The wave making resistance prediction of a mini-submarine by using tent function method

Aries Sulisetyono[1,*], and *Ardi* Nugroho Yulianto[2]

[1,2]Naval Architecture Department, Institut Teknologi Sepuluh Nopember, Indonesia

Abstract. This paper describes the wave making resistance solution of a mini submarine operating in under water surface with different level depth. The Thin ship theory was adopted to solve the problem for a case of the slenderness body. The source distribution along the centre plane of the body was expressed in Green's function of Havelock source potential under water surface. The Tent function method was proposed to illustrate the hull form based on offsets data, and to solve the Michell integral problem numerically. Four operational conditions were performed i.e. floating, snorkelling, and diving with 0.5m and 1m under water surface. The computational results for the mini submarine with length of 2m and diameter of 0.25m explained a more deeply operated under water surface cause to decrease a value of wave making resistance for all cases of Froude numbers. While in the diving conditions of 0.5m and 0.1m under the water surface, the wave making resistance were resulted about 64% and 74% less than the case of floating condition respectively. Furthermore, the effect of vertical fin on the body was investigated, where the wave making resistance could increase average 7.2% in snorkelling, 11.4% in 0.5m diving, and in the 1m diving about 9.07% for all Froude numbers. Over all the results of this approach shown a good agreement with the results come from Mitchell code.

1 Introduction

There were two total resistance components of ship such as a frictional resistance and a residuary resistance [1]. The frictional resistance of a hull ship was depend on the wetted surface area, Reynolds number, and form factor of hull. The wave-making resistance was the transfer of energy in the form of water waves, and manifests itself as a force opposing the forward motion.

The first theoretical solution of wave making resistance was given by [2] in term of the Integral equation. The Rankine source method was initiated to solve the Michell integral, in which the singularity have to be distributed on the water surface as well as the body surface [3]. Another method, the Havelock source needs only to be distributed on the body surface and it satisfied the free surface condition and far field conditions automatically [4]. The Michell integral equation was developed to calculate the wave making resistance on free surface conditions. For a submarine case, it was necessary to modify the Michell integral equation considers to a submarine mission under water surface [5].

In numerical computation, [6] introduce the 'tent' function to facilitate numerical integration over the ship hull for solving the wave resistance Michell integral. By using the tent function, the hull form can be easily expressed by the linear shape functions in terms of hull offsets that it is suitable for the case of single or multi hull ships. The wave-making resistance problem of multi-hull ships is solved by [7] using the tent function approach, and followed by [8] for single-hull fishing boat. The wave making resistance coefficient is used forward step in determining an optimized hull form of mini submarine case.

* Corresponding Author : sulisea@na.its.ac.id

The effect of vertical fin on the body need to be investigated in this paper since it device used to stabilize the platform of mini submarine. The stability and motion characteristics of a mini submarine was investigated by [9] numerically solving planar equations of a rigid-body motion subject to hydrodynamic force. The vertical fins effect due to the viscous resistance was predicted by using the wind tunnel test [10]. Generally, the appendage could increase the resistances value, but it had different effect consider to the operational condition of the mini submarine.

The intention of this paper was to propose the numerical approach of the tent functions in order to calculate the mave making resistance of under water vehicle. The effect of operasional depths under the free surface water was investigated as well as the the effect of vertical fin due to the wave making resistance.

2 Methodology

The body-fixed coordinate system for the problem was shown in Figure 1, Where *c* was steady advancing velocity, *D* was diameter, *Lwl* was length water line, *T* was draft, *Ss* was ship surface, and *Sf* was free surface. Assuming a fluid was an inviscid, incompressible, unbounded with free surface, and irrotational that satisfies Laplace's equation. Solutions satisfying particular boundary conditions could be superimposed to find the final solution based on the Green function method. The Green function which was the Havelock moving source have to satisfy the 1st order free surface boundary condition and the governing Laplace equation.

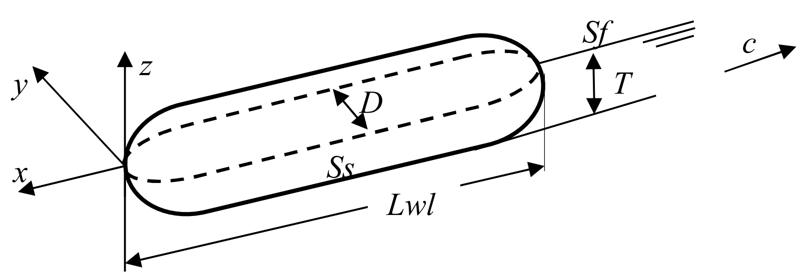


Fig. 1. Coordinate system

The wave resistance for a body was represented by a continuous distribution of sources and sinks moving in deep water was presented in equation 1 and 2 refer to [5].

$$R_w = 16\pi\rho k_o^2 \int_0^{\pi/2} \left[P(\theta)^2 + Q(\theta)^2 \right] \exp(-k_0 h \sec^2\theta) \sec^3\theta \, d\theta \tag{1}$$

$$\begin{Bmatrix} P(\theta) \\ Q(\theta) \end{Bmatrix} = \iint_S \sigma \ \exp(k_o z \sec^2\theta) . \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \left(k_o x \sec\theta + k_o y \tan\theta \sec\theta \right) dS \tag{2}$$

Where $k_0 = g/c^2$, θ was the dummy variables for integration, and the source strength $\sigma = -cf_x(x,z)/2\pi$ was assumed over the ship hull center plane. For the case of a submarine that not only operates under free surface conditions but also in submerged conditions, an additional formula was required that indicates the effect of depth in operation with the addition of equations i.e. exp ($-k_0h$ $\sec^2\theta$). h was the depth of water, and the deeper the mini-submarine operates the smaller wave resistance due to the negative (-) sign.

For numerical solution, the tent function was used to facilitate the numerical integration as shown in equation 1 and equation 2. From the hull function $f(x,z)$ was described under boundary condition, the hull slope function with respect to the *x*-direction could be represented as equation 3 and equation 4, and unit of tent function as shown in Figure 2.

$$f_x\left(x,z\right)=\sum_{ij} f_x^{ij}\frac{-c}{2\pi}\left(x,z\right) \tag{3}$$

$$\begin{aligned} f_x^{ij}\left(x,z\right)=&\left[\frac{1}{x_i-x_{i+1}}\right]\left[\frac{z-z_{j+1}}{z_j-z_{j+1}}\right]y_{i,j}+\left[\frac{1}{x_i-x_{i+1}}\right]\left[\frac{z-z_j}{z_{j+1}-z_j}\right]y_{i,j+1} \\ &+\left[\frac{1}{x_{i+1}-x_i}\right]\left[\frac{z-z_{j+1}}{z_j-z_{j+1}}\right]y_{i+1,j}+\left[\frac{1}{x_{i+1}-x_i}\right]\left[\frac{z-z_j}{z_{j+1}-z_j}\right]y_{i+1,j+1} \end{aligned} \tag{4}$$

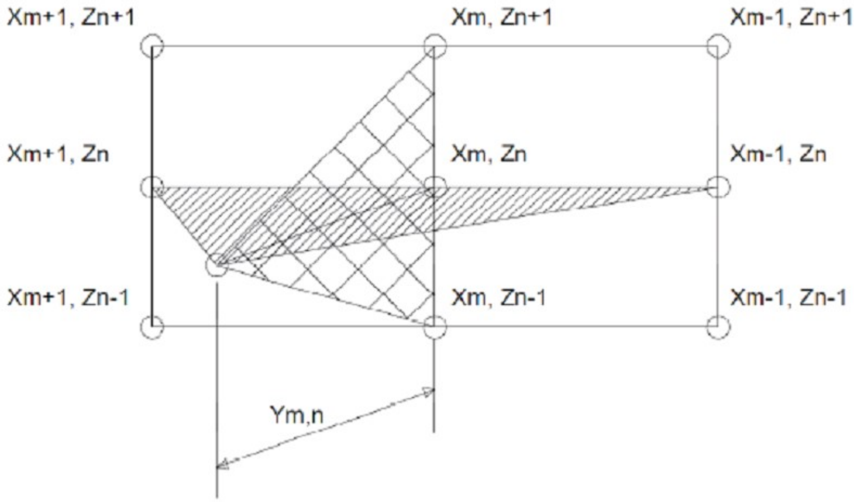


Fig. 2. Tent function unit

The *P* and *Q* in equation 2 could be expanded as,

$$\begin{aligned} \begin{Bmatrix} P(\theta) \\ Q(\theta) \end{Bmatrix} &= -\frac{c}{2\pi}\iint_S f_x\left(\text{y,z}\right)\ \exp\left(k_o z\sec^2\theta\right)\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}(k_o x\sec\theta)ds \\ &= -\frac{c}{2\pi}\sum_{ij}\int_{X_i}^{X_{i+1}}\begin{Bmatrix}\sin\\ \cos\end{Bmatrix}\left(k_o x\sec\theta\right)dx \\ &\left\{A_{ij}\int_{Z_j}^{Z_{j+1}}\left(z-z_{j+1}\right)\exp\left(k_o z\sec^2\theta\right)dz+B_{ij}\int_{Z_j}^{Z_{j+1}}\left(z-z_j\right)\exp\left(k_o z\sec^2\theta\right)dz\right\} \end{aligned} \tag{5}$$

Letting the following equations represented each of the separate integrals gives, and hence

$$\begin{Bmatrix} P(\theta) \\ Q(\theta) \end{Bmatrix} = -\frac{c}{2\pi}\sum_{ij}\begin{Bmatrix}E_i\\ F_i\end{Bmatrix}\left(A_{ij}C_j+B_{ij}D_j\right) \tag{6}$$

Where

$$A_{ij} = \left[\frac{1}{x_i - x_{i+1}}\right]\left[\frac{1}{z_j - z_{j+1}}\right] y_{i,j} + \left[\frac{1}{x_{i+1} - x_i}\right]\left[\frac{1}{z_j - z_{j+1}}\right] y_{i+1,j} \quad (7)$$

$$B_{ij} = \left[\frac{1}{x_i - x_{i+1}}\right]\left[\frac{1}{z_{j+1} - z_j}\right] y_{i,j+1} + \left[\frac{1}{x_{i+1} - x_i}\right]\left[\frac{1}{z_{j+1} - z_j}\right] y_{i+1,j+1} \quad (8)$$

$$\begin{aligned} C_j &= \int_{Z_i}^{Z_{i+1}} \left(z - z_{j+1}\right) \exp\left(k_o z \sec\theta\right) dz \\ &= \frac{1}{k_o \sec^2\theta} \left\{ \left(z_{j+1} - z_j\right) + \frac{1}{k_o \sec^2\theta} A_{ij} \right\} \exp\left(k_o z_j \sec\theta\right) - \frac{k_o z_j + \sec^2\theta}{k_o \sec^2\theta} \end{aligned} \quad (9)$$

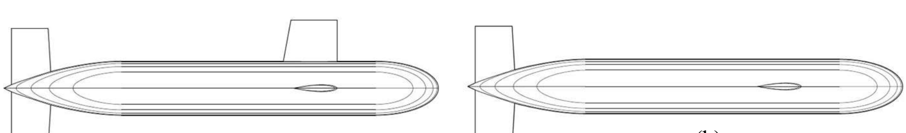
$$\begin{aligned} D_j &= \int_{Z_i}^{Z_{i+1}} \left(z - z_j\right) \exp\left(k_o z \sec^2\theta\right) dz \\ &= \frac{1}{k_o \sec^2\theta} \left\{ \left(z_{j+1} - z_j\right) + \frac{1}{k_o \sec^2\theta} \right\} \exp\left(k_o z_j \sec^2\theta\right) + \frac{\exp\left(k_o z_j \sec^2\theta\right)}{k_o \sec^2\theta} \end{aligned} \quad (10)$$

$$\begin{aligned} E_j &= \int_{X_i}^{X_{i+1}} \cos\left(k_o x \sec\theta\right) dx \\ &= \frac{1}{k_o \sec\theta} \left(\sin\left(k_o x_i + \sec^2\theta\right) - \sin\left(k_o x_i \sec\theta\right) \right) \end{aligned} \quad (11)$$

$$\begin{aligned} F_j &= \int_{X_i}^{X_{i+1}} \sin\left(k_o x \sec\theta\right) dx \\ &= \frac{1}{k_o \sec\theta} \left(\cos\left(k_o x_i \sec^2\theta\right) - \cos\left(k_o x_i + \sec\theta\right) \right) \end{aligned} \quad (12)$$

3 Numerical Results

The main body of a mini-submarine consisted of cylinder with length of body 2 m, and diameter 0.25, and the front body shape was a hemispherical. There were two vertical rudders at the behind of body to control body direction, and two side horizontal fins to increase the stability of body due to underwater current force. In this paper, the effect of vertical fin on the top of body was investigated, and two models were proposed including the mini-submarine with and without vertical fins as shown in Figure 3.



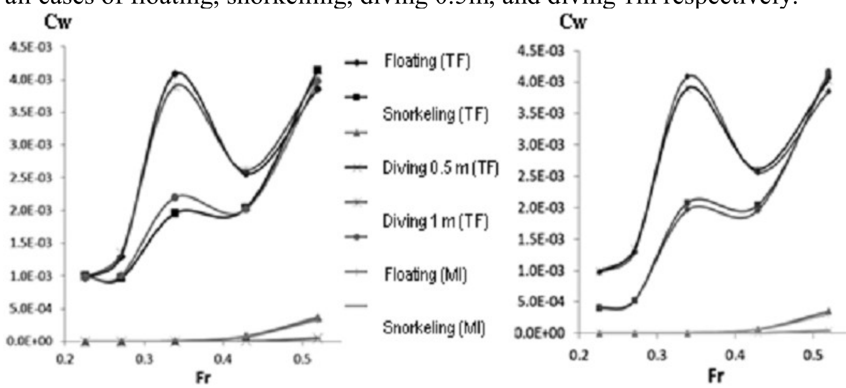
(a) (b)

Fig. 3. The mini-submarine model (a) with, (b) without vertical fin

The computational were conducted in four cases of the operational conditions such as floating, snorkelling, diving 0.5m, and diving 1m. The five different speed were performed in terms of Froude number such as 0.23, 0.27, 0.34, 0.43, and 0.52 for all cases, and it results was compared to the Mitchell code which was developed by [5].

The computational results were shown in figure 4, where the numerical approach of Tent function (TF) produced generally a good agreement results with the Mitchell code (MI) for all cases of floating, snorkelling, diving 0.5m, and diving 1m on the configuration of a mini-submarine with vertical fin. Since the operational of a mini submarine was considered in low speed, the first hump of curve was occurred at Froude number 0.34, and the second hump would be occurred at highest Froude number. For the case of floating condition on the mini-submarine with vertical fin, the Tent function approach had results up to Mitchell code with average of 3.29% for all Froude Number. This differences value shown also for other cases of snorkelling, diving 0.5 m, and diving 1 m which had different value of wave making resistance coefficient about average of 4.45%, 10.01%, and 15.06% respectively.

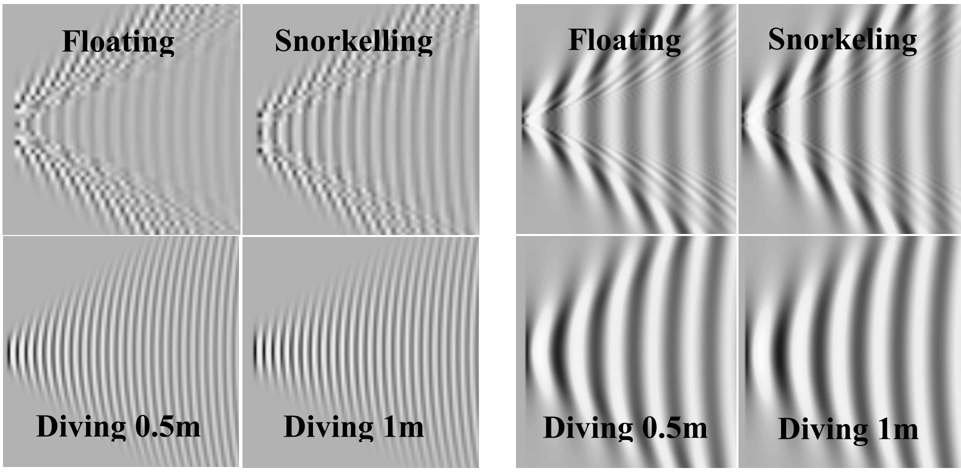
In Figure 4, the mini-submarine without vertical fin had the results by using Tent function approach about average of 3.29%, 3.28%, 9.15%, and 14.66% higher than Mitchell code for all cases of floating, snorkelling, diving 0.5m, and diving 1m respectively.



(a) (b)

Fig. 4. Wave making resistance results of Tent Function (TF) and Mitchell code (MI) for the mini-submarine (a) with and (b) without vertical fins

The wave patterns of four case conditions with Froude number 0.52 in which the diving condition of 0.5m and 1m had generate a small wave making as shown in Figure 5b. While the Froude number was decreased to 0.23, the elevation of wave surface was a very small for all cases of operational conditions as shown in Figure 5a. The value of wave making resistance might be zero if the mini-submarine operates at water depth more than 5 times of the ship's body diameter [11].



(a) (b)

Fig. 5. Wave pattern of mini-submarine without vertical fin at Froude number (a) 0.23, and (b) 0.52

The effect of vertical fin on body was very small as shown in Figure 6. The wave making resistance coefficient of a mini-submarine with vertical fin was increased about average 7.2% in snorkelling condition, 1.4% in 0.5m diving condition, and in 1m diving condition about average 9.1% higher than a mini-submarine without vertical fin. The biggest different shown at moment of Froude number 0.23 while it operated in snorkelling condition.

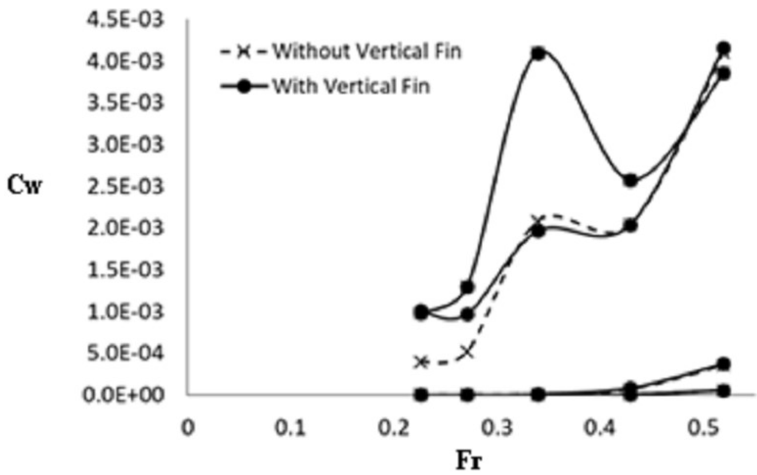


Fig. 6. Comparison results of Mini Submarine with and wihout vertical fin

4 Conclusions

In the diving conditions of 0.5m and 0.1m, the wave making resistance results were respectively about 64% and 74% less than the case of floating condition for all Froude numbers. The effect of vertical fin on the top of body was very small that could increase average 7.2% of the snorkelling condition, 11.4% of the 0.5m diving condition, and the 1m diving condition about 9.07%. Over all the results of the Ten Function approach shown a good agreement with the results came from Mitchell code.

References

1. W. Froude, The fundamental principles of the resistance of ships, *The Papers of William Froude The Institution of Naval architects* London (1955)
2. J. H. Michell, The wave-resistance of a ship, *Philosophical Magazine* Series 5 Vol **45** no 272 p 105-123 London (1898)
3. C. W. Dawson, A practical computer method for solving ship wave problems *Proceeding of the Second International Conference on Numerical Ship Hydrodynamics* p 30-38 University of California Berkeley (1977)
4. T. Havelock, The wave-making resistance of ships: theoretical and practical analysis, *The Collected Papers of Sir Thomas Havelock on Hydrodynamics Office,* Naval Research Department of the US Navy ONR/ACR **65** (1965)
5. E. O. Tuck, L. Lazauskas, and D. C. Scullen, Sea wave pattern evaluation (primary code and test results) University of Adelaide (1999)
6. C. C. Hsiung, Optimal ship forms for minimum wave resistance, *Journal of Ship Research* Vol **25** No 2 (1981)
7. H. Peng, Numerical computation of multi-hull ship resistance and motion, *Thesis at Dalhousie University* Halifax (2001)
8. A. Sulisetyono, Numerical computation for the wave making resistance prediction of ship without transom using tent function method, *Journal of Naval Architecture and Marine Engineering* (submitted) 2017
9. A. Sulisetyono and D. S. Purnomo, The mini-submarine design for monitoring of the pollutant and sewage discharge in coastal area, *Asian and Pacific Coasts 2009* p 252-258 Singapore (2009)
10. A. N. Yulianto, K. Suastika, A. Sulisetyono, Viscous-Resistance Calculation and Verification of Remotely Operated Inspection Submarine, *IPTEK The Journal for Technology and Science* Vol. **22** No. 4 (2014)
11. E.E. Allmendinger, Submersible Vehicles Systems Design, *Jersey: SNAME* (1990)