

Dynamic model elaboration for the planetary-precessional gear reducer against vibration

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Abstract. The purpose of modern-day machines in general and gear-box in particular is to carry out needs-based on maintenance, good working conditions and to minimize the repair and other servicing downtimes and disfunctionalities. This increases the overall equipment effectiveness and optimizes the cost. The mechanical condition of a machine or a machine components can be accurately diagnosed from the nature and extend of vibration they generate. The aim is to detect damage as it occurs, allowing scheduled repairs or maintenance to be carried out in time.

1 Vibration generating sources in the planetary-precessional gear

Precessional transmission is a mechanical system subjected to mechanical oscillations, generating vibrations and noise, and can be divided into the following groups [1]:

- a) Dimensional deviations of sliding contact elements;
- b) Dimensional deviations in machine parts that make rolling contact;
- c) Imbalance, when the center of mass of a rotating or precession element does not coincide with the center of rotation (or precession) [2];
- d) Disassembly - deviation from the collinearity of the axles of the electric motor shaft, the precession reducer and the brake, deviation which occurs at normal operating temperature;
- e) Vibration resulting from assembly errors [3].

2 The dynamic model of the planetary precessional reducer elastically bonded against forced vibrations

If we examine the dynamic model of the planetary precessional reducer, we conclude that the reducer is a rigid system with a vertical plane of symmetry. As with any other gear transmission, precessional planetary transmissions (later on the textPPT) are excited by various disturbing forces.

Figure 1 shows the 2D-simplified planetary 2K-H reducer simplified model. A summary analysis shows that, in general, the reducer consists of two cylinders with diameters D_1 , D_2 .

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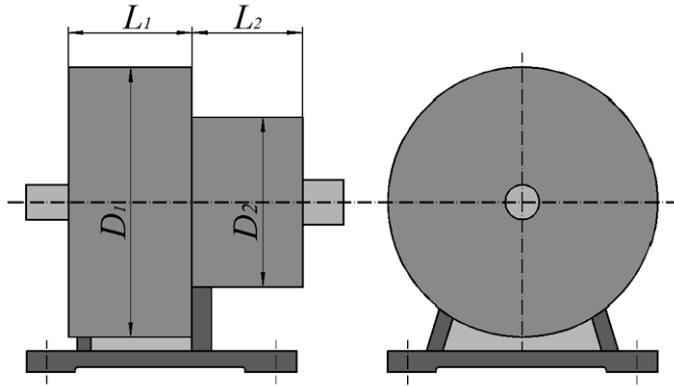


Fig. 1. Simplified 2K-H planetary precessional transmission model.

In this order, the planetary precessional reducer 2K-H can be approximated to a cylinder of diameter D_c and length L_c , like is shown in Figure 2. [4]

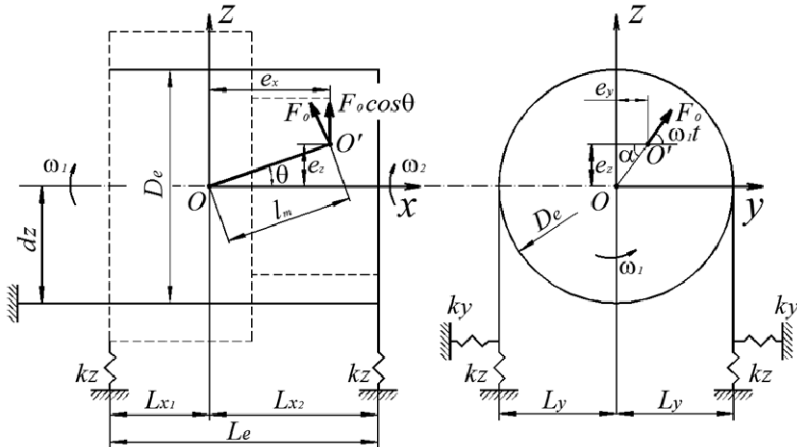


Fig. 2. 2K-H planetary precessional reducer simplified dynamic model.

This simplified model is characterized by the following features:

- the precessional planetary reducer is considered a solid body with two Oxy and Oxz symmetry planes;
- the resilience is performed in four points, each elastic system being characterized by two orthogonal elastic elements, having the rigidities kz and ky along the axes Ox and Oz ;
- the excitation of stationary forced vibrations have an inertial type due to an eccentric mass system (satellite block and crank shaft) with the rotation axis inclined with the angle θ from the main axle of the transmission (Ox axis) passing through the O' point displaced by the center the precession " O " at the distances e_y and e_z in the Oyz plane and e_x - in the Oxz plane.

$$\begin{cases} F_x(t) = F_0 \sin \theta \cos \omega_1 t = m_0 r \omega_1^2 \sin \theta; \\ F_y(t) = F_0 \cos \theta \cos \omega_1 t; \\ F_z(t) = F_0 \cos \theta \sin \omega_1 t = m_0 r \omega_1^2 \cos \theta. \end{cases} \quad (1)$$

The total static momentum of the eccentric masses is m_0r and the rotating disturbing force has the application point O' offset from the center of precession "O" at the distances e_x , e_y and e_z .

The external excitation forces acting on PPT are presented in the expression (1). Correspondingly, the external excitation moments are:

$$\begin{cases} M_{0x}(t) = (e_z \cos \omega_1 t - e_y \sin \omega_1 t)F_0 \cos \theta; \\ M_{0y}(t) = F_0 \cos \theta \cos \omega_1 t; \\ M_{0z}(t) = F_0 \cos \theta \sin \omega_1 t. \end{cases} \quad (2)$$

The moment $M_{0x}(t)$ can also be written in the form of $M_{0x}(t) = M_0 \sin(\omega_1 t - \alpha)$ where $M_0 = F_0 \cdot y = m_0 r \omega_1^2 \varphi$ is the amplitude of the excitatory moment, and α is the phase angle of the excitatory force vector:

$$\alpha = \operatorname{arctg} \frac{e_z}{e_y} = \operatorname{arctg} \frac{l_m \sin \theta}{l_m \sin \theta \sin \omega_1 t} = \operatorname{arctg} \frac{1}{\sin \omega_1 t}. \quad (3)$$

In this case summary excitation moment will be:

$$\begin{aligned} M_\Sigma &= M_{0x}(t) + M_{0y}(t) + M_{0z}(t) = F_0(e_z \cos \omega_1 t - e_y \sin \omega_1 t) + \\ &+ F_0 e_x \cos \theta \cos \omega_1 t + F_0 e_z \cos \theta \sin \omega_1 t = \\ &= F_0 l_m \cos \theta \left[\sin \theta \cos \omega_1 t - \operatorname{tg} \theta \sin^2 \omega_1 t + \cos \theta \cos \omega_1 t + \sin \theta \sin \omega_1 t \right] = \\ &= F_0 l_m \cos \theta \left[\sin(\theta - \omega_1 t) + \cos(\theta - \omega_1 t) - \cos \theta \sin \omega_1 t \left(1 + \frac{\sin \theta \sin \omega_1 t}{\cos^2 \theta} \right) \right]. \end{aligned} \quad (4)$$

2.1 Vibrations in the precession reducer with a vertical plane of symmetry

Regarding simplified scheme of the dynamic PPT model (Figure 2), the dynamic model is characterized by three degrees of freedom y , z , φ corresponding to movements (y , φ_x) and vertical displacement z .

Differential equations of motion are:

$$\begin{cases} m\ddot{Y} + 4k_x Y - 4k_y a_z \varphi_x = F_0 \cos \theta \sin \omega_1 t; \\ J_x \ddot{\varphi}_x + 4(k_z a_y^2 + k_y a_z^2) \varphi_x - 4k_y a_z Y = F_0 \sin \theta e \sin(\omega_1 t - \alpha); \\ m\ddot{Z} + 4k_z Z = F_0 \sin \theta \sin \omega_1 t. \end{cases} \quad (5)$$

2.1.1 Own vibrations

For movements (Y , φ_x), the vibration equations are:

$$\begin{cases} m\ddot{Y} + 4k_y Y - 4k_z a_z \varphi_x = 0; \\ J_x \ddot{\varphi}_x + 4(k_z a_y^2 + k_y a_z^2) \varphi_x - 4k_y a_z Y = 0. \end{cases} \quad (6)$$

The solutions with following form $Y = Y_0 \sin pt$, $\varphi_x = \varphi_0 \sin pt$ and

$\ddot{Y} = -p^2 Y_0 \sin pt$, $\ddot{\varphi}_x = -p^2 \varphi_0 \sin pt$, lead to the system of equations expressed by Y_0 and

$$\varphi_0: \begin{cases} (-mp^2 - \tilde{k}_y) Y_0 - \tilde{k}_x a_z \varphi_0 = 0; \\ -\tilde{k}_y a_z Y_0 + (-J_x p^2 + \tilde{k}_z a_y^2 + \tilde{k}_y a_z^2) \varphi_0 = 0, \end{cases} \quad (7)$$

$$\text{Where: } \begin{cases} \tilde{k}_x = \Sigma k_y = 4k_y; \\ \tilde{k}_z = \Sigma k_z = 4k_z; \\ \omega_z^2 = \frac{k_z}{m}; \end{cases} \quad \begin{cases} J_x = m \rho_x^2 = \tilde{k} \frac{\rho_x^2}{\omega_z}; \\ J_x m = \tilde{k}_z^2 \frac{\rho_x^2}{\omega_z^2}, \end{cases} \quad (8)$$

and ω_z is the own pulsation of the precessional planetary reducer, $\rho_x = l_m - \sin\theta$ the radius of the body with mass m .

The compatibility condition is that the determinant of the equation system is null

$$m J_x p^4 - m(\tilde{k}_z a_y^2 - \tilde{k}_y a_z^2) p^2 - J_x \tilde{k}_y p^2 + \tilde{k}_y \tilde{k}_z a_y^2 = 0. \quad (9)$$

2.1.2 Forced vibrations

The dynamic response of the precession reducer in the stabilized mode, for known ω_l , is described with the expressions of vibration amplitudes as exciting pulse ω_l functions:

- amplitude of linear vibrations on the y axis:

$$A_y(\omega_1) = \frac{m_0 r \omega_1^2}{\tilde{k}_z} \cdot \frac{B(\omega_1)}{D(\omega_1)}, \quad (10)$$

Where B and D are determined with the relations:

$$B(\omega_1) = \sqrt{\left[\frac{\tilde{k}_y}{\tilde{k}_z} \cdot \frac{a_z}{\rho_x} \left(\frac{a_z}{\rho_x} - \frac{e_z}{\rho_x} \right) + \frac{a_y^2}{\rho_x^2} - \frac{\omega_1^2}{\omega_z^2} \right]^2 + \left[\frac{\tilde{k}_y}{\tilde{k}_z} \cdot \frac{e_y}{\rho_x} \cdot \frac{a_z}{\rho_x} \right]^2} \quad (11)$$

And :

$$D(\omega_1) = \left[\frac{\omega_1}{\omega_z} \right]^4 - \left[\frac{\omega_1}{\omega_z} \right]^2 \left[\frac{a_y^2}{\rho_x^2} + \frac{\tilde{k}_y}{\tilde{k}_z} \cdot \frac{a_z^2}{\rho_x^2} + \frac{\tilde{k}_y}{\tilde{k}_z} \right] + \frac{\tilde{k}_y}{\tilde{k}_z} \cdot \frac{a_y^2}{\rho_x^2}; \quad (12)$$

- the magnitude of the angular vibrations around the Ox axis (pitching) determined with the relation:

$$A_{\varphi}(\omega_1) = \frac{m_0 r \omega_1^2 C(\omega_1)}{\tilde{k}_z \cdot \rho_x \cdot D(\omega_1)}, \tag{13}$$

Where C is determined by the relationship

$$C(\omega_1) = \sqrt{\left[\frac{\tilde{k}_y}{\tilde{k}_z} \left(\frac{a_z}{\rho_x} - \frac{e_z}{\rho_x} \right) + \frac{e_z}{\rho_x} \cdot \frac{\omega_1^2}{\omega_z^2} \right]^2 + \left[\frac{e_y}{\rho_x} \left(\frac{\tilde{k}_y}{\tilde{k}_z} - \frac{\omega_1^2}{\omega_z^2} \right) \right]^2}; \tag{14}$$

- the amplitude of linear vibrations along the Oz axis:

$$A_z = \frac{m_0 r \omega_1^2}{\tilde{k}_z} \cdot \frac{1}{1 - \frac{\omega_1^2}{\omega_z^2}}. \tag{15}$$

3 Identifying resonant frequencies

To analyze the vibrations of precessional planetary transmissions, which are actually more or less complex mechanical systems, the energy and vibration sources on which this energy is propagated must be taken into account. Energy is always transferred as energy flows from the vibration source to the energy absorber where it is converted to heat. In some cases, the propagation path may be very short, but in others the energy flow can be propagated over relatively long distances before it is absorbed.

An oscillating mechanical system can be excited by harmonic, periodic or random forces. The oscillating mechanical system vibrates in a certain way, at each of these oscillations its movement can be characterized by a given number of parameters, the determination of the values of which means finding out the response of the whole system to the excitation to which it is subjected. The response will depend on the characteristics of the oscillating system and the excitation parameters.

Resonance occurs when the frequency of an excitation force applied to a system coincides with the system's own frequency, causing a great amplification of system vibrations. This gain depends on system damping.

Own frequencies can be excited by a harmonic oscillation if it has enough energy to excite the system and if its frequency has a value around its own frequency.

The resonance phenomenon is recognized by the large amplitude variation and the phase angle change at low speed variations, which depend on system damping and can be determined in several ways:

- by calculation using modal analysis by computer programs, where the resonance frequencies obtained are not accurate and their values are very close to the real ones;
- by practical test with a hammer, when the mechanical structure is struck and at the same time the impact response of the system is measured, followed by the analysis of the obtained data, which usually results in the first resonance frequency;

- by actual measurements, by analyzing vibration data measured during start-up or shutdown of the machine being studied, in stationary operation at a speed higher than the speed or critical speeds.

4 Constructive-technological solutions for noise and vibration reduction

Practically in vibro-acoustical diagnostics [8,9] there are various methods to minimize the vibration and noise level in the technical and energetic machines and devices. In the other way the mechanical transmissions used in various machines, installations are sources of vibration and high frequency noise. The most efficient but also the most expensive method for obtaining silent transmissions is the method of processing the parts with very high precision or the static and dynamic balancing of the moving parts.

As conclusion most simpler and cheapest is constructivist-technological methods such as:

- a) Manufacture of a plastic gear or sintered metallic powder wheel [5,7];
- b) Compensation of manufacturing and assembly errors [3,10];
- c) Use of intermediate elastic thin- layered rubber metal located between the contact surfaces of the gear [6];

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