

Geometrical analysis, for rapid prototyping, of the compressor's helical conic rotor model

Nicușor Baroiu, Viorel Păunoiu, Virgil Gabriel Teodor*, Florin Susac and Nicolae Oancea

“Dunarea de Jos” University of Galati, Department of Manufacturing Engineering, Domneasca 111, Galati, Romania

Abstract. The conical rotary compressor represents a solution for gases compression, for relatively small flow, in conditions of a superior energetic efficiency. Although the theoretical solution for the helical conical compressor was elaborated decades ago, a viable technology solution become possible at industrial level only after the development of machine tools with numerically controlled axes. In this paper is proposed a study, analytical and graphical, concerning the geometry of conical rotary compressor's rotors, modelled by rapid prototyping technology. The propose modelling allow to elaborate various constructive forms for conical compressor's rotors.

1 Introduction

The conical rotary compressor [1] represents a solution for gases compression, in the domain of low flows and high pressures [2]. The constructive solution for such compressor with active elements of the conical helix surfaces has been known for over half a century, but the industrial applications of these constructions become possible, from technological point of view, only after development of machine tools with several numerically controlled axis [1]. The effectiveness for this compressor type was theoretically and experimentally proven, from the energy point of view [2], [3].

An analytical and graphical modelling is proposed in this paper. Also, a 3D model by rapid prototyping for the worms was made.

2 Analytical form of conical rotary compressor — crossing sections

For the crossing section of driving rotor of the helical conic compressor is accepted a form composed from an assembly of curves, epicycloids—hypocycloid, see figure 1. The reference systems are defined: xy is the fixed reference system, with origin onto the rotor's axis, $x_e y_e$ — auxiliary fixed reference system, joined with the r_e roulette axis; $x_i y_i$ — auxiliary fixed reference system joined with the r_i roulette axis; $X_e Y_e$ — mobile reference system, joined with r_e roulette and $X_i Y_i$ — mobile reference system joined with r_i roulette.

*Corresponding author: virgil.teodor@ugal.ro

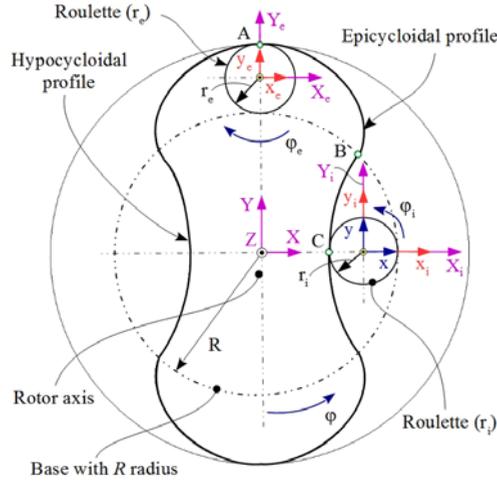


Fig. 1. Crossing profile of helical conic rotor.

The rotor's crossing profile is composed from two cycloid curves, generated by points *A* and *C*, see figure 1. The *A* point has coordinates, in $X_e Y_e$ Cartesian reference system:

$$X_e = 0; \quad Y_e = r_e, \tag{1}$$

And point *C* has coordinates:

$$X_i = -r_i; \quad Y_i = 0. \tag{2}$$

The relative motion of the mobiles spaces is given by:

$$X = \omega_3(\varphi) \left[\omega_3^T(-\varphi_e) \cdot X_e + A \right], \tag{3}$$

With $A = (0 \quad -(R+r_e))^T$

After replacements, the equations of the epicycloid in XY mobile reference system are, for $r_e = r_i = r$:

$$\begin{cases} X = r \cdot \sin(\varphi + \varphi_e) - (R+r) \cdot \sin \varphi; \\ Y = r \cdot \cos(\varphi + \varphi_e) - (R+r) \cdot \cos \varphi. \end{cases} \tag{4}$$

* From condition $X^2 + Y^2 = R^2$, (R – basis radius), (5)

The limit for angular parameter results

$$\varphi_e = \arccos \left[\frac{R^2 - (R+r)^2 - r^2}{2r \cdot (R+r)} \right]. \tag{6}$$

** Between φ and φ_e angular parameters exists the relation:

$$\varphi = (r/R) \cdot \varphi_e. \tag{7}$$

Similarly, the BC flank is described.

3 Driven worm helical conic flank's equations

The conic worm assumes a basis with variable radius, R , starting from value R_0 in frontal plane. The results, see also figure 2:

$$R = R_0 - 2 \cdot p \cdot \varphi_1 \cdot \operatorname{tg} \alpha \tag{8}$$

Where α is the cone's generatrix angle; p is helical parameter of conical worm; φ_1 is the twisting angle in helical motion of the conic rotor's crossing profile and R_0 is basis radius.

We assumes that the roulette radius $r_e = r_i = r$ is a variable value.

For $r = 1/4 \cdot R$ results $r = 1/4 \cdot [R_0 - p \cdot \varphi_1 \cdot \operatorname{tg} \alpha]$, with R_0 , α and p constructive constants values. In this way, in $X_1Y_1Z_1$ reference system, see figure 2, the generating motion of helical conic worm is given by:

$$X_1 = \omega_3^T(\varphi_1) \cdot X - p \cdot \varphi_1 \cdot \bar{k}. \tag{9}$$

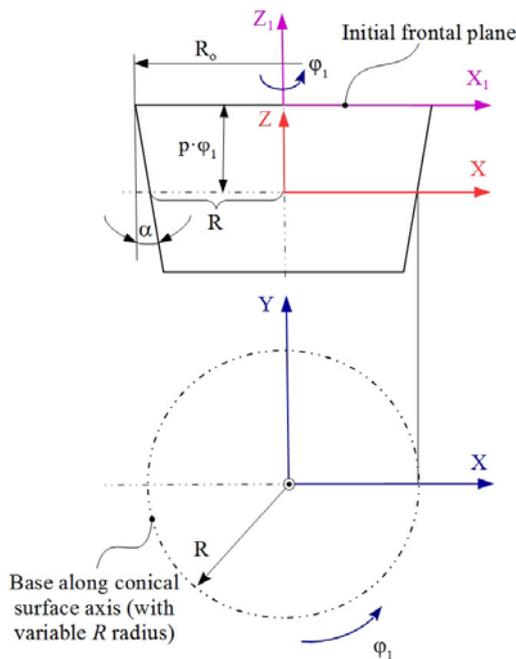


Fig. 2. Conical worm generatrix.

After replacements of cycloid profiles,(4) the Σ_{AB} helical conic surface's equations result:

$$\Sigma_{AB} \begin{cases} X_1 = [r \cdot \sin(\varphi + \varphi_e) - (R + r) \cdot \sin \varphi] \cdot \cos \varphi_1 - [r \cdot \cos(\varphi + \varphi_e) - (R + r) \cdot \cos \varphi] \cdot \sin \varphi_1; \\ Y_1 = [r \cdot \sin(\varphi + \varphi_e) - (R + r) \cdot \sin \varphi] \cdot \sin \varphi_1 + [r \cdot \cos(\varphi + \varphi_e) - (R + r) \cdot \cos \varphi] \cdot \cos \varphi_1; \\ Z_1 = -p \cdot \varphi_1, \end{cases} \tag{10}$$

Right hand worm, for conical cycloid flank \square_{AB} . Similarly for the Σ_{BC} surface.

4 Driven worm flank's equations — crossing sections

They are defined the reference systems: XYZ is the mobile reference system, joined with the internal (driving) worm, with origin in O_i ; $X_1Y_1Z_1$ — mobile reference system, joined with the external (driven) worm, with origin in O_e ; $x_1y_1z_1$ — fixed reference system joined with the external worm axis' base and xyz — fixed reference system, joined with internal worm axis base, see figure 3. The ratio between R_e and R (gearing ratio) is:

$$i_{12} = (R_e / R) = 3 / 2 \tag{11}$$

The angles of rotation motions around the bases axes, ψ_1, ψ_2 :

$$R_e \cdot \psi_2 = R \cdot \psi_1 \rightarrow i = \psi_1 / \psi_2 = R_e / R \tag{12}$$

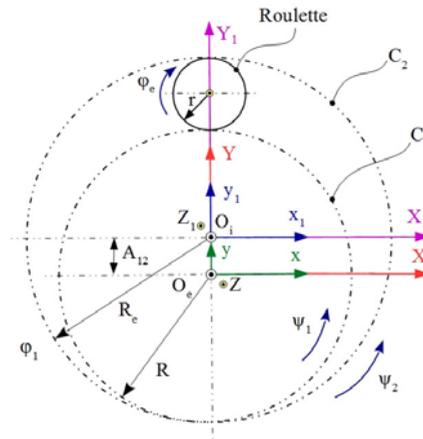


Fig. 3. Rolling centredes, C_1 and C_2 — identically with bases circles with radii R and R_e of compressor's rotors, in crossing section.

The relative motion between the two centredes is defined:

$$X_1 = \omega_3(\psi_2) \cdot [\omega_3^T(\psi_1) \cdot X + B], \quad B = (0 \quad -A_{12} \quad 0)^T. \tag{13}$$

If, in (13), the X matrix represents geometric locus of cycloid profiles \overline{AC} , equation (4), with remark $r_e = r$, then transformation (13) represents the cycloid curves' family, in $X_1Y_1Z_1$ reference system. So, from (4) and (13), the surface generated by \overline{AB} arc results:

$$\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = \omega_3(-\psi_2) \cdot \omega_3^T(\psi_1) \cdot \begin{pmatrix} r \cdot \sin(\varphi + \varphi_e) - (R + r) \cdot \sin \varphi \\ r \cdot \cos(\varphi + \varphi_e) - (R + r) \cdot \cos \varphi - A_{12} \\ 0 \end{pmatrix}, \tag{14}$$

In $X_1Y_1Z_1$ reference system joined with compressor's driven worm.

Similarly, for surface generated by arc \overline{BC} .

Joining these equations with the enwrapping condition specific for one of the complementary methods, for example method of "minimum distance", [4], the profiles in crossing sections of driving worms are determined.

In figure 4, the relative positions between driving and driven worms are presented.

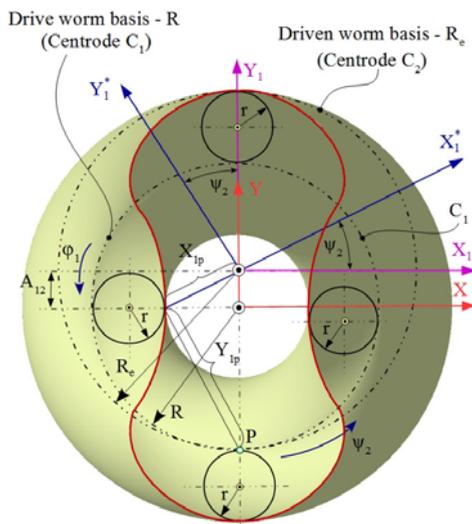


Fig. 4. Rolling centres C_1, C_2 – the method of “minimum distance”.

In figure 4, is presented a position (for a value of ψ_2 angle) of $X_1Y_1Z_1$ reference system, in rolling time. In this position, the coordinates of P gearing pole, in $X_1Y_1Z_1$ reference system, are:

$$X_{1p} = -R_e \cdot \sin \psi_2; Y_{1p} = -R_e \cdot \cos \psi_2. \tag{15}$$

In this way, the distance from the current point on the cycloid profile to the gearing pole, expressed in $X_1Y_1Z_1$ reference system is:

$$d = \sqrt{(X_1 + R_e \cdot \cos \psi_2)^2 + (Y_1 + R_e \cdot \sin \psi_2)^2}, \tag{16}$$

with X_1, Y_1 from (14). The minimum value of this distance is obtained from condition:

$$d_{\min} = \left(\frac{\partial d}{\partial \psi_2} \right) = 0. \tag{17}$$

So, the condition (17) become:

$$(X_1 + R_e \cdot \cos \psi_2) \cdot X_{\psi_2} + (Y_1 + R_e \cdot \sin \psi_2) \cdot Y_{\psi_2} = 0, \tag{18}$$

Representing the enveloping condition, specific for the method of “minimum distance”, [4].

The (16) and (17) equations assembly, and condition (18) applied for \overline{AB} and \overline{BC} profile, see figure 1, represents the crossing profile of external worm, in $X_1Y_1Z_1$ reference system. Regarding the definition of roulette radius, $r_e = r$ and $r_i = r$, consecutive sections of driven worm can be determined along the driving worm’s axis, see figure 4.

Also, considering the gearing ratio between the two centrodes with radii R and R_e ,

$$i_{\psi_1, \psi_2} = \psi_2 / \psi_1 = R / R_e. \tag{19}$$

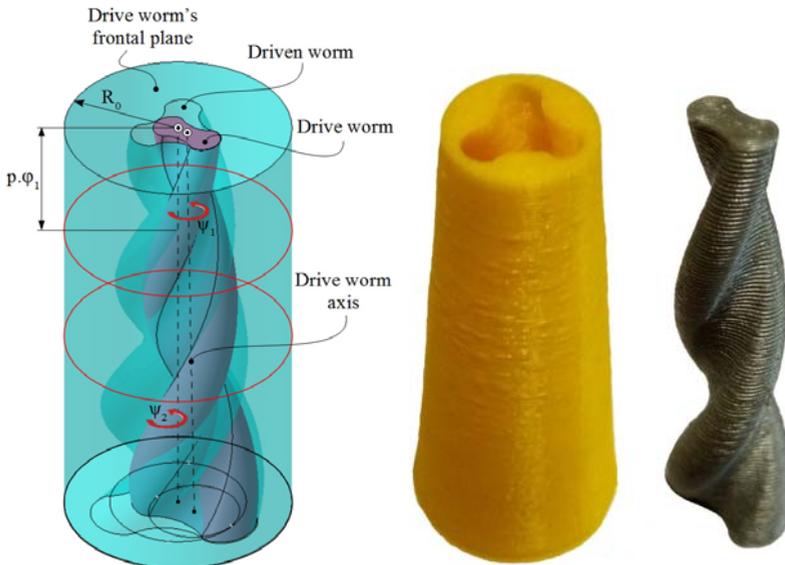


Fig. 5. Positions of three-lobed surface's center against the driving worm's revolving axis; 3D print of the two worms.

5 Conclusions

The paper presents the model of a conical cycloid worm determined by analytical geometry. At the same time, based on the complementary theorem of the surface enwrapping, the “minimum distance” method [4], has determined the conjugate worm within the conical helical compressor.

Based on the analytical models of the two worms the solid models of the two worms representing the movable elements of the conical helical compressor were made in CATIA.

These solid models were exported as “*stl*” file format and were printed on a 3D printer, model Ultimaker3 Extended, using Cura slicing software.

In the future, it is intended to develop an original technology for compressor manufacturing.

References

1. C. Abagnale, M. Cardone, B. Gargiulo, R. Marialto, *Energy Procedia*, **101**, 710-717 (2016)
2. O. Dmitriev, E. Tabota, *EFMC 12*, **103-108** (2014)
3. O. Dmitriev, E. Tabota, I. Arbon, *IOP Conf. Series: Mat. Science and Eng.*, **90**(2015)
4. N. Oancea, *Surfaces generation by winding*, (Galati Univ. Press, Galati, 2004)