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Global Existence of the Three Dimensional Heat-conductive Incompressible Viscous Fluids

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Abstract. In this paper, we consider the Cauchy problem of non-stationary motion of heat-conducting incompressible viscous fluids in $\mathbb{R}^3$. About the heat-conducting incompressible viscous fluids, there are many mathematical researchers study the variants systems when the viscosity and heat-conductivity coefficient are positive. For the heat-conductive system, it is difficulty to get the better regularity due to the gradient of velocity of fluid own the higher order term. It is hard to control it. In order to get its global solutions, we must obtain the a priori estimates at first, then using fixed point theorem, it need the mapping is contracted. We can get a local solution, then applying the criteria extension. We can extend the local solution to the global solutions. For the two dimensional case, the Gagliardo-Nirenberg interpolation inequality makes use of better than the three dimensional situation. Thus, our problem will become more difficulty to handle. In this paper, we assume the coefficient of viscosity is a constant and the coefficient of heat-conductivity satisfying some suitable conditions. We show that the Cauchy problem has a global-in-time strong solution $(\boldsymbol{u},\theta)$ on $\mathbb{R}^3\times(0,+\infty)$.

1 Introduction

We continue to consider the Cauchy problem of the system (1), the model of a heat-conducting incompressible viscous fluid is

$$\begin{cases} \partial_t \boldsymbol{u}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}-div(\mu(\theta)\nabla\boldsymbol{u})+\nabla P=0, \\ \partial_t\theta+(\boldsymbol{u}\cdot\nabla)\theta-div(\kappa(\theta)\nabla\theta)-\mu(\theta)\left|\nabla\boldsymbol{u}\right|^2=0, \\ div\,\boldsymbol{u}=0, \\ \boldsymbol{u}\big|_{t=0}=\boldsymbol{u}_0,\quad \theta\big|_{t=0}=\theta_0, \end{cases} \tag{1}$$

with $x\in\mathbb{R}^3$ and $t>0$. Here, the unknowns $\boldsymbol{u},\theta,P$ denote the velocity, temperature and pressure of the fluid, respectively. $\mu(\theta)$ is the coefficient of viscosity, we assume that $\mu(\theta)\equiv\mu$ is a constant, $\kappa(\theta)$ denotes the coefficient of heat conductivity and assumes that functions of temperature satisfying

$$\kappa(\cdot)\in C^1([0,+\infty)),\ 0<\underline{\kappa}\le\kappa(\theta)\le\bar{k},\ \kappa'(\theta)>0,\ \frac{\bar{\kappa}}{\underline{\kappa}}<\kappa_1, \tag{2}$$

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for some positive constant $\mu, \underline{\kappa}, \bar{\kappa}, \kappa_1$.

There are many mathematical investigators for the variants system when the viscosity and heat-conductivity coefficient are positive constants in [1-3]. Benes [1] proved that the system has a local existence in time and global uniqueness of the strong solution in two dimension bounded domain. Kagei [2] considered adding the external force field $f(\theta)$ in the system the first equation of (1) , he give the attractors of the initial-boundary value problem in $\mathbb{R}^2$. Kakizawa [3] proved the exixtence of mild solution for general initial data in local time and global solution for the initial value under a smallness assumption in time. Shilkin [4] considered a simple uni-directional Poiseuille-type flow of a incompressible Newtonian fluid :

$$\begin{cases} \partial_t \boldsymbol{u} - div(\mu(\theta)\nabla \boldsymbol{u}) = f, \\ \partial_t \theta - \Delta\theta = \mu(\theta)\left|\nabla \boldsymbol{u}\right|^2. \end{cases} \tag{3}$$

He proved existence of weak solutions in spatial dimension three and a classical solution on an arbitrary interval of time in two dimension. Bulicek et al [5] investigated the maximal L^2 regularity theory of system (1) under κ is a positive constant and certain structural assumptions of $\mu(\theta)$ without the convective terms and [6] studied $\mu = \mu(\theta, \nabla \boldsymbol{u})$ the regularity properties of unsteady flows in a two dimension periodic establishing under certain structural assumption on the Cauchy stress. However, the system (1) is difficulty to get the better regularity of $\boldsymbol{u}$ and θ, moreover, in the most general case that become much more complication. Very recently, Ye [7] investigated global existence in time under some smallness condition of initial data or the $|\mu'(\theta)|$ is small enough, or the $\underline{\kappa}$ is suitably large in two dimensional case.

In this paper, we aim to study global existence under some smallness condition of initial data or the $\underline{\kappa}$ is suitably large in three dimensional situation.

2 A Priori Estimates

In order to obtain the global existence, the main purpose of this section is to prove the following key a priori estimate. For getting the priori estimates of Lemma 2.3-Lemma 2.5, we give the following two important inequalities.

Lemma 2.1 (**Gronwall's inequality**; see [8])

(i) Let $\eta(\cdot)$ be a nonnegative, absolutely continuous function on $[0,T]$, which satisfies for a.e. t the differential inequality

$$\eta'(t) \le \phi(t)\eta(t) + \psi(t), \tag{4}$$

where $\phi(t)$ and $\psi(t)$ are nonnegative, summable functions on $[0,T]$. Then

$$\eta(t) \le e^{\int_0^T \phi(s)ds}[\eta(0) + \int_0^T \psi(s)ds], \tag{5}$$

for all $0 \le t \le T$.

(ii) In particular, if

$$\eta'(t) \le \phi(t)\eta(t) \text{ on } [0,T] \text{ and } \eta(0) = 0, \tag{6}$$

then

$$\eta(t)=0, \text{ on } [0,T]. \tag{7}$$

Lemma 2.2 (**Gagliardo-Nirenberg Interpolation Inequality** ; see[9], Proposition 3)

Let j,k be any integers satisfying $0 \le j < k$, and let $1 \le S, Q \le \infty$ and $R \in \mathbb{R}$, $\frac{j}{k} \le \theta \le 1$, such that

$$\frac{1}{R}=\frac{j}{k}+\theta(\frac{1}{s}-\frac{k}{n})+(1-\theta)\frac{1}{Q}. \tag{8}$$

Then

1. For any $h\in W^{k,S}(\mathbb{R}^n)\bigcap L^Q(\mathbb{R}^n)$, there is a positive constant C depending only on n,k,j,Q,S,θ such that the following inequality holds :

$$\left|D^j h\right|_R \le C\left|D^k h\right|_S^{\theta}\left|h\right|_Q^{1-\theta} \tag{9}$$

with the following exception: If $1<S<\infty$ and $k-j-\dfrac{n}{S}$ is a nonnegative integer, then (9) holds only for θ satisfying $\dfrac{j}{k}\le\theta<1$.

2. For any $h\in W^{k,S}(\Omega)\bigcap L^Q(\Omega)$ where $\Omega\subset\mathbb{R}^n$ is a bounded domain with smooth boundary,there are two positive constants C_1,C_2 such that the following inequality holds :

$$\left|D^j h\right|_R \le C\left|D^k h\right|_S^{\theta}\left|h\right|_Q^{1-\theta}+C_2\left|h\right|_Q \tag{10}$$

with the same exception as in part 1. In particular, for any $h\in W_0^{k,S}(\Omega)\bigcap L^Q(\Omega)$, the constant C_2 in (10) can be taken as zero.

Lemma 2.3 Assume that $(\boldsymbol{u}_0,\theta_0)\in H^1(\mathbb{R}^3)$. Let $(\boldsymbol{u},\theta)$ be a solution of (1) on $\mathbb{R}^3\times(0,T)$. Then,

$$\sup_{0\le t\le T}\|\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2+2\mu\int_0^T\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2dt=\|\boldsymbol{u}_0\|_{L^2(\mathbb{R}^3)}^2. \tag{11}$$

Proof. Multiplying the first equation of (1) by $\boldsymbol{u}$ and integrating by parts over $\mathbb{R}^3$, we derive that

$$\frac{1}{2}\frac{d}{dt}\|\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2+\mu\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2=0. \tag{12}$$

Integrating (12) with respect to time from 0 to T, we can get (11).

Lemma 2.4 Assume that $(\boldsymbol{u}_0,\theta_0)\in H^1(\mathbb{R}^3)$. Let $(\boldsymbol{u},\theta)$ be a solution of (1) on $\mathbb{R}^3\times(0,T)$ satisfying

$$\int_0^T\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^6dt\le K_1 \text{ and } \int_0^T\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^4dt\le K_2, \tag{13}$$

where $K=\max\{K_1,K_2\}$ is a constant independent of T.

Then,

$$\|\nabla\theta\|_{L^2(\mathbb{R}^3)}^2\le\frac{C}{\underline{\kappa}}\exp\{\frac{1}{\underline{\kappa}^2}\}(\frac{1}{\underline{\kappa}}+\bar{\mu}^2\kappa_1^2) \tag{14}$$

and

$$\int_0^T\|\Delta\theta\|_{L^2(\mathbb{R}^3)}^2dt\le\frac{C}{\underline{\kappa}^3}\exp\{\frac{1}{\underline{\kappa}^4}\}(1+4\mu^2\bar{\kappa}\kappa_1) \tag{15}$$

Proof. We use the similar technique as in [7] to introduce a new quantity $\hat{\theta}=\int_0^\theta\kappa(z)dz$, which satisfies the following Cauchy problem

$$\begin{cases}\partial_t\hat{\theta}+(\boldsymbol{u}\cdot\nabla)\hat{\theta}=\kappa(\theta)\Delta\hat{\theta}+\mu\kappa(\theta)\left|\nabla\boldsymbol{u}\right|^2,\\ \hat{\theta}(x,0)=\int_0^{\theta_0}\kappa(z)dz:=\hat{\theta}_0(x).\end{cases} \tag{16}$$

Multiplying (16) by $\Delta\hat{\theta}$ and then integrating over $\mathbb{R}^3$, we derive from Hölder's inequality and Young inequality that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\left\|\nabla\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2+\left\|(\kappa(\theta))^{1/2}\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2\le\int_{\mathbb{R}^3}\left|\boldsymbol{u}\nabla\hat{\theta}\Delta\hat{\theta}\right|dx+\mu\int_{\mathbb{R}^3}\left|\kappa(\theta)\left|\nabla\boldsymbol{u}\right|^2\Delta\hat{\theta}\right|dx\\
&\le\|\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}\left\|\nabla\hat{\theta}\right\|_{L^4(\mathbb{R}^3)}\left\|\Delta\hat{\theta}\right\|_{L^4(\mathbb{R}^3)}+\mu\bar{\kappa}\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^2\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}\\
&\le\frac{2}{\underline{\kappa}}\|\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^2\left\|\nabla\hat{\theta}\right\|_{L^4(\mathbb{R}^3)}^2+2\mu^2\bar{\kappa}\kappa_1\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^4+\frac{\underline{\kappa}}{4}\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2\\
&\le\frac{C_1}{\underline{\kappa}}\|\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^{\frac{1}{2}}\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^{\frac{3}{2}}\left\|\nabla\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^{\frac{1}{2}}\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^{\frac{3}{2}}+2\mu^2\bar{\kappa}\kappa_1\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^4+\frac{\underline{\kappa}}{4}\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2\\
&\le\frac{C_2}{\underline{\kappa}^4}\|\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^6\left\|\nabla\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2+2\mu^2\bar{\kappa}\kappa_1\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^4+\frac{\underline{\kappa}}{2}\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2
\end{aligned}\tag{17}$$

This combined with (11) and Gronwall's inequality implies that

$$\begin{aligned}
&\sup_{0\le t\le T}\left\|\nabla\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2+\underline{\kappa}\int_0^T\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2dt\\
&\le\exp\left\{\frac{2C_2}{\underline{\kappa}^4}\|\boldsymbol{u}_0\|_{L^2(\mathbb{R}^3)}^2\int_0^T\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^6dt\right\}\left(\left\|\nabla\hat{\theta}_0\right\|_{L^2(\mathbb{R}^3)}^2+4\mu^2\bar{\kappa}\kappa_1\int_0^T\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^4dt\right)\\
&\le C\exp\{\frac{1}{\underline{\kappa}^4}\}(1+4\mu^2\bar{\kappa}\kappa_1).
\end{aligned}\tag{18}$$

By virtue of $\nabla\theta=\frac{1}{\kappa(\theta)}\nabla\hat{\theta},\Delta\theta=\frac{1}{\kappa(\theta)}\Delta\hat{\theta}-\frac{\kappa'(\theta)}{\kappa^3(\theta)}\left|\nabla\hat{\theta}\right|^2\le\frac{1}{\underline{\kappa}}\Delta\hat{\theta}$ and (2), (18), we have

$$\|\nabla\theta\|_{L^2(\mathbb{R}^3)}^2\le\frac{C}{\underline{\kappa}^2}\exp\left\{\frac{1}{\underline{\kappa}^2}\right\}(1+\bar{\mu}^2\bar{\kappa}\kappa_1)\le\frac{C}{\underline{\kappa}}\exp\left\{\frac{1}{\underline{\kappa}^2}\right\}(\frac{1}{\underline{\kappa}}+\bar{\mu}^2\kappa_1^2),\tag{19}$$

and

$$\int_0^T\|\Delta\theta\|_{L^2(\mathbb{R}^3)}^2dt\le\frac{1}{\underline{\kappa}^2}\int_0^T\left\|\Delta\hat{\theta}\right\|_{L^2(\mathbb{R}^3)}^2dt\le\frac{1}{\underline{\kappa}^3}\exp\left\{\frac{1}{\underline{\kappa}^4}\right\}(1+4\mu^2\bar{\kappa}\kappa_1).\tag{20}$$

Lemma 2.5 Assume that $(\boldsymbol{u}_0,\theta_0)\in H^1(\mathbb{R}^3)$. Let $(\boldsymbol{u},\theta)$ be a solution of (1) on $\mathbb{R}^3\times(0,T)$ satisfying (13). Then, we have the following energy inequality

$$\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2+\mu\int_0^T\|\Delta\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2dt\le 2C_5K+\|\nabla\boldsymbol{u}_0\|_{L^2(\mathbb{R}^3)}^2.\tag{21}$$

Proof. Multiplying the first of (1) by $\Delta\boldsymbol{u}$ and integrating by parts over $\mathbb{R}^3$, we derive that

$$-\int_{\mathbb{R}^3}\nabla\boldsymbol{u}\cdot\nabla\boldsymbol{u}_t dx+\int_{\mathbb{R}^3}\Delta\boldsymbol{u}\cdot\boldsymbol{u}\cdot\nabla\boldsymbol{u}dx-\mu\int_{\mathbb{R}^3}|\Delta\boldsymbol{u}|^2dx+\int_{\mathbb{R}^3}\Delta\boldsymbol{u}\cdot\nabla Pdx=0,\tag{22}$$

And using Hölder's inequality and Young inequality, we have

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2+\mu\|\Delta\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2\le\int_{\mathbb{R}^3}|\Delta\boldsymbol{u}||\boldsymbol{u}||\nabla\boldsymbol{u}|dx\\
&\le\|\boldsymbol{u}\|_{L^6(\mathbb{R}^3)}\|\nabla\boldsymbol{u}\|_{L^3(\mathbb{R}^3)}\|\Delta\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}\le C_3\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}\|\nabla\boldsymbol{u}\|_{L^3(\mathbb{R}^3)}\|\Delta\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}\\
&\le C_4\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^{\frac{3}{2}}\|\Delta\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^{\frac{3}{2}}\le\frac{\mu}{2}\|\Delta\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2+C_5\|\nabla\boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^6
\end{aligned}\tag{23}$$

That is

$$\frac{d}{dt}\|\nabla \boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2 + \mu\|\Delta \boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^2 \le 2C_5\|\nabla \boldsymbol{u}\|_{L^2(\mathbb{R}^3)}^6 \tag{24}$$

We apply Gronwalls inequality to (15) obtain (14).

3 Proof of Theorem 3.2

We purpose is to obtain the estimate about $\|\theta\|_{L^\infty(0,T;L^2(\mathbb{R}^3))}$. We give the following lemma.

Lemma 3.1 Assume that $(\boldsymbol{u}_0,\theta_0)\in H^1(\mathbb{R}^3)$ and (2), (13) hold. Let $(\boldsymbol{u},\theta)$ be a solution of (1) on $\mathbb{R}^3\times(0,T)$. Then

$$\sup_{0\le t\le T}\|\theta\|_{L^2(\mathbb{R}^3)}^2 + \int_0^T\|\nabla\theta\|_{L^2(\mathbb{R}^3)}^2\,dt \le C. \tag{25}$$

Proof. Multiplying the second equation of (1) with θ, we have

$$\int_{\mathbb{R}^3}\partial_t\theta\cdot\theta\,dx + \int_{\mathbb{R}^3}\boldsymbol{u}\cdot\nabla\theta\cdot\theta\,dx - \int_{\mathbb{R}^3}div(\kappa(\theta)\nabla\theta)\cdot\theta\,dx - \mu\int_{\mathbb{R}^3}|\nabla\boldsymbol{u}|^2\cdot\theta\,dx = 0. \tag{26}$$

We apply integrating by parts over $\mathbb{R}^3$, one has

$$\frac{1}{2}\frac{d}{dt}\|\theta\|_{L^2(R^3)}^2 + \int_{R^3}\kappa(\theta)|\nabla\theta|^2\,dx \le \mu\int_{\mathrm{R}^3}|\nabla u|^2\cdot\theta\,dx \tag{27}$$

That is

$$\begin{aligned}\frac{d}{dt}\|\theta\|_{L^2(\mathbb{R}^3)}^2 + 2\underline{\kappa}\int_{\mathbb{R}^3}|\nabla\theta|^2\,dx &\le 2\mu\int_{\mathbb{R}^3}|\nabla\boldsymbol{u}|^2\cdot\theta\,dx\\ &\le 2\mu\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^2\|\theta\|_{L^2(\mathbb{R}^3)}\\ &\le \mu^2\|\nabla\boldsymbol{u}\|_{L^4(\mathbb{R}^3)}^4 + \|\theta\|_{L^2(\mathbb{R}^3)}^2\end{aligned} \tag{28}$$

By the Gronwall's inequality and (18), we have

$$\|\theta\|_{L^2(\mathbb{R}^3)}^2 + 2\underline{\kappa}\int_0^T\|\nabla\theta\|_{L^2(\mathbb{R}^3)}^2\,dt \le e^T\left(\|\theta\|_{L^2(\mathbb{R}^3)}^2 + \mu^2 K\right). \tag{29}$$

The proof of Lemma 3.1 is finished.

Theorem 3.1 (Local existence) Assume that $(\boldsymbol{u}_0,\theta_0)\in H^1(\mathbb{R}^3)$ and (2), (13) hold. Then there exists a small time classical solution $(\boldsymbol{u},\theta)$ to the Cauchy problem (1) such that

$$(\boldsymbol{u},\theta)\in L^2(0,T;H^2(\mathbb{R}^3)),\qquad (\nabla\boldsymbol{u},\nabla\theta)\in L^\infty(0,T;L^2(\mathbb{R}^3)). \tag{30}$$

Theorem 3.2 (Global existence) Assume that $(\boldsymbol{u}_0,\theta_0)\in H^1(\mathbb{R}^3)$ and (2), (13) hold. Then there is a global strong solution $(\boldsymbol{u},\theta)$ with the following regularity

$$(\boldsymbol{u},\theta)\in L^2(0,+\infty;H^2(\mathbb{R}^3)),\quad (\nabla\boldsymbol{u},\nabla\theta)\in L^\infty(0,\infty;L^2(\mathbb{R}^3)). \tag{31}$$

Proof. With all the priori estimates established at hand, due to Theorem 3.1, there exists a T_0 such that the system (1) has a local strong solution $(\boldsymbol{u},\theta)$ on $[0,T_0]$. We expect to extend the local solution to global one. We know that there exists a positive time $T_1\in(0,T_0]$ such that (13) hold for $T=T_1$. Set

$$\overline{T}:\sup\left\{T\middle|\,(13)\text{ hold}\right\}. \tag{32}$$

It is clear that $\overline{T}\ge T_1$. Next, we claim that $\overline{T}=+\infty$.

Otherwise, $\bar{T} < \infty$. Then it follows from Lemma2.3-Lemma2.5 and (13) hold for all $0 < T < \bar{T}$. So, there exist some $\tilde{T} > \bar{T}$ such that (13) hold for all $0 < T < \tilde{T}$, which contradicts (32). Thus, $\bar{T} = \infty$. We can obtain $(\boldsymbol{u}, \theta)$ is a strong solution from Lemma 2.3-Lemma 2.5 on $\mathbb{R}^3 \times (0, \infty)$.

4 Conclusion

With all the priori estimates established at hand, we can obtain the global existence by extend the time. Meanwhile, we use the Gagliardo-Nirenberg interpolation inequality and some skills to obtain the higher regularity of $(\boldsymbol{u}, \theta)$.

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