. This work develops a penalty approach by adding spring and damper in the contact interface between inner and outer bodies to the modeling of unilateral contact conditions. Denoting the magnitude of normal contact forces as f_n , the normal contact forces applied to bodies k and l can be written as

$$\mathbf{F}_n^k = f_n \mathbf{n}, \quad \mathbf{F}_n^l = -f_n \mathbf{n} \quad (5)$$

In the case of friction in the contact interface there are possibilities of sticking and sliding. When sliding occurs, the relative velocity between material points which coincide with the contact candidates will compute from time derivatives of positions, $\dot{\mathbf{z}}^k$ and $\dot{\mathbf{z}}^l$, as

$$\mathbf{v}^r = \dot{\mathbf{v}}^l + \boldsymbol{\omega}^l \times \rho^l \mathbf{n} - \dot{\mathbf{v}}^k - \boldsymbol{\omega}^k \times \rho^k \mathbf{n} \quad (6)$$

The component of relative velocity in the null space of normal direction $\mathbf{n}$ is readily obtained from

$$\mathbf{v}_t = (\mathbf{I} - \mathbf{n}\mathbf{n}^T) \mathbf{v}^r \quad (6)$$

and the normalization of $\mathbf{v}_t$ produces the tangent vector $\mathbf{t}$ indicating the direction of sliding. The friction force and moment applied to bodies k and l are obtained from the associated virtual work done by the friction force, f_t , like

$$\mathbf{F}_t^k = f_t \mathbf{t}, \quad \mathbf{M}_t^k = f_t \rho^k \mathbf{n} \times \mathbf{t}, \quad \mathbf{F}_t^l = -f_t \mathbf{t}, \quad \mathbf{M}_t^l = -f_t \rho^l \mathbf{n} \times \mathbf{t} \quad (7)$$

2.2 Normal contact force based on Hertz model

For penalty method, the relationship between normal contact force and inner-penetration will be given by a constitutive law during contact. Conventional approach applies Hertz model[22], a spring model as the constitutive law, to derive the magnitude of normal contact force f_n from a potential, $V(a)$, like

$$f_n = \partial V / \partial a \quad (8)$$

In a generic sense, this work separates the normal force of contact into its elastic and dissipative components and a suitable expression is given as

$$f_n = f_e + f_c = \partial V / \partial a (1 + \mu \dot{a}) \quad (9)$$

where $\mu \dot{a}$ accounts for energy dissipation during contact. In the case of a Hertzian contact, the nonlinear modeling of elastic contact force is applied

$$f_e = k_h a^{3/2} \quad (10)$$

then the damping contact force becomes

$$f_c = \mu k_h a^{3/2} \dot{a} \quad (11)$$

The definitions of stiffness parameter k_h and damping coefficient μ can be found in Refs[18, 22].

2.3 Modeling of friction force

This work models the friction force based on the Coulomb's friction law, which states the friction force f_t is proportional to the magnitude of normal contact force f_n like

$$f_t = -\mu_k f_n \quad (12)$$

where μ_k is the coefficient of dynamic friction. If the relative velocity vanishes, sticking or rolling takes place. In this case, the friction force becomes

$$|f_t| \leq \mu_s f_n \quad (13)$$

where μ_s is the coefficient of static friction. This highly nonlinear phenomenon of friction coefficient switching between sliding and sticking conditions makes it very hard to deal with classic Coulomb's law numerically[23]. In current implementation, the friction coefficient is regularized to obtain a continuous friction law[24, 25], successfully treat sticking, sliding and transition regions, like

$$f_t = -\mu_k f_n \tanh(v_t/v_c) \quad (14)$$

where v_c is a characteristic velocity.

3 Bio-inspired flight flapping wing

This example deals with the kinetic problem of a stick model of flying robot, which is designed to simulate the flapping motion of birds. The internal structure of wing has been simplified to the four-bar linkages. Due to symmetry, only a half of the basic geometric configuration is depicted in Fig.2. Note that 16 points are numbered in the figure to determine the geometric topology of the four-bar linkage, where the left linkage will connect to the right one through points 13 and 16. The whole structure is clamped at point 16. The coordinates of all the points are given in table 1 with the unit of length cm.

Figure 2. Configuration of four-bar linkage.

In current implementation, the numerical model of four-bar linkage is consisted of beam elements[26] and revolute joints. A user-defined function, $\phi = \omega t$, with constant angular speed $\omega = 12.56$ rad/s, prescribes rotation of crank shaft, and the kinematic constraint[27] is enforced to the kinematic model for revolute joint located at point 1.

Table 1. Coordinates of the four-bar linkages control points.

No.	Left wing	No.	Right wing
L 1	(-4.91071, 0.00000, 0)	R 1	(4.91071, 0.00000, 0)
L 2	(-2.25663, 2.38975, 0)	R 2	(2.25663, 2.38975, 0)
L 3	(-2.22156, 9.08609, 0)	R 3	(2.22156, 9.08609, 0)
L 4	(-0.76893, 12.3488, 0)	R 4	(0.76893, 12.3488, 0)
L 5	(-11.4767, 12.7227, 0)	R 5	(11.4767, 12.7227, 0)
L 6	(-44.0461, 13.8600, 0)	R 6	(44.0461, 13.8600, 0)
L 7	(-45.4988, 10.5974, 0)	R 7	(45.4988, 10.5974, 0)
L 8	(-47.5828, 13.3630, 0)	R 8	(47.5828, 13.3630, 0)
L 9	(-48.0479, 10.7251, 0)	R 9	(48.0479, 10.7251, 0)
L10	(-52.9394, 13.4378, 0)	R10	(52.9394, 13.4378, 0)
L11	(-100.256, 14.0985, 0)	R11	(100.256, 14.0985, 0)
L12	(-11.5390, 10.9380, 0)	R12	(11.5390, 10.9380, 0)
L13	(0.00000, 8.75726, 0)	R13	(0.00000, 8.75726, 0)
L14	(-4.91071, 4.39570, 0)	R14	(4.91071, 4.39570, 0)
L15	(-7.36607, 4.39570, 0)	R15	(7.36607, 4.39570, 0)
L16	(0.00000, 0.00000, 0)	R16	(0.00000, 0.00000, 0)

The beam elements have shared the same material properties as following: mass per unit length, $\rho = 0.1032$ kg/m, bending stiffness, $I_{22} = I_{33} = 29.856$ N • m², torsional stiffness, $J = 23.303$ N • m². Two different test cases have been performed. For case *I*, all the revolute joints have been modeled by using kinematic constraints. The governing equations of motion for discrete model will be DAE of index-3 with degrees of freedom, $N = 1368$, containing 42 Lagrange multipliers. The 2-stage Radau IIA algorithm[28] is applied to integrate the governing equations and the predicted trajectory of four-bar linkage is shown in Fig.3.

Figure 3. Trajectory of four-bar linkage.

The cyclic flapping of four-bar linkage apparently mimics the flapping motion of birds. For case *II*, the kinematic constraints at point 4 for both left and right linkages are replaced by two contact models, leaving the rest unchanged. The contact models with the same properties have been applied, and their physical properties are defined as: the radii of inner and outer races are 50 and 50.1 mm, respectively. The stiffness parameter for Hertz model is $k_h = 1 \times 10^9 \text{ N/m}^{3/2}$. The Coulomb's law has been applied to predict the friction force, where $\mu_k = 0.6$, and $v_c = 2 \text{ m/s}$. The numerical simulation results show that the clearance joints seriously affect the dynamic response of the flexible linkage and degrade the dynamic performance of connected bodies. Fig.4, presents the time histories of dynamic response of point 11 located at the tip of linkage. The apparent deviation of the simulation results of case *I* and *II* can be observed from the figure.

Figure 4. Time histories of position for point 11 at the tip of linkage, contact model: ○, kinematic constraints: ◇.

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References

1. P. Flores, J.C.P. Claro, A systematic and general approach to kinematic position errors due to manufacturing and assemble tolerances. Proceedings of ASME 2007 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Las Vegas, NV, 4-7 Sep. (2007).
2. E. Askari, P. Flores, D. Dabirrahmani, R. Appleyard, Dynamic modeling and analysis of wear in spatial hard-on-hard couple hip replacements using multibody systems methodologies. *Nonlinear Dynamics*, **82** (1-2), 1039--1058 (2015).
3. S. Bing, J. Ye, Dynamic analysis of the reheat-stop-valve mechanism with revolute clearance joint in consideration of thermal effect. *Mechanism and Machine Theory*, **43**(12), 1625--1638 (2008).
4. H. Funabashi, K. Ogawa, M. Horie, A dynamic analysis of mechanisms with clearances. *Bulletin of the JSME*, **21**(161), 1652--1659 (1978).
5. P. Flores, J. Ambrósio, Revolute joints with clearance in multibody systems. *Computers & structures*, **82**(17), 1359--1369 (2004).

6. P. Flores, P., Ambrósio, J., Claro, J.C.P., Lankarani, H.M.: Spatial revolute joints with clearances for dynamic analysis of multi-body systems. *Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics*, **220**(4), 257--271 (2006).
7. Q. Tian, Y. Sun, C. Liu, H. Hu, P. Flores, ElastoHydroDynamic lubricated cylindrical joints for rigid-flexible multibody dynamics. *Computers and Structures*, **114**, 106--201 (2013).
8. P. Flores, R. Leine, C. Glocker, Modeling and analysis of rigid multibody systems with translational clearance joints based on the nonsmooth dynamics approach. *Multibody System Dynamics*, **23**(2), 165--190 (2010).
9. S. Mukras, N.H. Kim, N.A. Mauntler, T.L. Schmitz, W.G. Sawyer, Analysis of planar multibody systems with revolute joint wear. *J. Wear*, **268**(5-6), 643--652 (2010).
10. B.A.M. Abdallah, I. Khemili, N. Aifaoui, Numerical investigation of a flexible slider-crank mechanism with multijoints with clearance. *Multibody System Dynamics*, **38**(2), 173--199 (2016).
11. F. Marques, F. Isaac, N. Dourado, P. Flores, An enhanced formulation to model spatial revolute joints with radial and axial clearances. *Mechanism and Machine Theory*, **116**, 123--144 (2017).
12. N. Akhadkar, V. Acary, B. Brogliato, Multibody systems with 3D revolute joints with clearances: an industrial case study with an experimental validation. *Multibody System Dynamics*, **81**, 1--34 (2017).
13. H. Tan, Y. Hu, L. Li, A continuous analysis method of planar rigid-body mechanical systems with two revolute clearance joints. *Multibody System Dynamics*, **40**(4), 347--373 (2017).
14. G. Gilardi, I. Sharf, Literature survey of contact dynamics modelling. *Mechanism and Machine Theory*, **37**(10), 1213--1239 (2002).
15. C. Pereira, J. Ambrósio, A. Ramalho, Dynamics of chain drives using a generalized revolute clearance joint formulation. *Mechanism and machine Theory*, **92**, 64--85 (2015).
16. P. Flores, J. Ambrósio, On the contact detection for contact-impact analysis in multibody systems. *Multibody System Dynamics*, **24** (1), 103--122 (2010).
17. F. Marques, P. Flores, H.M. Lankarani, On the frictional contacts in multibody system dynamics. *Multibody Dynamics*, Springer International Publishing (2016).
18. K.H. Hunt, F.R.E. Crossley, Coefficient of restitution interpreted as damping in vibroimpact. *Journal of Applied Mechanics*, **42**(2), 440--450 (1975).
19. F. Pfeiffer, C. Glocker, *Multibody Dynamics with Unilateral Contacts*. John Wiley & Sons, New York (1996).
20. J.C. Oden, J.A.C. Martins, Models and computational methods for dynamic friction phenomena. *Computer Methods in Applied Mechanics and Engineering*, **52**(1-3), 527--634 (1985).
21. J.E. Shigley, C.R. Mischke, *Mechanical Engineering Design*. McGraw-Hill Book Company, New York (1989).
22. J. Alves, N. Peixinho, M.T. Silva, P. Flores, H. Lankarani, A comparative study on the viscoelastic constitutive laws for frictionless contact interfaces in multibody dynamics. *Mechanism and Machine Theory*, 85 (Supplement C), 172--188 (2015).
23. Z. Qi, X. Luo, Z. Huang, Frictional contact analysis of spatial prismatic joints in multibody systems. *Multibody System Dynamics*, **26**(4), 441--468 (2011).
24. O.A. Bauchau, C. Ju, Modeling Friction Phenomena in Flexible Multibody Dynamics. *Computer Methods in Applied Mechanics and Engineering*, **195**(50-51), 6909--6924 (2011).
25. E. Pennestrì, V. Rossi, P. Salvini, P.P. Valentini, Review and comparison of dry friction force models. *Nonlinear Dynamics*, **83**(4), 1785--1801 (2016).
26. J. Wang, Implementation of geometrically exact beam element for nonlinear dynamics modeling. *Multibody System Dynamics*, **35**(4), 377--392 (2015).
27. J. Wang, Modified models for revolute joints coupling flexibility of links in multibody systems. *Multibody System Dynamics*, 1-19 (2018).
28. J. Wang, Application of Radau IIA algorithms to flexible multibody system with holonomic constraints. *Nonlinear Dynamics*, 88(4), 2391-2401, (2017).