Two Improved Homomorphic Techniques Used in Functional Encryption Schemes

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Abstract. In this paper, we analyze the key homomorphic technique used in constructions of functional encryption schemes and point out its weakness in efficiency. Based on this, we propose two improved homomorphic techniques and show their advantages and weaknesses through the method of comparison.

1 Introduction

With the development of cloud computing and other complex networks, important progress has been witnessed recently in the field of computing on encrypted data. A number of concepts and constructions of cryptographic primitives have been raised, such as Attribute Based Encryption (Gorbunov et al. 2013; Boneh et al. 2014), Fully Homomorphic Encryption (Gentry 2009), Functional Encryption (Boneh et al. 2011; Gary et al. 2013; Agrawal 2017).

Among them, functional encryption was formalized by Boneh et al. (2011) to unify and extend the notions of identity based encryption, attribute-based encryption and Predicate Encryption. Since then, considerable progress has been made towards constructing functional encryption scheme. In 2014, Boneh et al. (2014) proposed an attributed based encryption scheme for general circuits. Based on this, Gorbunov et al. (2015) designed a predicate encryption. However, in the construction, there are two sources of leakage that prevent the scheme from achieving the strong attribute-hiding guarantee (Agrawal 2017). In order to conquer these draw-backs, Brakerski et al. (2017) proposed an improved predicate encryption scheme.

In this paper, we abstract the key homomorphic technique used in the construction of this series of functional encryption scheme. Based on this, we propose two improved homomorphic techniques and show their advantages and weaknesses through the method of comparison.

2 Basic homomorphic Techniques

In this section, we first review BGGHNSVV attribute-based encryption scheme briefly (Boneh et al. 2014), and then analyze the basic homomorphic techniques used in its construction.

2.1 Review Of BGGHNSVV Scheme

Let n be an integer, and $q = q(n)$, $m = \Theta(n \log q)$. Let $G \in \mathbb{Z}_q^{n \times m}$ be the gadget matrix. We review the attribute-based encryption scheme as follows:

- $\text{Setup}(1^\lambda, l)$: Sample a matrix with associated trapdoor $(A, T_A) \leftarrow \text{TrapGen}(1^m, 1^n, q)$; Choose random matrices $A_i \in \mathbb{Z}_q^{n \times m}$ for $i = 1, \dots, l$, and $D \in \mathbb{Z}_q^{n \times m}$. Set $\text{mpk} = (A, D, A_1, \dots, A_l)$, and $\text{msk} = (\text{mpk}, T_A)$.
- $\text{Enc}(\text{mpk}, x \in \mathbb{Z}_q^l, \mu \in \{0, 1\}^m)$: Choose a secret vector $s \leftarrow \mathbb{Z}_q^n$ and error terms $e_0, e_1 \leftarrow \chi^m$, and set $H = (A \mid x_1 G + A_1 \mid \dots \mid x_l G + A_l) \in \mathbb{Z}_q^{n \times (l+1)m}$, $e = (I_m \mid S_1 \mid \dots \mid S_l)^T \cdot e_0 \in \mathbb{Z}_q^{(l+1)m}$. Set $c = (H^T s + e, D^T s + e_1 + \lceil q/2 \rceil \mu) \in \mathbb{Z}_q^{(l+2)m}$.
- $\text{KeyGen}(\text{msk}, f)$: Compute $A_f = \text{Eval}_{\text{pk}}(f, (A_1, \dots, A_l))$; Run the algorithm $\text{SampleRight}(A, T_A, A_f, D, \sigma)$ to generate low-norm matrix R_f such that $(A \mid A_f) \cdot R_f = D$. Output $\text{sk}_f := R_f$.
- $\text{Dec}(\text{sk}_f, (x, c))$: If $f(x) \neq 0$, output $\perp$. Otherwise, parse the ciphertext as $c = (c_{in},$

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$c_1, \dots, c_t, c_{out}) \in \mathbb{Z}_q^{(t+2)m}$, and compute $c_f = \text{Eval}_{ct}$
 $(f, \{(x_i, B_i, c_i)\}_{i=1}^t) \in \mathbb{Z}_q^m$. Output
 $\text{Round}(c_{out} - R_f^T(c_{in} \mid c_f)) \in \{0, 1\}^m$.

2.2 Basic Homomorphic Techniques

In this subsection, we abstract the homomorphic techniques used in construction of the BGGHNSVV attribute-based encryption scheme. In literature, the technique is as one of basic tools to make the fine-grain access to the encrypted data feasible in the field of functional encryption.

For convenience, we define some notions as follows: Set parameters n, q, m and G as above. Choose uniform randomly matrices $B_1, B_2, \dots, B_t \in \mathbb{Z}_q^{n \times m}$, and values $x_1, x_2, \dots, x_t \in \mathbb{Z}_q$. Compute $c_i = B_i + x_i G$ for $i = 1, 2, \dots, t$.

2.2.1 Homomorphic Technique For Weighted Addition Gates

For a weighted addition gate $f : \mathbb{Z}_q^t \rightarrow \mathbb{Z}_q$,

$$f(x) = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_t x_t$$

Do the following computation:

$$\begin{aligned} B_f &= \text{Eval}_{pk}(f, B_1, \dots, B_t) \\ &= A_1 G^{-1}(\alpha_1 G) + \dots + A_t G^{-1}(\alpha_t G) \\ c_f &= \text{Eval}_{ct}(f, B_1, \dots, B_t, c_1, \dots, c_t, x_1, \dots, x_t) \\ &= c_1 G^{-1}(\alpha_1 G) + \dots + c_t G^{-1}(\alpha_t G) \\ &= [A_1 G^{-1}(\alpha_1 G) + \dots + A_t G^{-1}(\alpha_t G)] \\ &\quad + (\alpha_1 x_1 + \dots + \alpha_t x_t) G \\ &= B_f + (\alpha_1 x_1 + \dots + \alpha_t x_t) G \\ &= B_f + f(x_1, \dots, x_t) G \end{aligned}$$

This type of representation is from Boneh et al. (2014). In order to describe the basic homomorphic more clearly, we make a change of the presentation as follows:

$$\begin{aligned} H_f &= \text{Eval}F(f, B_1, \dots, B_t) \\ &= [G^{-1}(\alpha_1 G), \dots, G^{-1}(\alpha_t G)]^T \end{aligned}$$

Thus, we have

$$\begin{aligned} B_f &= A_1 G^{-1}(\alpha_1 G) + \dots + A_t G^{-1}(\alpha_t G) \\ &= (A_1, \dots, A_t) \begin{pmatrix} G^{-1}(\alpha_1 G)^T & \dots & G^{-1}(\alpha_t G)^T \end{pmatrix}^T \\ &= (A_1, \dots, A_t) H_f \\ H_{f,x} &= \text{Eval}FX(f, B_1, \dots, B_t, c_1, \dots, c_t, x_1, \dots, x_t) \\ &= [G^{-1}(\alpha_1 G), \dots, G^{-1}(\alpha_t G)]^T \end{aligned}$$

Similarly, we have the following formula:

$$\begin{aligned} c_f &= c_1 G^{-1}(\alpha_1 G) + \dots + c_t G^{-1}(\alpha_t G) \\ &= (c_1, \dots, c_t) \begin{pmatrix} G^{-1}(\alpha_1 G)^T & \dots & G^{-1}(\alpha_t G)^T \end{pmatrix}^T \\ &= (c_1, \dots, c_t) H_{f,x} \end{aligned}$$

From above formulations, we may realize that the matrices H_f and $H_{f,x}$ play a vital role of homomorphically transforming the original resource B_1 ,

$B_2, \dots, B_t$ and $c_1, c_2, \dots, c_t$ to B_f and c_f , respectively. Therefore, the matrices H_f and $H_{f,x}$ represent the basic homomorphic technique for weighted addition gates.

2.2.2 Homomorphic Technique for Multi-plication Gates

For a weighted multiplication gate $f : \mathbb{Z}_q^t \rightarrow \mathbb{Z}_q$,

$$f(x) = \alpha_1 x_1 x_2 \dots x_t$$

Since process of this homomorphic technique is complex for general t , we set $t=3$ here to illustrate the technical detail of computational process as follows:

$$\begin{aligned} c_1(x_2 x_3) G^{-1}(\alpha G) &= (x_2 x_3) B_1 G^{-1}(\alpha G) + (\alpha x_1 x_2 x_3) G \\ c_2(x_3) G^{-1}(-B_1 G^{-1}(\alpha G)) \\ &= x_3 B_2 G^{-1}(-B_1 G^{-1}(\alpha G)) - (\alpha x_2 x_3) B_1 \\ c_3 G^{-1}(-B_2 G^{-1}(-B_1 G^{-1}(\alpha G))) \\ &= B_3 G^{-1}(-B_2 G^{-1}(-B_1 G^{-1}(\alpha G))) \\ &\quad - x_3 B_2 G^{-1}(-B_1 G^{-1}(\alpha G)) \end{aligned}$$

Summing above three equations, we obtain

$$\begin{aligned} c_f &= c_1 x_2 x_3 G^{-1}(\alpha G) + c_2 x_3 G^{-1}(-B_1 G^{-1}(\alpha G)) \\ &\quad + c_3 G^{-1}(-B_2 G^{-1}(-B_1 G^{-1}(\alpha G))) \\ &= B_3 G^{-1}(-B_2 G^{-1}(-B_1 G^{-1}(\alpha G))) \\ &\quad + (\alpha x_1 x_2 x_3) G \\ &= B_f + (\alpha x_1 \dots x_t) G \end{aligned}$$

From the equation, we are able to obtain

$$\begin{aligned} H_f &= \text{Eval}F(f, B_1, B_2, B_3) \\ &= \begin{pmatrix} \alpha x_2 x_3 E \\ \alpha x_3 G^{-1}(-B_1 G^{-1}(\alpha G)) \\ G^{-1}(-B_2 G^{-1}(-B_1 G^{-1}(\alpha G))) \end{pmatrix} \\ H_{f,x} &= \text{Eval}FX(f, B_1, B_2, B_3, c_1, c_2, c_3, x_1, x_2, x_3) \\ &= \begin{pmatrix} O \\ O \\ G^{-1}(-B_2 G^{-1}(-B_1 G^{-1}(\alpha G))) \end{pmatrix} \end{aligned}$$

For general t , the outputs of algorithm $\text{Eval}F(\cdot)$ and $\text{Eval}FX(\cdot)$ should be

$$\begin{aligned} H_f &= \text{Eval}F(f, B_1, \dots, B_t) \\ &= \begin{pmatrix} (x_2 \dots x_t) G^{-1}(\alpha G) \\ (x_3 \dots x_t) G^{-1}(-B_1 G^{-1}(\alpha G)) \\ \vdots \\ G^{-1}(-B_{t-1} G^{-1}(-B_{t-2} G^{-1}(\dots G^{-1}(-B_1 G^{-1}(\alpha G)))) \end{pmatrix} \\ H_{f,x} &= \text{Eval}FX(f, B_1, \dots, B_t, c_1, \dots, c_t, x_1, \dots, x_t) \\ &= \begin{pmatrix} O \\ O \\ G^{-1}(-B_{t-1} G^{-1}(-B_{t-2} G^{-1}(\dots G^{-1}(-B_1 G^{-1}(\alpha G)))) \end{pmatrix} \end{aligned}$$

2.2.3 Homomorphic Technique for General Arithmetic Circuits

For general arithmetic circuits $f: \mathbb{Z}_q^t \rightarrow \mathbb{Z}_q$, the input wires are indexed 1 to t , the internal wires have indices $t+1, t+2, \dots, |f|-1$ and the output wire has index $|f|$, which also denotes the size of the circuit. Each gate $g_\omega: \mathbb{Z}_q^{k_\omega} \rightarrow \mathbb{Z}_q$ is indexed as a tuple $(\omega_1, \dots, \omega_{k_\omega}, \omega)$ where k_ω is the fan-in of the gate.

By the algorithms for single gates, which compute weighted additions and multiplications as described above, we can conduct the required matrices H_f and $H_{f,x}$ inductively input to output gate-by-gate.

3 Proposed homomorphic techniques

In this section, we proposed two new homomorphic methods. In both of them, the way to deal with cases of addition gate and general circuits is partially same to corresponding processes of the basic homomorphic techniques described in section 2. For simplicity, we only point out the different part in our proposed techniques, that is, how to evaluate multiplication gate.

3.1 Proposed Technique I

For a weighted multiplication gate $f: \mathbb{Z}_q^t \rightarrow \mathbb{Z}_q$,

$$f(x) = \alpha_1 x_1 x_2 \cdots x_t$$

We compute matrices H_f and $H_{f,x}$ as follows:

$$\begin{aligned} H_f &= \text{EvalF}(f, B_1, \dots, B_t) \\ &= \begin{pmatrix} (\alpha x_2 \cdots x_t)E \\ (\alpha x_3 \cdots x_t)G^{-1}(-B_1) \\ \vdots \\ \alpha G^{-1}(-B_{t-1}G^{-1}(-B_{t-2}G^{-1}(\cdots G^{-1}(-B_1)))) \end{pmatrix} \\ H_{f,x} &= \text{EvalFX}(f, B_1, \dots, B_t, c_1, \dots, c_t, x_1, \dots, x_t) \\ &= \begin{pmatrix} O \\ O \\ \alpha G^{-1}(-B_{t-1}G^{-1}(-B_{t-2}G^{-1}(\cdots G^{-1}(-B_1)))) \end{pmatrix} \end{aligned}$$

3.2 Proposed Technique II

For a weighted multiplication gate $f: \mathbb{Z}_q^t \rightarrow \mathbb{Z}_q$,

$$f(x) = \alpha_1 x_1 x_2 \cdots x_t$$

We compute matrices H_f and $H_{f,x}$ as follows:

$$\begin{aligned} H_f &= \text{EvalF}(f, B_1, \dots, B_t) \\ &= ((\alpha x_2 \cdots x_t)E \quad O \quad O \quad O)^T \\ H_{f,x} &= \text{EvalFX}(f, B_1, \dots, B_t, c_1, \dots, c_t, x_1, \dots, x_t) \\ &= ((\alpha x_2 \cdots x_t)E \quad O \quad O \quad O)^T \end{aligned}$$

4 Comparison of these three Techniques

In this section, we compare our two proposed homomorphic techniques with that used in BGGHNSVV attribute-based encryption scheme (Boneh et al. 2014) in two aspects of efficiency and functionality.

4.1 Comparison in Efficiency

The basic homomorphic technique described above is a classical tool in constructions of functional encryption scheme. However, in the process of evaluating multiplication gate, the efficiency is a bottleneck since the procedure cannot be implement-ed in a parallel way. By the construction of the basic homomorphic technique, one has to do the following computation:

$$G^{-1}(-B_{t-1}G^{-1}(-B_{t-2} \cdots G^{-1}(-B_1G^{-1}(\alpha G))))$$

when evaluating the multiplication gate. In above equation, we have to compute the matrix step by step inside and out which is the key problem to result in low efficiency. In order to conquer the weakness of sequential computation, a natural approach is to reduce the number of nests in above computational formula.

In our proposed homomorphic technique I, we need to compute

$$\alpha G^{-1}(-B_{t-1}G^{-1}(-B_{t-2} \cdots G^{-1}(-B_1)))$$

Thus, we reduce the number of nests in the computation from n to $n-1$. Hence, our proposed homomorphic technique is more efficient than the basic one.

Though efficiency of the proposed technique I is improved, the advantage is only one time computation of G^{-1} . In our proposed homomorphic technique II, we substantially improve the efficiency since we only need to compute $(\alpha x_2 \cdots x_t)E$.

4.2 Comparison in Functionality

One of vital properties of the basic homomorphic technique which make functional encryption scheme (including attribute-based encryption scheme and predicate encryption scheme) feasible is that during process of computing the matrix H_f , algorithm $\text{EvalF}(\cdot)$ only takes as input the predicate function f and the public parameters $B_1, \dots, B_t$, not including the attribute information $x_1, \dots, x_t$. In our proposed homomorphic technique I, this vital property is maintained since H_f has nothing to do with the attribute information $x_1, \dots, x_t$. However, our proposed homomorphic technique II does not succeed in keeping this property. Therefore, we cannot substitute the basic homomorphic technique with the proposed homomorphic technique II although the latter is more efficient. Nevertheless, replacing with the proposed homomorphic technique I not only ensures the functionality, but also improves the efficiency.

5 Conclusions

In this paper, we analyze the key homomorphic techniques used in the constructions of functional encryption schemes, and point out its weakness in efficiency. In addition, we propose two improved homomorphic techniques and show their advantages and weaknesses through the method of comparison.

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