Manipulate Vibration Propagation by Anisotropic Honeycomb Structure

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Abstract. As a new type of acoustic metamaterial, the pentamode material has extensive application prospect in controlling acoustic wave propagation because of its fluid properties. Firstly, a kind of pentamode material unit cell is designed, which is a two-dimensional honeycomb truss structure. Then, the asymptotic homogenization method is used to calculate static parameters of the unit cell, and also the influence of the geometric parameters and material composition of the unit cell on its mechanical properties is studied. Besides, based on transformation acoustics and the design method of the cylindrical cloak proposed by Norris, an acoustic cloak with isotropic density and gradient elastic modulus is constructed by periodically assembling the unit cell to guide the wave to bypass obstacles. Finally, the full displacement field analysis is carried out to prove the stealth effect of the acoustic cloak.

1 Introduction

In natural, fluctuation is a common way of energy transmission. As an example, mechanical vibration is an elastic wave propagating in mechanical structures. Therefore, exploring the relationship between materials and wave propagation has an extraordinary significance for practical engineering and scientific researches. For many years, some researchers have been trying to design a kind of acoustic cloak with stealth effect. At the beginning, researchers mainly study the acoustic cloak based on inertial materials with isotropic stiffness and anisotropic density[1]. However, because the cloak possesses an infinite density at the inner surface of the cloak, it is difficult to fabricate. In order to overcome the problem, Norris proposes to use pentamode materials with anisotropic stiffness and isotropic density to design[2]. Pentamode materials can be realized by periodic materials with micro-truss unit cell configuration similar to a diamond cell, and the two-dimensional configuration is a hexagonal honeycomb structure[3]. Now, the pentamode materials are classified as a new kind of acoustic metamaterials, and the main characteristic is that the ratio of shear modulus to bulk modulus is very small, so the coupling of shear wave and expansion wave can be broken up. Recently, the pentamode metamaterials were successfully developed and a pentamode material model was fabricated by the laser direct writing technology[4]. Now the pentamode materials have become an ideal choice for controlling the propagation path of acoustic waves.

Aiming at unit cells of the pentamode materials or the two-dimensional honeycomb truss structures, the asymptotic homogenization method is applied to explore the influence of structural parameters of unit cells on the static effective elastic modulus, especially, the anisotropic material characteristics. And the relationship between structural parameters of unit cells and the effective elastic modulus is figured out. Then based on the transformation acoustics, a cylindrical cloak with the two-dimensional honeycomb structure configuration is designed. Finally, the full displacement field analysis is carried out to illustrate the stealth effect of the acoustic cloak.

2 PM Material Acoustic Cloaking Theory

According to Norris's transformation acoustics theory [2], when designing a cylindrical invisibility cloak, the media parameters can be either the anisotropic density and the isotropic modulus, or the anisotropic modulus and the isotropic density. In order to construct the cylindrical cloak with the anisotropic modulus and the isotropic density, there are kinds of transformation functions to be chosen to realize the design. For example, the mapping function Eq.(1) can be chosen to achieve a constant material density, and the material parameters of the PM material cloak can be obtained by Eq.(2) [5]:

$$f\left(r\right)=\left[\left(\frac{b^2-\delta^2}{b^2-a^2}\right)r^2-\left(\frac{a^2-\delta^2}{b^2-a^2}\right)b^2\right]^{\frac{1}{2}} \tag{1}$$

$$\rho=\rho_0\frac{f(r)f'(r)}{r},\quad E_r=E_0\frac{f(r)}{rf'(r)},\quad E_t=E_0\frac{rf'(r)}{f(r)} \tag{2}$$

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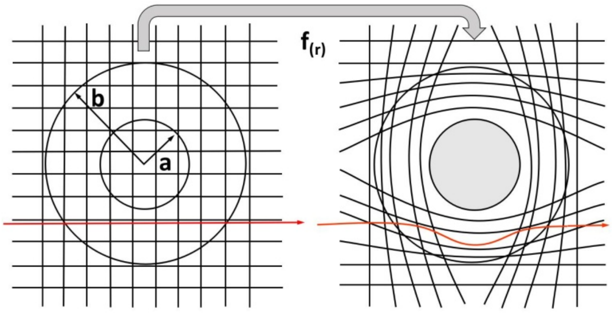


(a) Virtual space (b) Physical space

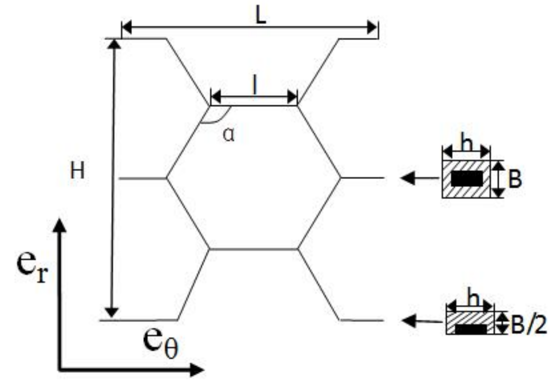
Figure 1. Coordinate transformation

Where, ρ is the density of the cloak, E_r is the radial modulus of the cloak, E_t is the tangential modulus of the cloak, ρ_0 is the density of the background material surrounding the cloak, E_0 is the modulus of the background material, r is the radius of the cloak, $f(r)$ is the coordinate transformation function, a is the inner diameter of the cloak, and b is the outer diameter of the cloak, δ is an infinitesimal amount. As shown in Fig.1, $f(r)$ represents the mapping transformation function from the physical space to the virtual space, and the acoustic wave can propagate along the coordinate curve and bypass the cavity smoothly.

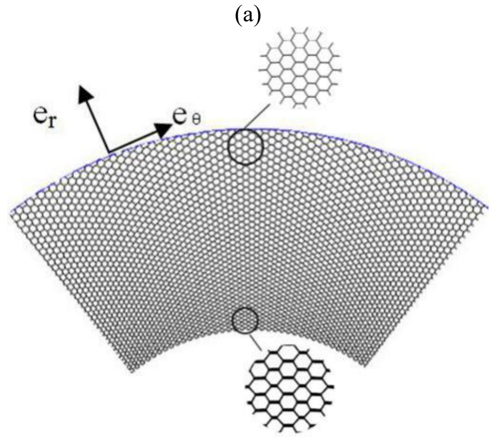
If $r = b$, $f(b) = b$, then at the interface between cloak material and the background material, we have $\sqrt{\rho E r} = \sqrt{\rho_0 E_0}$ and $\sqrt{Et / \rho} = \sqrt{E_0 / \rho_0}$. In general, when the parameter $\delta \ll a$, let $f(a) = \delta$, by this way, the singularity of material density can be avoided, and the transformation is equivalent to a mapping from the annular region $\delta < r < a$ into the annular region $a < r < b$, and the scattering characteristic of the cloak is the same as the scatters that radius is δ, and the scattering intensity reduces greatly.

3 Relationship between Equivalent Material Parameters and Geometric Configuration of Unit Cells

3.1 Calculatiing Equivalent Material Parameters



(a)



(b)

Figure 2. Unit cell structure and annular cloak

Fig.2 (a) is a schematic diagram of a unit cell structure which constitutes the cloak region. The total length of the unit cell is L, and the total height is H. The side length of the hexagon is *l*, and the interior angle is α. The non-uniform annular cloak is composed of a cyclical array of the unit cell material. In the process of designing the cyclical array, the structural parameters of the unit cell, such as α and the material composition of each beam, change gradually to satisfy the transformation acoustics. Figure 2(b) is a partial view of the cloak. The cylindrical cloak consists of 180 segments, and each segment consists of 18 layers of unit cells. In this paper, the physical parameters satisfying design requirements of the cylindrical cloak are constructed by changing the parameters of unit cells, including structural dimensions and material properties of unit cells.

The asymptotic homogenization method is adopted to calculate equivalent elastic parameters of unit cells, using two-scale to define the stress field and displacement field of periodic composite materials. In this way, the equivalent elastic parameters of periodic materials can be obtained under the macroscopic scale. The expression for the equivalent elastic parameters of unit cells can be written as follows[6]:

$$E^H_{ijkl} = \frac{1}{|Y|}\int_Y (E_{ijkl}\delta_{km}\delta_{\ln} - E_{ijpq}\frac{\partial \chi_p^{kl}}{\partial y_q})dY \tag{3}$$

$$\int_Y \frac{\partial V_i^1}{\partial y_j} E_{ijpq} \frac{\partial \chi_p^{kl}}{\partial y_q} dY = \int_Y \frac{\partial V_i^1}{\partial y_j} E_{ijpq} \tag{4}$$

Where, $|Y|$ is volume of the unit cell， E_{ijkl} is elastic modulus of material in the unit cell, δ_{km} is the kronecker symbol, V_i^1 is the defined displacement, χ^{kl} is the characteristic displacement, which is determined by Eq.(4). Substituting χ^{kl} into Eq.(4), the equivalent elastic tensor E^H_{ijkl} can be obtained, and then, by comparing with the Hooke's law, the equivalent elastic modulus E_e and Poisson's ratio v_e of the unit cell can be gotten. In a commercial finite element software, Eq.(3) can be implemented as [7]:

$$E^H_{ijkl} = \frac{1}{|Y|}\int_Y (\chi^{0(ij)} - \chi^{*(ij)})^T (f^{0kl} - f^{*(kl)}) \qquad (5)$$

Where, $\chi^{0(ij)}$ is the nodal displacement field corresponding to the unit strain field; $\chi^{*(ij)}$ is the displacement field corresponding to the characteristic strain field; $f^{0(ij)}=K\chi^{0(ij)}$, $f^{*(ij)} = K\chi^{*(ij)}$, K is the stiffness matrix of the cell.

3.2 Relationship between Equivalent Material Parameters and Geometric Configuration

As shown in Fig. 2(a), the matrix material of each cell is composed of rubber and aluminum (Al). The density and elastic modulus of the beam are adjusted by changing the volume fraction of Al. Supposing the rubber density ρ_{rub} and the Al density ρ_{Al} are $1380Kg/m^3$ and $2700Kg/m^3$, separately, the average density ρ_c is

$$\rho_c = \rho_{Al} \times \eta_v + \rho_{rub} \times (1\text{-}\eta_v) = 1320 \times \eta_v + 1380 \qquad (6)$$

where η_v is the volume fraction of Al. The effective elastic modulus can be obtained by finite element analysis of a single beam. The elastic moduli and Poisson's ratios of rubber and Al are given as $E_{rub} = 0.0078\text{GPa}$, $E_{Al} = 69\text{GPa}$, $v_{rub} = 0.47$, $v_{Al} = 0.3$. The finite element model of beam is built as shown in Fig.2 (a), and the equivalent material parameters of the beam can be obtained by the strain energy method,

$$E_c = 68.9922e^3\eta_v + 0.0078e^3 \qquad (7)$$

Fig.3 shows the relationship between E_c , ρ_c and η_v , and the structural and material parameters of the unit cell are given in Table 1. Where, B is the width of beam , h is the height of beam, E is the Young's modulus of material, v is the Poisson's ratio of material, and ρ is the density of material.

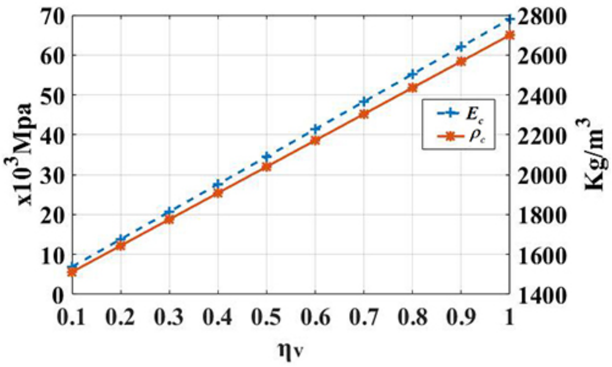


Figure 3. Relationship between E_c , ρ_c and η_v

Table 1. Structural and material parameters of unit cell

L[mm]	H[mm]	l[mm]	α[rad]	B[mm]
3000	$2000\sqrt{3}$	1000	$2\pi/3$	35
h[mm]	**E[Mpa]**	**v**	**ρ [g/mm³]**	/
80	Ec	vc	ρ_c	/

In general, unit cell equivalent material parameters can be adjusted by geometric parameters of the unit cell and the material composition of each beam. Several different structures and material adjustment schemes are proposed, and the asymptotic homogenization method is used to analyze the relationship between equivalent elastic moduli of unit cells and the adjusted structural parameters of unit cells. The adjustment schemes include: (1) only changing the topology angle of the unit cell, (2) simultaneously increasing the width B of all beams in the unit cell, (3) only increasing the width *Bh* of all horizontal beams in the unit cell, (4) simultaneously increasing the volume fraction of Al of all beams in the unit cell. And the analysis results are illustrated from Fig.4 to Fig.7

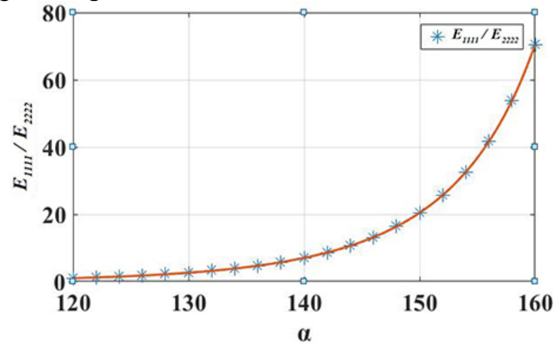


Figure 4. Relationship between E1111/E2222 and α

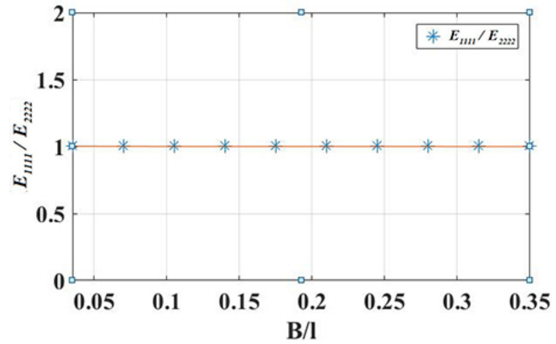


Figure 5. Relationship between E1111/E2222 and B/l

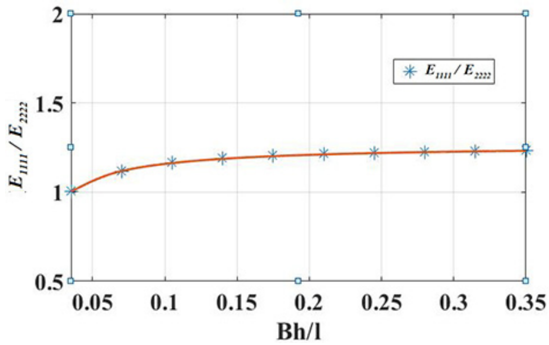


Figure 6. Relationship between E1111/E2222 and Bh/l

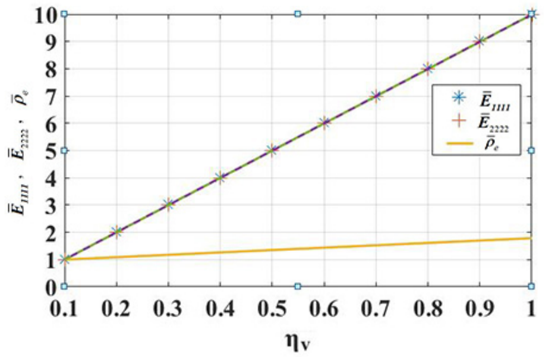


Figure 7. Relationship between $\bar{E}_{1111}$, $\bar{E}_{2222}$, $\bar{\rho}_e$ and η_v

In the unit cell, E1111/E2222 is defined as the anisotropy ratio of the equivalent elastic moduli in orthogonal directions, B/l is the width-to-length ratio of the beam and Bh/l is the width-to-length ratio of the horizontal beam. In Fig.4, it can be seen that the anisotropy ratio is greatly affected by the topological angle. When α increases, the equivalent elastic moduli present anisotropy. And the ratio of E1111/E2222 can be greater than 70, therefore, the distribution of material physical properties required by transformation acoustics can be achieved by designing topological angle. Fig.5 shows that the anisotropy ratio is not affected by the width-to-length ratio of the beam. In Fig.6, it can be seen that E1111/E2222 is less affected by the width-to-length ratio of the horizontal beam, when Bh/l is increased to a certain value, the anisotropy ratio of the equivalent elastic moduli in orthogonal directions tends to remain unchanged.

In Fig.7, the vertical axis denotes the ratio of the equivalent parameter of the adjusted unit cell to the equivalent parameter of the initial unit cell, which are defined as $\bar{E}_{1111} = E_{1111} / E^*_{1111}$, $\bar{E}_{2222} = E_{2222} / E^*_{2222}$, $\bar{\rho}_e = \rho_e / \rho^*$. Here, $E^*_{1111}, E^*_{2222}, \rho^*$ are the equivalent parameters of the initial unit. From the curves, it can be concluded that the equivalent elastic moduli in orthogonal directions are always equal, and they are linearly dependent on the volume fraction of Al, which can be expressed as $E_{2222} = (828.9 \times \eta_v + 0.09371)\,MPa$. The formula can be used to design the desired equivalent elastic modulus of the unit cell.

During the design process, in order to ensure the topological relationship of each layer in the unit cell, the unit cell will be enlarged proportionally with the increase of the radius, and the amplification ratio between adjacent layers is set as $n = 1.041$. Therefore, it is necessary to study the effect of the amplification factor on the equivalent elastic modulus. Fig.8 offers the variation curves of the unit cell equivalent elastic moduli with n. It can be seen that $\bar{E}_{1111}$ and $\bar{E}_{2222}$ almost do not change with the amplification factor.

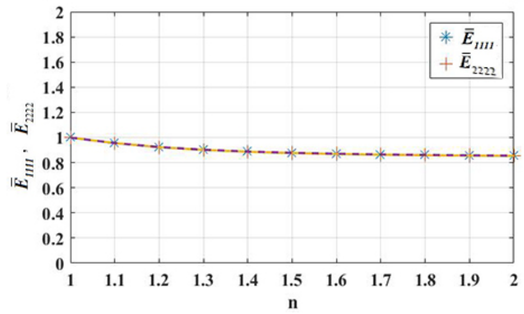


Figure 8. Relationship between $\bar{E}_{1111}$, $\bar{E}_{2222}$ and n

4 Numerical Simulation of the Cloak

In this section, COMSOL Multiphysics finite element simulation software is employed to simulate and validate the design by the full-wave simulation method. Choosing $\delta = a/10$, b=2a, the material parameters in the design domain of the cloak can be calculated and determined according to Eq.(1,2). Theoretically, the material parameters in the design structure should keep continuous distribution. Here, this is implemented by layered approximation with anisotropic honeycomb structure, and the material parameters are shown in Fig.9. The anisotropy ratio is greater in the innermost layer, and the parameters of the first two layers are designed according to parameters of the next layer. According to the curves or the function relationships obtained by using the asymptotic homogenization method, the geometric and material parameters of unit cell in each layer of the design region are calculate by virtue of the interpolation method.

$$\rho = 1.33 \times \rho_0$$
$$E_r = \left(\frac{1.33r^2 - 1.32a^2}{1.33r^2} \right) \times E_0 \tag{8}$$
$$E_t = \left(\frac{1.33r^2}{1.33r^2 - 1.32a^2} \right) \times E_0$$

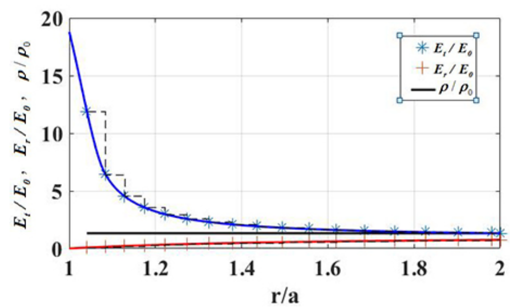


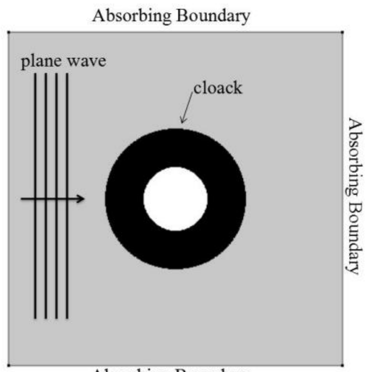
Figure 9. When b=2a, $\delta = a/10$, the material parameters in accordance with the radius of cloak

Firstly, the topological angles of the unit cell in each layer are determined according to the anisotropic ratio of the orthogonal elastic moduli given by transformation acoustics. Then, the material composition of each beam in the unit cell is determined according to the elastic modulus value, and finally according to the density distribution, the width of the beam is adjusted further under the premise that the elastic modulus remains invariable, so as to satisfy the requirement that the equivalent density of the unit cell in each layer remains unchanged.

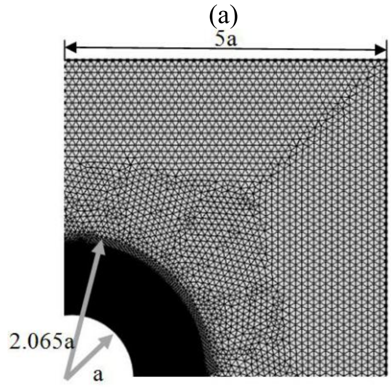
Table.2 shows the parameters of every layer which satisfy the requirement of the constant density cloak. And it can be seen that in the cloak the anisotropy of the innermost layer is the greatest. Therefore, the topology angle of the innermost unit cell needs to be designed larger. According to the layered approximation method, the unit cells in each layer are constructed, the innermost layer is composed of two layers of unit cells, and the other layers are composed of one layer of unit cells. The design radius is determined by the radius of the outer edges of the unit cells. As shown in Fig.10 (a), in order to avoid the effect of reflection at the boundary, the absorption boundary conditions are applied around the model, which is used to simulate a semi-infinite region. On the left side of the model, a sinusoidal excitation is impinged for full-wave simulation. The parameters of the background material are determined by the equivalent elastic modulus and equivalent density of the outermost unit cell. The aluminum volume fraction of the outermost unit cells is 0.752, and the topology angle of the outermost unit cell is $\alpha = 125.9^{\circ}$, the width-to-length ratio of beam is 0.035. The equivalent elastic moduli of the outermost unit cell are E1111=807.470MPa, E2222=457.680MPa, and the equivalent density is 95.89 Kg/m^3. The background material is an isotropic material with an elastic modulus of 608.714MPa and a density of 72.10Kg/m^3. The propagation velocity of the elastic wave in the background material is 2905.6m/s, which can be calculated though $\sqrt{E/\rho}$. The applicable conditions of the quasi-static parameters in the dynamic analysis are related to the microstructure size and wavelength. When the ratio of the microstructure size to the wavelength is $\xi \leq 0.1$, the quasi-static and dynamic parameters are almost the same, when the ratio $\xi \geq 0.3$, quasi-static parameters are no longer suitable for dynamic analysis [8]. If the frequency range of the incident wave is $\lambda \leq 42Hz$, the statically calculated equivalent parameters will be effective in the dynamic analysis. The frequencies of input sinusoidal excitation are 20Hz, 30Hz, and 40Hz, and the incident wave amplitude is $A = l / 1000 = 0.001m$. When designing the cloak area, the inside diameter of the cloak is a=84.26m. To ensure the structural integrity of the outermost unit cell, the outside diameter of the cloak is b=173.67m=2.061a, and the space occupied by background material is $10a \times 10a$. Fig.10 (b) is a grid division diagram of the 1/4 model.

Table 2 Parameters of the cloak domain (b=2a, $\delta = a / 10$)

(layer) r/a	$\alpha[\circ]$	ηv	B	Et/Er
(1)1.041	/	/	/	/
(2)1.084	155.95	0.155	0.0524	41.422
(3)1.128	150.09	0.220	0.0497	20.664
(4)1.174	145.69	0.280	0.0475	12.763
(5)1.222	142.26	0.335	0.0456	8.891
(6)1.273	139.42	0.387	0.0439	6.658
(7)1.325	137.12	0.435	0.0425	5.292
(8)1.379	135.21	0.478	0.0413	4.375
(9)1.436	133.57	0.519	0.0402	3.717
(10)1.495	132.16	0.556	0.0393	3.235
(11)1.556	130.9	0.590	0.0385	2.872
(12)1.620	129.86	0.622	0.0377	2.586
(13)1.686	128.88	0.651	0.0371	2.361
(14)1.755	127.78	0.678	0.0365	2.177
(15)1.827	127.29	0.703	0.0360	2.025
(16)1.902	126.62	0.726	0.0355	1.899
(17)1.980	126.06	0.747	0.0351	1.793
(18)2.000	125.9	0.752	0.0350	1.769



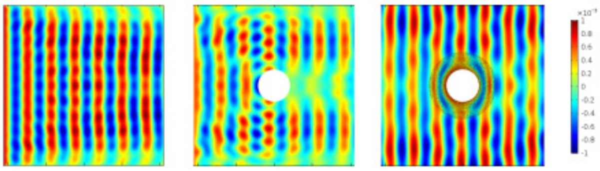
(a)



(b)

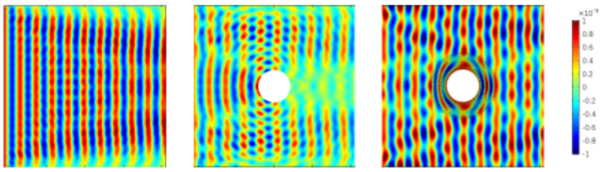
Figure 10. Comsol model and 1/4 model mesh diagram

The results of the simulation are shown in Fig.11, Fig.12 and Fig.13, where the contours of displacement field are drawn. Fig.11, Fig.12 and Fig.13 are the full displacement filed at an incident frequency of 20 Hz, 30Hz and 40Hz, respectively, where (a) is a plate made of background material, (b) is a plate with a cavity, (c) is a plate with a cavity covered by the acoustic cloak.



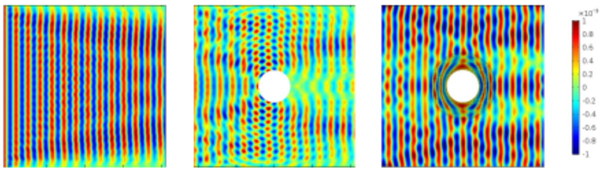
(a) (b) (c)

Figure 11. Incident plane wave with the frequency of 20Hz



(a) (b) (c)

Figure 12. Incident plane wave with the frequency of 30Hz



(a) (b) (c)

Figure 13. Incident plane wave with the frequency of 40Hz

Seeing from the simulation results, we could find that because of the existence of the cavity in plate there is scattering in front of the cavity and behind the cavity, the cavity leads to the breakup and dislocation of the wave front; however, the cavity covered by the cloak is obviously different, the plane wave still maintains the propagation characteristics of the plane wave, and there is a clear wave bending in the cloak region (a<r<b) , which proves that the radial gradient of material parameters in the cloak region can guide the wave to smoothly bypass the obstacle and continue to propagate forward. This shows that the designed two-dimensional honeycomb cloak has the characteristics of perfect acoustic cloaking. However, due to the layered approximation, the stealth effect is weakened, so some scattering is generated and the wave front has small disturbances. Comparing the full-wave simulation diagrams of each frequency, it can be concluded that the invisibility cloak can keep its wave front intact after bypassing the cavity under the applicable conditions of quasi-static parameters, which verifies the validity of the static parameter. Comparing the field with the different frequencies, it can be seen that the control effect of the plane wave on the low frequency is better than that of the plane wave on the high frequency. It is due to the ratio of the wavelength to the cell size. The greater the ratio of wavelength to cell size is, the better the stealth effect.

In order to study the stealth effect of the cloak when loading incident wave from different directions, it is effective to apply the cylinder wave as an initial condition in the model, as shown in Fig.14. Here is the full displacement filed and the incident wave is a 40 Hz cylindrical wave. The position of the wave source is (x=-400, y=0). As shown in the Fig.14, the incident cylindrical wave smoothly bypasses the cavity under the guidance of the cloak and maintains its original propagation characteristics, and only small scattering occurs. Wave bending can also be clearly seen in the cloak region. It can be shown that the cloak is not affected by the direction of the incident wave.

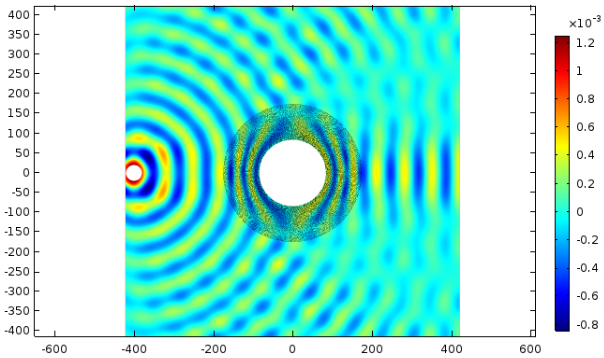


Figure 14. Displacement contours under the incident cylindrical wave at the frequency of 40 Hz

5 Conclusions

The static equivalent material parameters of two-dimensional honeycomb-typed pentamode material are investigated by the asymptotic homogenization method, and the relationship between the static equivalent parameters and the structural parameters of periodic unit cells are presented. Furthermore, by making use of the relationship a two-dimensional acoustic cloak is designed by the transformation acoustics method. Meanwhile, the commercial software COMSOL Multiphysics is used to analyze and verify the design.

(1) The equivalent elastic modulus of the honeycomb structure unit cell can be designed by changing the geometric parameters and material composition of the unit cell, where the topological angle of the unit cell has a significant effect on the anisotropy ratio of the equivalent elastic modulus of the unit cell, so the periodic material with anisotropy modulus can be designed by changing the topological angle of the unit cell.

(2) The static equivalent parameters calculated by the asymptotic homogenization method can be effectively used for structural design in dynamic simulation within a certain range;

(3) Incorporating the transformation acoustics into the two-dimensional anisotropy honeycomb material design method, a two-dimensional pentamode material cloak is constructed successfully, and the finite element simulation shows that effective control of the propagation of the acoustic wave is achieved, so that the wave can smoothly bypass the obstacle and continue to propagate forward unaffected.

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