Application of Extreme Learning Machine in GPS Positioning Process

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Abstract. In the positioning process of GPS, the linear least squares algorithm and Kalman filtering algorithm are widely used but still have shortcomings. Application of extreme learning machine in this area is proposed in this paper, which breaks through the limitations of the traditional method of positioning based on mathematical models. Two simulation experiments of ELM in GPS positioning process are presented in this paper while the latter is a supplement to the former. Each one contains three phases, including simulation data generation, network training and network prediction, each of which is considered carefully. The feasibility of extreme learning machine is verified through experimental simulation. A more accurate positioning result can be obtained.

1 Introduction

In the positioning process of the global positioning system (GPS), the linear least squares (LS) algorithm and Kalman filtering (KF) algorithm are usually used [1]. The LS algorithm is sensitive to observation errors. The KF algorithm depends on the statistical properties of the noise of the state model and the measurement model, which may lead to the divergence of the filter [2, 3]. Although both algorithms have good performance practically, there are still some problems to solve.

Artificial neural network (ANN) is a nonlinear dynamic system composed of a large number of simple processing units (neurons) interconnected by a certain structure [4]. It is the simplification and simulation of human brain information processing mechanism. Neural networks have been shown to approximate continuous and differentiable functions with arbitrary accuracy [5]. Since GPS positioning equations are continuous and differentiable, which do not exist any analytic solution, the equations can be approximated by neural networks [6]. Including the back propagation (BP) neural network, the traditional single hidden layer feedforward neural network (SLFN) have some disadvantages, including the slow training speed, the poor generalization ability and the sensitivity to the learning rate.

Extreme learning machine (ELM) is one of SLFN with high precision and fast training speed [7]. ELM is applied to the GPS positioning process in this paper. The training data is composed of observation data from the simulation and real locations of nine receivers arranged in the grid. The network may predict the positions of the receivers, which move in a certain path in the area. The

result of ELM is compared to that of least linear squares method and Kalman filtering method.

2 Positioning observation equation

Pseudo distance positioning observation equation [8] can be expressed as

$$Pr = |SV - U| + B + Er \quad (1)$$

where Pr is the observation pseudorange of the receiver, SV is the coordinates of the visible satellites in the earth-centered/earth-fixed (ECEF) coordinate system, U is the position of the receiver in the ECEF coordinate system, B is the clock error of the receiver, Er is the pseudorange measurement error, including the satellite clock error, the ephemeris error, the atmospheric delay error, the multi-path error, the noise of receivers and other unknown error.

In (1), SV and Pr is known, and the known part of Er can be calculated by models, while U and B are unknown. Without considering the unknown error, the essence of the traditional positioning algorithms is to solve the nonlinear equation set composed of (1). They are usually linearized and calculated by LS or KF method. It is obvious that the ignorance of the original nonlinear characteristics of the observation equations may result in a reduction in the accuracy. Meanwhile, the models of atmospheric delay error and other known errors are empirical models, so the true value of errors in one epoch are unknown.

From (1), the relationship among the observation pseudorange of the receiver Pr , the coordinates of the visible satellites SV and the position of the receiver U

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can be considered to be a multi-input and multi-output nonlinear mapping F which has a fixed form as

$$U = F(SV, Pr) \quad (2)$$

There is no explicit expression for the nonlinear mapping F , which is continuous and differentiable. It can be approximated with arbitrary accuracy by the neural network with enough nodes of the hidden layer.

3 Extreme learning machine

ELM is one of SLFN, which have good learning ability. The typical SLFN structure can be seen in Fig. 1.

Fig. 1. The typical single hidden layer feedforward neural network structure.

Yet some learning algorithms of SLFN like BP algorithm have slow training speed and adopt the gradient descent method, leading to poor generalisation ability and sensitivity to the learning rate. ELM randomly generates weights w of connections between the input layer and the hidden layer, and the bias b of the neurons in the hidden layer. w and b keep the same in the training process. The weights β of connections between the hidden layer and the output layer can be obtained by solving the LS solution of the following equation as

$$\min_{\beta} \|\mathbf{H}\beta - \mathbf{T}'\| \quad (3)$$

where $\mathbf{H}$ is the output matrix of the hidden layer and $\mathbf{T}'$ is the transpose of the network output.

The solution is unique and its norm is minimal. Compared to other neural networks of SLFN, ELM has fast learning speed, good generalization performance and high training precision in theory. In the GPS positioning process, the coordinates of receivers are on the order of 10^7 meters. In order to achieve centimeter-level positioning accuracy, high training precision is required.

4 Application of ELM in GPS positioning process

There are two simulation experiments of application of ELM in GPS positioning process. The second one is a

supplement to the first one. Each one contains three phases, including simulation data generation, network training and network prediction.

4.1 Simulation experiment I

4.1.1 Simulation data generation

In the simulation data generation phase, the simulation data is composed of the positions and corresponding observation data of receivers. Since the systematic errors can be basically eliminated by the known model, they are not considered, including the earth's rotation error, the relativistic effect and the satellite clock error. The positions of satellites are true value in one epoch. The ionosphere error obtained by the 8-Klobuchar model and the tropospheric error obtained by the Hopfield model is added to the true pseudorange as the observation pseudorange.

Nine receivers are arranged in the 3×3 layout, which can be seen in Fig. 2. The difference in latitude and longitude between adjacent nodes is $0.5'$, and the height of each node is 312 meters.

Fig. 2. The layout of 9 training receivers and the motion route of the testing receiver of the first simulation experiment.

The training data is the simulation data of the receivers in 40 minutes. Each epoch is one second. So the number of samples for the training set is $9 \times 40 \times 60 = 21600$. The input of the network is composed of coordinates and the pseudorange of nine continuous visible satellites, of which the cut-off angle is five degrees. The input of ELM is a 36-dimensional vector, which can be expressed as $(x_1, y_1, z_1, p_1, \dots, x_9, y_9, z_9, p_9)$. The output of the network is the exact location (x, y, z) of the receivers corresponding to the input.

4.1.2 Network training and prediction

In the network training phase, in order to achieve higher training precision and a larger network scale, the number of neurons in the hidden layer is taken as 2000, and the neuron activation function takes the 'sin' function.

In the network prediction phase, we consider the motion of the testing receiver. The input of the trained

network is the observation data of the testing receiver that perform circular motion in the grid area. The circular motion route can be seen in Fig. 2. The output is the 3-dimensional coordinates of the receivers' position. The motion time of the receivers is ten minutes, and the line velocity of the motion is 6m/s.

With no noise, the result of network prediction performs high accuracy, which shows that ELM can approximate the nonlinear mapping of the input and the output, even if there is the existence of the ionospheric error and the tropospheric error.

Then the random noise is added to the pseudorange, of which the standard deviation is 5 meters and the mean value is 0. After prediction, we compare the 3-dimensional positioning error of the prediction result with that of LS method and KF method. The results are shown in Fig. 3.

Fig. 3. The 3-dimensional positioning error of LS algorithm, KF algorithm and ELM.

Table 1. The mean square errors of 3-dimensional positioning error of LS algorithm, KF algorithm and ELM.

Positioning algorithm	LS	Kalman	ELM
Mean square error/m	2.83	1.33	1.06

As can be seen in Table 1, the mean value of the 3-dimensional error of LS algorithm is 2.83m, and that of KF algorithm is 1.33m, while that of ELM is 1.06m, which performs higher accuracy than the others.

We can also see in Fig. 3 that the curve of the error of ELM varies regularly with the circular motion of the testing receiver, which may be related to the position of the testing receiver in the area. This may be due to the lack of training data in the blank area of the 2×2 grid. So we conduct following simulation experiment.

4.2 Simulation experiment II

In the simulation data generation phase, 4 training receivers are newly added to the center of each grid, which can be seen in Fig. 4. Other experimental settings remain unchanged.

Fig. 4. The layout of 13 training receivers and the motion route of the testing receiver of the second simulation experiment.

As can be seen in Fig. 5 and Table 2, the 3-dimensional positioning error of 13 training receivers is smaller and more stable than Simulation Experiment I. The mean value of positioning error reduces to 0.0755m, which is acceptable.

Fig. 5. The 3-dimensional positioning error of ELM with 9 receivers and 13 receivers.

Table 2. The mean square errors of 3-dimensional positioning error of ELM with 9 receivers and 13 receivers.

Number of receivers	9	13
Mean square error/m	1.06	0.0755

5 Conclusion

Applications of ELM in GPS positioning process are proposed in this paper. The results of simulation experiments are more accurate than LS and KF algorithms. The results of experiments show that the more complete the training data is, the more accurate and stable the positioning results are.

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