A Specific Sliding Mode Control for Autopilot Design of Bank-to-Turn Missiles

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Abstract. This paper studies the BTT missile's sliding mode flight controller. Firstly, the control model is established according to dynamic inverse method and the general dynamic equation of the missile flying in the atmosphere. Then, the sliding mode controller is designed and it adopts the time-varying sliding mode surface, the self-defined reaching law and the adaptive control parameters. Finally, three sets of simulation are carried out to prove the flight controller designed in this paper for BTT missile can follow the guidance instructions and meet operational needs.

1 Introduction

The Back-to-turn steering is realized by rolling and pitching the missile while maintaining (ideally) a zero sideslip angle[1]. The most important advantage of BTT missile is high maneuverability[2], which is desperately needed under certain flying conditions such as air-to-air combat[3]. The bank-to-turn steering brings BTT missile many advantages, but also leads to the strong coupling between pitch channel and roll channel[4]. In actual flight, the sideslip angle is a small value around zero, and it cannot strictly guarantee the sideslip angle is zero. So the dynamics model of BTT missile is strongly nonlinear equations, this poses a severe challenge to the control system.

This paper firstly builds the control model by the general dynamics equation of the cruise missile, which include the dynamic inverse process. Then a specific sliding mode controller is designed. Finally, the effectiveness of the controller is verified through simulation.

2 Control model of BTT missile

The dynamics model of missiles flying in the atmosphere is established as follows:

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Generally for BTT missile, the controlled variable of system, which means the guidance command, is 

\[ y_V = \frac{P \cos \alpha \cos \beta - X - mg \sin \theta}{m} \]

\[ \dot{\theta} = \frac{P(\sin \alpha \cos \gamma_v + \cos \alpha \sin \beta \sin \gamma_v) + Y \cos \gamma_v - Z \sin \gamma_v - mg \cos \theta}{mV} \]

\[ \dot{\gamma}_v = \frac{P(\sin \alpha \sin \gamma_v - \cos \epsilon \alpha \sin \beta \cos \gamma_v) + Y \sin \gamma_v + Z \cos \gamma_v}{-mV \cos \theta} \]

\[ \dot{w}_x = \frac{M_x + (J_y - J_z)w_y w_z}{J_x} \]

\[ \dot{w}_y = \frac{M_y + (J_z - J_x)w_z w_y}{J_y} \]

\[ \dot{w}_z = \frac{M_z + (J_x - J_y)w_x w_y}{J_z} \]

\[ \dot{\gamma} = w_y \sin \gamma + w_z \cos \gamma \]

\[ \dot{\psi} = \frac{w_y \cos \gamma - w_z \sin \gamma}{\cos \theta} \]

\[ \dot{\vartheta} = w_x - \tan \vartheta(w_y \cos \gamma - w_z \sin \gamma) \]

\[ \sin \beta = \cos \theta \left[ \cos \gamma \sin(\psi - \psi_v) + \sin \vartheta \sin \gamma \cos(\psi - \psi_v) \right] - \sin \theta \cos \vartheta \sin \gamma \]

\[ \sin \alpha = \cos \theta \left[ \sin \vartheta \cos \gamma \cos(\psi - \psi_v) - \sin \gamma \sin(\psi - \psi_v) \right] - \sin \theta \cos \vartheta \cos \gamma \]

\[ \sin \gamma_v = \frac{\cos \alpha \sin \beta \sin \vartheta - \sin \alpha \sin \beta \cos \gamma \cos \vartheta + \cos \beta \sin \gamma \cos \vartheta}{\cos \theta} \]

Equation (1) is dealt with by dynamic inverse method as follows:

It is found that six equations in system (1) forms partial closure, which are
\[ \dot{\omega} = F_\theta \omega \]  
\[ \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad U_M = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}, \quad F_w = \begin{bmatrix} (J_y - J_z)w_y w_z \\ (J_x - J_z)w_x w_z \\ (J_x - J_y)w_x w_y \end{bmatrix} \]  
\[ F_\theta = \begin{bmatrix} \theta \\ \psi \\ \gamma \end{bmatrix}, \quad J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \]  
\[ B_w = \begin{bmatrix} \dot{\omega}_y \cos \gamma - \omega_z \sin \gamma \\ \omega_y \cos \gamma - \omega_z \sin \gamma \tan \theta - \dot{\gamma} \left( \omega_y \sin \gamma + \omega_z \cos \gamma \right) \\ \omega_y \sin \gamma + \omega_z \cos \gamma \tan \theta - \dot{\gamma} \omega_y \cos \gamma - \omega_z \sin \gamma \cos^2 \theta \end{bmatrix} \]  
\[ \dot{v} = \ddot{\theta} \]  
\[ U_M = JF_\theta^{-1}(v - B_w) - F_w \]
Therefore, what we should do below is to design the new control quantity $\nu$.

For BTT missile, the controlled variable is usually $T y V n \beta \gamma$. So we need derive the relationship between the controlled variable $T y V n \beta \gamma$ of system (1) and the controlled variable $\theta$ of system (4), which includes two instruction translations:

Instruction translation 1: the guidance command $n y_d \beta_d \gamma V_d$ is translated to the expected input $\theta_d$ of system (4).

Instruction translation 2: the actual output $\theta$ of system (4) is translated to actual output $n y \beta \gamma$ of system (1).

System (1) is shown in Figure 1.

The formulas of two instruction translations are shown below:

1) Instruction translation 1

The missiles in equilibrium state satisfy the equation:

\[
\begin{align*}
M y &= n y_d \\
M z &= 0
\end{align*}
\]  

(10)

According to equation (10) and aerodynamic data, the expected pitch overload $n y_d$ can be translated to expected attack angle $\alpha_d$.

Then translate $\alpha_d \beta_d \gamma V_d$ to $\theta_d = [\psi_d \gamma_d]$ by equation (11).

\[
\begin{align*}
\sin \alpha_d &= \cos \beta_d \cos \alpha_d \sin \theta + \sin \alpha_d \cos \theta \cos \gamma V_d + \sin \beta_d \cos \alpha_d \cos \theta \sin \gamma V_d \\
\cos \alpha_d \cos \psi_d &= \cos \beta_d \cos \alpha_d \cos \theta \cos \psi V + \sin \alpha_d \left( \sin \psi V \sin \gamma V_d - \sin \theta \cos \psi V \cos \gamma V_d \right) \\
&\quad - \sin \beta_d \cos \alpha_d \left( \sin \psi V \cos \gamma V_d + \sin \theta \cos \psi V \sin \gamma V_d \right) \\
\cos \alpha_d \sin \psi_d &= \cos \beta_d \cos \alpha_d \cos \theta \sin \psi V - \sin \alpha_d \left( \cos \psi V \sin \gamma V_d + \sin \theta \sin \psi V \cos \gamma V_d \right) \\
&\quad + \sin \beta_d \cos \alpha_d \left( \cos \psi V \cos \gamma V_d - \sin \theta \sin \psi V \sin \gamma V_d \right) \\
\cos \alpha_d \cos \gamma_d &= \cos \beta_d \cos \theta \cos \gamma V_d - \cos \beta_d \cos \alpha_d \sin \theta - \sin \beta_d \sin \alpha_d \cos \theta \sin \gamma V_d \\
\cos \alpha_d \sin \gamma_d &= \cos \beta_d \cos \theta \sin \gamma V_d - \sin \beta_d \sin \theta
\end{align*}
\]  

(11)

The following equation need introduce trajectory inclination angle $\theta$ and trajectory deflection angle $\psi V$. Because of the uncertainty of the quadrant of $\psi$ and $\gamma$, each angle needs two equations to determine its sign and magnitude.
2) Instruction translation2
The formula calculating pitch overload $n_y$, sideslip angle $\beta$ and aerodynamic roll angle $\gamma_v$ is as follows:

$$n_y = \frac{P(\sin \alpha \cos \gamma_v + \cos \alpha \sin \beta \sin \gamma_v) + Y \cos \gamma_v - Z \sin (\gamma_v)}{mg}$$

$$\sin \beta = \cos \theta \left[\cos \gamma \sin (\psi - \psi_v) + \sin \theta \sin \gamma \cos (\psi - \psi_v)\right] - \sin \theta \cos \theta \sin \gamma$$

$$\sin \gamma_v = \frac{\cos \alpha \sin \beta \sin \theta - \sin \alpha \sin \beta \cos \gamma \cos \theta + \cos \beta \sin \gamma \cos \theta}{\cos \theta}$$

So the complete closed-loop control system(1) is shown in Figure 2.

The process of building control model has been completed in this section, and the controller is to be designed in the next section.

3 Sliding Mode Controller Design

The essence of design of controller is the calculation of control quantity, which is $\nu$ in this paper. And the sliding mode control(SMC) can calculate a appropriate control quantity which can make system has a good dynamic performance.

SMC not only includes the idea of error feedback, but also includes the idea of designing error reduction process. When there is an error between the expected state and the actual state of control system, the control quantity given by classical PID control just makes the actual state approaching the expected state and the process of convergence cannot be strictly designed and controlled. On the contrary, the control quantity given by SMC not only makes the actual state approaching the desired state, but also makes the process of reaching the expected state is strictly designed, that is, the point representing the actual state of the system approaches the sliding mode surface at the rate of the sliding mode reaching law and finally slides on the sliding mode surface, which is the expected state of the system. In this process, we can flexibly design sliding mode surface and reaching law, of course, the premise is to ensure that the system will eventually stabilize in sliding mode surface. The following is shown in Figure 3.
The sliding mode surface is designed as time-varying sliding mode surface:

\[
S = \begin{bmatrix}
\left( \frac{d}{dt} + \lambda_{\theta} \right) \theta + A\theta e^{-a_{\theta}t} \\
\left( \frac{d}{dt} + \lambda_{\psi} \right) \psi + A\psi e^{-a_{\psi}t} \\
\left( \frac{d}{dt} + \lambda_{\gamma} \right) \gamma + A\gamma e^{-a_{\gamma}t}
\end{bmatrix}
\]  

As for sliding mode reaching law, we adopt the self-defined reaching law:

\[
\dot{S} = \begin{bmatrix}
-\varepsilon_{\theta} \text{sgn}(S_{\theta}) - k_{\theta} \left( e^{S_{\theta}} - 1 \right) \\
-\varepsilon_{\psi} \text{sgn}(S_{\psi}) - k_{\psi} \left( e^{S_{\psi}} - 1 \right) \\
-\varepsilon_{\gamma} \text{sgn}(S_{\gamma}) - k_{\gamma} \left( e^{S_{\gamma}} - 1 \right)
\end{bmatrix}
\]

The parameters above are all positive numbers except \( A_{\theta} \), \( A_{\psi} \), and \( A_{\gamma} \).

Take the derivative of equation(15), and bring the results into equation(16):

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\psi} \\
\dot{\gamma}
\end{bmatrix} = \begin{bmatrix}
\lambda_{\theta} \dot{\theta} \\
\lambda_{\psi} \dot{\psi} \\
\lambda_{\gamma} \dot{\gamma}
\end{bmatrix} - \begin{bmatrix}
A_{\theta} \theta e^{-a_{\theta}t} \\
A_{\psi} \psi e^{-a_{\psi}t} \\
A_{\gamma} \gamma e^{-a_{\gamma}t}
\end{bmatrix} + \begin{bmatrix}
-\varepsilon_{\theta} \text{sgn}(S_{\theta}) - k_{\theta} \left( e^{S_{\theta}} - 1 \right) \\
-\varepsilon_{\psi} \text{sgn}(S_{\psi}) - k_{\psi} \left( e^{S_{\psi}} - 1 \right) \\
-\varepsilon_{\gamma} \text{sgn}(S_{\gamma}) - k_{\gamma} \left( e^{S_{\gamma}} - 1 \right)
\end{bmatrix}
\]

Equation(16) is the sliding mode controller of BTT missile.

Then prove two conclusions and analyse the reason why the sliding mode surface and reaching law are designed as shown above.

Conclusion1: If system(1) adopts equation(16) as the controller, the closed-loop system is globally exponentially stable.

The following is the proof:

Define Lyapunov function as follows:

\[
V = \frac{1}{2} S^{T} S
\]

Obviously \( V \) is positive definite function. Take the derivative of equation(17):

\[
\dot{V} = S^{T} \dot{S} = S_{\theta} \dot{S}_{\theta} + S_{\psi} \dot{S}_{\psi} + S_{\gamma} \dot{S}_{\gamma}
\]

Because the parameters in \( \dot{S} \) are all positive numbers, \( S_{\theta} \dot{S}_{\theta} \leq 0 \), if and only if \( S_{\theta} = 0 \), \( S_{\theta} \dot{S}_{\theta} = 0 \). It has similar conclusions for other two directions of \( S \). So \( \dot{V} \) is negative definite function.
According to Lyapunov second stability criterion, system(1) exists a positive definite function $V$ and $\dot{V}$ is negative definite function, so system(1) is asymptotic stability. And also when $\|\mathbf{S}\|_2 \rightarrow \infty$, $V \rightarrow \infty$. So system(1) is global asymptotic stability.

Conclusion1: If system(1) adopts equation(16) as the controller, the closed-loop system is globally exponentially stable.

Then prove two conclusions and analyse the reason why the sliding mode surface and reaching law are designed as shown above.

The following is the proof:

If

$$A_i = -\lambda_i \cdot e_{i_o}$$

When $t = 0$,

$$S_i = 0 + \lambda_i e_{i_o} + A_i e^{-\alpha \cdot \tau_0} = \lambda_i e_{i_o} - \lambda_i e_{i_o} = 0$$

This lead to

$$V_0 = V\big|_{t=0} = 0$$

In conclusion1, it has been proved that $\dot{V} \leq 0$, and also $V \geq 0$, so $V \equiv 0$. Also because $V = \frac{1}{2} \mathbf{S}^T \mathbf{S}$, so

$$\mathbf{S} \equiv \mathbf{0}$$

It is worth mentioning that $\mathbf{S} \equiv \mathbf{0}$ is the result under the ideal condition. If some practical limitations such as the delay of rudder and the error of numerical calculation are considered, $\mathbf{S}$ is not always $\mathbf{0}$ but changes in a small range around $\mathbf{0}$, which means $\mathbf{S}$ is very close to zero vector.

The following analyses the advantages of time-varying sliding mode surface and self-defined reaching law:

According to conclusion2, when parameter $A_i$ satisfies $A_i = -\lambda_i \cdot e_{i_o}$, the sliding mode control above can guarantee the system state is always on the sliding mode surface, which can eliminate the reaching phase of sliding mode control. This can enhances the global robustness when the system has parametric uncertainty or external disturbance[5].

In this paper, the approach of controlling system(1) is achieved by controlling its subsystem(4), which needs calculating input instruction of subsystem(4) by real-time flight parameters. Being different from the system whose input instruction is imported directly, system(1) would produce a greater degree of oscillation, which presents a high demand on the sliding mode reaching law. Typical reaching law cannot reduce the chattering of SMC. Therefore, the self-defined reaching law is designed as follows:

$$\dot{S}_i = -\varepsilon_i \text{sgn}(S_i) - k_i \left(e^{S_i} - 1\right)$$

where, the switching function $\text{sgn}()$ is replaced by the saturation function $\text{sat}()$ in order to further reduce the chattering:

$$\dot{S}_i = -\varepsilon_i \text{sat}(S_i / \eta_i) - k_i \left(e^{S_i} - 1\right)$$

where, the saturation function $\text{sat}()$ is:

$$\text{sat}(S_i / \eta_i) = \begin{cases} \text{sgn}(S_i) \mid S_i \mid > \eta_i \\ S_i / \eta_i \mid S_i \mid < \eta_i \end{cases}$$

The superiority in reducing the sliding mode chattering by self-defined reaching law comparing to typical reaching law is reflected through simulation in section5.
4 Adaptive control parameters

Different parameters in section 3 have different control effects. It should be divided into two situations:

Situation 1: When the error between the actual state and expected state is biggish, the settling time and overshoot of the step response is determined by parameters $\lambda, a, \epsilon$ and $k$ together. In this situation, the actual state should reach expected state fast, which requires that parameters of the sliding surface and approach law are biggish.

Situation 2: When the actual state is close to expected state, the settling time and overshoot of the step response is mainly determined by $\lambda$ and $a$. In this situation $\epsilon$ and $k$ have little influence on the settling time and overshoot, and they start having great influence on the degree of chattering. If the value of $\epsilon$ and $k$ are still biggish, the chattering phenomenon will be severe. So it requires both $\epsilon$ and $k$ are smaller to reduce the chattering.

Synthesize these two situations and the control parameters can be designed in the following way. The parameters included in sliding mode surface can be designed as fixed value while the parameters included in sliding mode reaching law can be designed as variable value, which means $\epsilon$ and $k$ are the adaptive parameters and their value are depended on the error between the actual state and expected state. Only in this way, it can be guaranteed that not only the controller meets the performance requirements but also the chattering degree is acceptable. The superiority of adaptive parameters comparing to fixed parameters is reflected through simulation in section 5.

And the missile parameters used in this paper’s simulation are given in Table 1.

<table>
<thead>
<tr>
<th>Missile Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>50000</td>
</tr>
<tr>
<td>$m$</td>
<td>500</td>
</tr>
<tr>
<td>$J_x$</td>
<td>7.7</td>
</tr>
<tr>
<td>$J_y$</td>
<td>1500</td>
</tr>
<tr>
<td>$J_z$</td>
<td>1400</td>
</tr>
</tbody>
</table>

After design, the control parameters are given in the Table 2.

<table>
<thead>
<tr>
<th>Control Parameters</th>
<th>Value</th>
<th>Control Parameters</th>
<th>Value</th>
<th>Control Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_g$</td>
<td>0.69</td>
<td>$a_g$</td>
<td>1.5</td>
<td>$k_g$</td>
<td>0.02*coe</td>
</tr>
<tr>
<td>$\lambda_\psi$</td>
<td>0.7</td>
<td>$a_\psi$</td>
<td>4</td>
<td>$k_\psi$</td>
<td>0.05*coe</td>
</tr>
<tr>
<td>$\lambda_\gamma$</td>
<td>0.7</td>
<td>$a_\gamma$</td>
<td>2.5</td>
<td>$k_\gamma$</td>
<td>0.05*coe</td>
</tr>
<tr>
<td>$A_g$</td>
<td>-0.69*$e_{\delta_h}$</td>
<td>$e_g$</td>
<td>1e-4*coe</td>
<td>$\eta_g$</td>
<td>0.005</td>
</tr>
<tr>
<td>$A_\psi$</td>
<td>-0.7*$e_{\psi_0}$</td>
<td>$e_\psi$</td>
<td>1e-4*coe</td>
<td>$\eta_\psi$</td>
<td>0.005</td>
</tr>
<tr>
<td>$A_\gamma$</td>
<td>-0.7*$e_{\gamma_0}$</td>
<td>$e_\gamma$</td>
<td>1e-4*coe</td>
<td>$\eta_\gamma$</td>
<td>0.005</td>
</tr>
</tbody>
</table>

$\text{coe}$ is adaptive coefficient. After design, its optimal value is as follows:

$$\text{coe}=1+\frac{999}{n_\gamma-3.212|n_\gamma-n_{\gamma^*}|}$$ (26)

The number 3.212 is because that according to the aerodynamic data used in this paper, the pitch overload is 3.212 when the attack angle and sideslip angle is zero, the speed is 500 m/s and the air density is 1.17 kg/m$^3$. 

8
5 Simulation

Take guidance commands as \( n_{\gamma_d} = 5 \), \( \beta_d = 0 \), \( \gamma_{\nu_d} = 60^\circ \). And three sets of simulations are performed below.

Simulation1: the response under the condition of adaptive parameters, time-varying sliding mode surface and self-defined reaching law.

Simulation2: the response under the condition of fixed parameters (coe = 1000), time-varying sliding mode surface and self-defined reaching law.

Simulation3: the response under the condition of adaptive parameters, time-varying sliding mode surface and typical reaching law.

It can be found that:

1) The chattering degree of fixed parameters is more severe than the chattering degree of adaptive parameters according to Figure 4 and Figure 5;

2) The chattering degree of typical reaching law is more severe than the chattering degree of self-defined parameters according to Figure 4 and Figure 6;

3) According to Figure 4, when time of simulation is more than 5 seconds, the actual pitch overload will stabilize at 5, the sideslip angle will be less than 0.1° and the aerodynamic roll angle will be more than 58° and stabilize at 60°.

4) During the whole turning process, the maximum of sideslip angle is 1.2°. And this maximum is produced by a great maneuvering condition, which means if the missile is flying under a smaller maneuvering condition, the sideslip angle will be smaller than 1.2°. And in most time of flying, the
guidance command of missile is smoother, so the sideslip angle just has a very little error comparing with zero. So the sideslip angle corresponds to the feature of BTT missile.

Therefore, we can make a conclusion that:
The response speed is fast, the overshoot is small and the sideslip angle maintain near zero under the condition of time-varying sliding mode surface, self-defined reaching law and adaptive parameters.

6 Conclusion

This paper studies the sliding mode flight controller of BTT missile. In the second section, the control model is established according to the general dynamic equation of missile and the dynamic inverse method. Since the control model studied in this paper is derived from the general dynamic equations of missile, the control model is applicable for any missile that flies in the atmosphere, which means what this paper studies has a wider application range in engineering. In the third and fourth sections, the sliding mode controller is designed. The main contents of the design include time-varying sliding mode surface, self-defined reaching law and adaptive control parameters. The time-varying sliding mode surface is designed to satisfy the control performance requirements. Self-defined reaching law and adaptive parameters are used to solve the chattering problem. In the fifth section, three sets of simulations are carried out. The result is the sideslip angle is very close to zero, both the pitch overload and aerodynamic roll angle are able to follow the guidance instructions faster and with small overshoot, and the chattering is also very weak, which are all proves the good performance in the time domain response and the effectiveness of reducing sliding mode chattering. The control method studied in this paper is universally applicable to any BTT missile. That is, for BTT missiles with different aerodynamic layouts, the structure of the sliding mode controller need not be changed and only the control parameters need to be changed.

References

3. Liqun Ma, Chaoyang Duan, Gongping Zhang, 29th CCDC, A(2017)