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# Parameter Estimation of Chirp for Underwater Acoustic Channel

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**Abstract.** To overcome the performance degradation of conventional Chirp parameters estimation methods in underwater acoustic multipath channels, a novel parameters estimation method based on Fractional Fourier transform (FRFT) and Fourier transform (FFT) was proposed. Firstly, the Chirp rate was estimated by searching for the best degree of Chirp after Fractional Fourier transform. Secondly, the Chirp signal turned into a single-frequency signal by means of Chirp rate equalization. Finally, FFT was applied to estimate the initial frequency. The simulation experiment show that the proposed algorithm enhanced about 1dB RMSE performance on Chirp initial frequency compared with FRFT while the computational complexity is similar to FRFT.

## 1 Introduction

Chirp, a mature non-stationary signal, is widely used in radar and underwater acoustic communication. The Chirp rate and initial frequency are the key parameters for the Chirp signal and the research on the estimation of the two parameters has been widely studied. In the context of traditional wireless channels, many domestic and foreign scholars have proposed many methods of parameter estimation for Chirp signals, such as ML[1], the short-time Fourier transform[2], wavelet transform[3], Wigner-Hough transform[4], Wigner-Ville distribution[5] and FRFT[6]. FRFT can be understood as Chirp decomposition, is particularly fit for the process of Chirp signals. The multi-component Chirp signal was estimated by FRFT in [7]. [8] analyzed the estimation performance of FRFT for Chirp parameter estimation and the estimation performance of different discrete algorithms. [4] utilized the FRFT origin moment as statistic to estimate the parameter of Chirp. [9] proposed a method for detecting and estimating the Chirp signal via simple fractional Fourier under the condition of low signal to noise ratio. [10] proposed an improved FRFT algorithm to improve the precision of parameter estimation in the case of low duty ratio.

The above algorithm has achieved good results in traditional wireless channels. However, in practical applications, Chirp signals are widely applied in underwater acoustic communication. The proposed algorithm has an estimated performance decline under the background of underwater acoustic multipath channel. [11] proposed a way to transmit Sine-Chirp signals to detect channels and signal parameters and channel information were estimated, but not suitable for passive sonar signal processing.

To solve the above problems, this paper proposes a parameter estimation method based on FRFT and FFT for underwater acoustic channel and it does not need to transmit training sequence. Theoretical analysis and simulation results show that the proposed algorithm can improve the performance of parameter estimation.

## 2 Signal model

The expression of the single component Chirp and the expression of the time-varying impact response of the underwater acoustic multipath channel are expressed as:

$$s(t)=\begin{cases} A\exp(j(2\pi f_0 t+k\pi t^2)), t\in[-T/2,T/2] \\ 0 \qquad\qquad others \end{cases} \tag{1}$$

$$h(t,\tau)=\sum_{l=1}^{L}A_l(t)\delta(\tau-\tau_l) \tag{2}$$

Where $A$ , $f_0$ and $k$ are the amplitude, initial frequency and Chirp rate, and $A_l$ , $\tau_l$ are the fading and delay of the channel. The received signal is as follows:

$$\begin{aligned} r(t) &= h(t,\tau)*s(t)+n(t) \\ &= \sum_{l=1}^{L}AA_l s(t-\tau_l)+n(t) \\ &= \sum_{l=1}^{L}AA_l\exp(\mathrm{j}(2\pi f_0't+k'\pi t^2+\varphi'))+n(t) \end{aligned} \tag{3}$$

$$\begin{aligned} k' &= k \\ f_0' &= f_0-k\tau_l \\ \varphi' &= \pi k\tau_l^2-2\pi f_0\tau_l \end{aligned} \tag{4}$$

Where $n(t)$ is the white Gaussian noise, obviously, $r(t)$ is still a Chirp signal, the Chirp rate and initial frequency convert to $k'$ and $f_0'$ .For the N component Chirp, the received signal is as follows:

It can be seen that the received signal is the sum of the signal which passed multipath channel. The aim in this paper is the parameter estimation of Chirp signals through multipath channels.

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$$
\begin{aligned}
R(t) &= \sum_{i=1}^{N}\sum_{l=1}^{L} A_i A_l \exp(\mathrm{j}(2\pi f_{0i}(t-\tau_l)+k_i\pi(t-\tau_l)^2))+n_i(t) \\
&= \sum_{i=1}^{N}\sum_{l=1}^{L} A_i A_l \exp(\mathrm{j}(2\pi f_{0i}{}'t+k_i{}'\pi t^2+\varphi_i{}'))+n_i(t)
\end{aligned} \tag{5}
$$

## 3 The parameter estimation algorithm based on FRFT and FFT

### 3.1 The estimation of Chirp rate

In recent years, as a new time-frequency analysis tool, FRFT is widely accepted by researchers in the field of signal processing. For a given Chirp signal, there exists an optimal fractional order to make its energy aggregate to a maximum value. Therefore, FRFT is especially suitable for the process of Chirp signals.

The FRFT of the time signal is defined as follows:

$$X_p(u)=\{F^p x(t)]\}(u)=\int_{-\infty}^{+\infty}\tilde{K}_p(u,t)x(t)dt \tag{6}$$

Where $\tilde{K}_p(u,t)=A_\alpha \exp[\mathrm{j}\pi(u^2\cot\alpha-2ut\csc\alpha+t^2\cot\alpha)]$ is called the kernel of FRFT, meanwhile $A_\alpha=\sqrt{1-j\cot\alpha}$, $\alpha=p\pi/2, p\neq 2n$. For the convenience of calculation, through variable substitution equation (6) can be expressed as:

$$
\begin{aligned}
X_p(u) &= \{F^p x(t)]\}(u)=\int_{-\infty}^{+\infty}K_p(u,t)x(t)dt \\
&= \begin{cases} B_\alpha\int_{-\infty}^{+\infty}\exp(\mathrm{j}\dfrac{t^2+u^2}{2}\cot\alpha-\dfrac{jtu}{\sin\alpha})x(t)dt & \alpha\neq n\pi \\ x(t) & \alpha=2n\pi \\ x(-t) & \alpha=(2n+1)\pi \end{cases}
\end{aligned} \tag{7}
$$

Where $B_\alpha=\sqrt{\dfrac{1-j\cot\alpha}{2\pi}}$ is called amplitude factor, after substituting equation (1) in equation (7):

$$F^p\{s(t)\}(u)=AB_\alpha\int_{-\infty}^{\infty}\exp\{\mathrm{j}\pi[u^2\cot\alpha-2(u\csc\alpha-f_0)t+(\cot\alpha+k)t^2]\}dt \tag{8}$$

While $k=-\cot(\dfrac{p\pi}{2})$, $F^p\{s(t)\}(u)$ takes the maximum value and $p=2\alpha/\pi$ is called best order. According to [12], the Chirp signal has a huge value to adapt to underwater acoustic channel, FRFT is sensitive to the variation of Chirp rate, that is the basics to estimate Chirp rate from multi-component Chirp signals.

### 3.2 The estimation of initial frequency

In order to estimate the initial frequency, we use equation (9) to transform the Chirp signal into single frequency signal.

$$
\begin{cases}
R_1(t)=R(t)*\exp(-\mathrm{j}k_1\pi t^2) \\
R_2(t)=R(t)*\exp(-\mathrm{j}k_2\pi t^2) \\
\qquad\vdots \\
R_N(t)=R(t)*\exp(-\mathrm{j}k_N\pi t^2)
\end{cases} \tag{9}
$$

Taking $R_1(t)$ for an example, the rest can also be proved.

$$
\begin{aligned}
R_1(t) &= R(t)*\exp(-\mathrm{j}k_1\pi t^2) \\
&= \sum_{l=1}^{L} A\,A_l \exp(\mathrm{j}(2\pi(f_{01}-k_1\tau_l)t-2\pi f_{01}\tau_l+k_1\pi\tau_l{}^2)) \\
&+ \sum_{i=2}^{N}\sum_{l=1}^{L} A_i A_l \exp(\mathrm{j}(2\pi f_{0i}(t-\tau_l)+(k_i-k_1)\pi(t-\tau_l)^2))+n_i{}'(t)
\end{aligned} \tag{10}
$$

The first item in equation (10) corresponds to the converted component of the first Chirp signal and the location of the peak position corresponding to the initial frequency. The second items correspond to the output signals of other Chirp signals, and the spectrum is distributed throughout the frequency domain.  is the product of Gauss white noise and frequency modulation component, also distributed in the whole frequency domain. So the initial frequency can be obtained by extracting FFT peak.

### 3.3 Joint estimation of Chirp parameters and channel time-delay

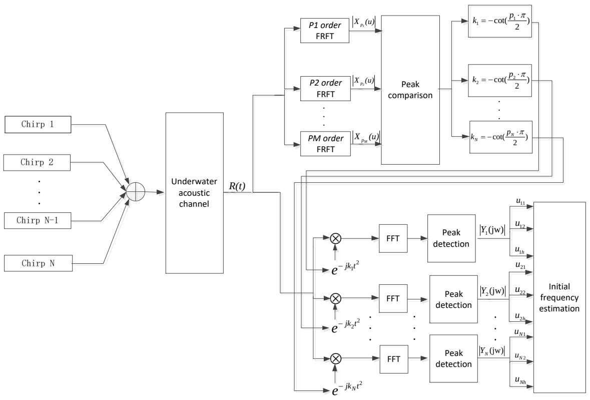

**Fig. 1.** The structure for the estimation of Chirp parameter and channel time-delay

The frame is shown in the diagram, we process the signal in two steps. Firstly, after the process of FRFT, the peak can be obtained by the coarse to fine method in literature [7]. The peak number is the number of the Chirp signal, and the peak order is the best order of the corresponding signal. Thus, we get the Chirp rate. Secondly, after getting the product of signal and the frequency component, signal become a single frequency signal. The maximum frequency is the estimated value of the starting frequency after FFT.

## 4 Simulation

In this section, the performance of the proposed algorithm was investigated and compared with the traditional FRFT algorithm.

### 4.1 Experiment 1

In order to verify the validity of the proposed algorithm, we consider three Chirp signals are sent, all of the amplitude is 1, the frequency modulation and the starting frequency are 10000Hz/s, 20000Hz/s, 30000Hz/s and 1500Hz 2000Hz, 2500Hz. The simulation channel using Bellhop ray model to simulate shallow underwater acoustic channel. The signal to noise ratio is -5dB and the impulse response is shown in Figure 2. Figure 3 show the search of peak for FRFT. Experimental results are shown as follows.

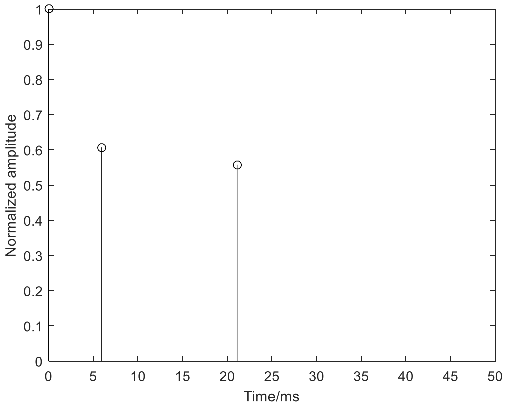

**Fig. 2.** The normalized impulse response

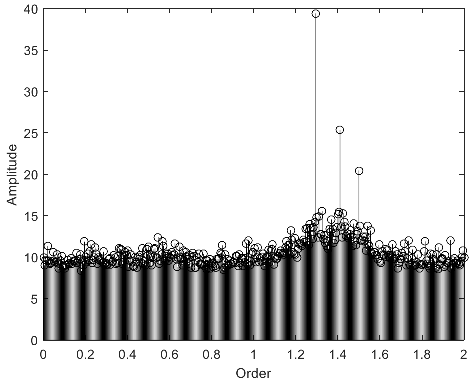

**Fig. 3.** The maximum amplitude of FRFT

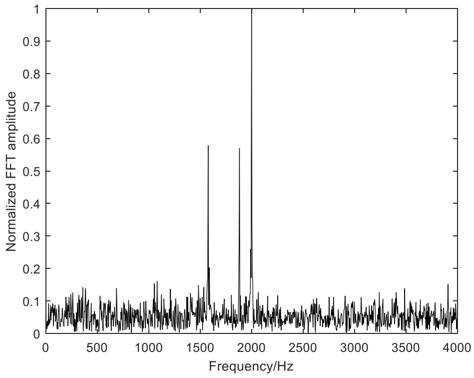

**Fig. 4.** The spectrogram after compensating by the estimated Chirp rate

**Table 1.** The parameters estimation of multiple Chirp

| Chirp signal | Method | Computing time/s | $f_0$ | |
|---|---|---|---|---|
| | | | Actual value | Estimated value |
| Signal 1 | FRFT | 2.81 | 1500 | 1497.5 |
| | FRFT+FFT | 2.82 | 1500 | 1500.1 |
| Signal 2 | FRFT | 2.81 | 2000 | 1995.4 |
| | FRFT+FFT | 2.83 | 2000 | 2000.1 |
| Signal 3 | FRFT | 2.81 | 2500 | 2494.7 |
| | FRFT+FFT | 2.82 | 2500 | 2499.8 |

From table 1, we can find that the proposed algorithm is more accurate than FRFT in the estimation of the starting frequency with the approximate calculation time. FRFT [7] loses much information in the frequency domain when it rotates to the best angle. The proposed algorithm in this paper obtains the initial frequency in the frequency domain axis after the frequency compensation, thus the proposed algorithm improves the performance of parameter estimation. The three peaks in figure 3 correspond to three Chirp rate, After Chirp rate compensation, we can get three frequency as shown in figure 4. As far as complexity is concerned, FRFT and the proposed algorithm is approximately equal, so the computation time is similar. The simulation results are consistent with the theoretical analysis.

## 4.2 Experiment 2

In order to explore the influence of the signal-to-noise ratio of data on the experiment, we add signal-to-noise varies from -10dB to 5dB, and the step length is 1dB. The sending signals are the same as signals in experiment 1. The result was presented on figure 5.

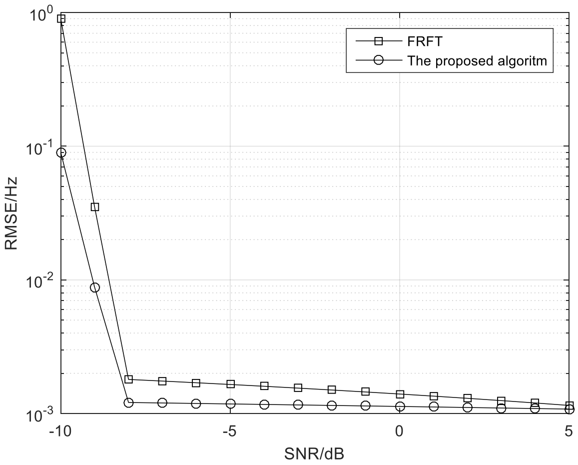

**Fig. 5.** RMSE of initial frequency

Figure 5 and figure 6 give the RMSE of the initial frequency and the maximum delay of the channel with the SNR. It is clear that all the RMSE of the algorithm decrease with the increase of SNR. When the SNR is lower than -8dB, RMSE increases rapidly with the decrease of the SNR, the proposed algorithm enhance about 1dB RMSE compared with FRFT, therefore, the proposed algorithm can effectively improve the estimation performance of initial frequency and channel delay under different signal-to-noise ratio. The curves in figure 5 and figure 6 is similar, and the reason is that time delay is determined by the frequency value and Chirp rate and Chirp rate is precise, so the error is mainly from the frequency. Figure 7 give the RMSE of the channel Amplitude.

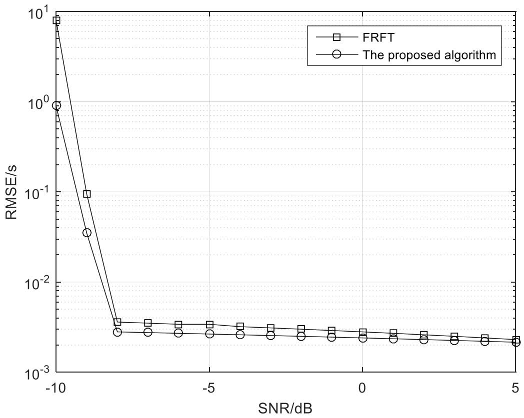

**Fig. 6.** RMSE of channel maximum time-delay

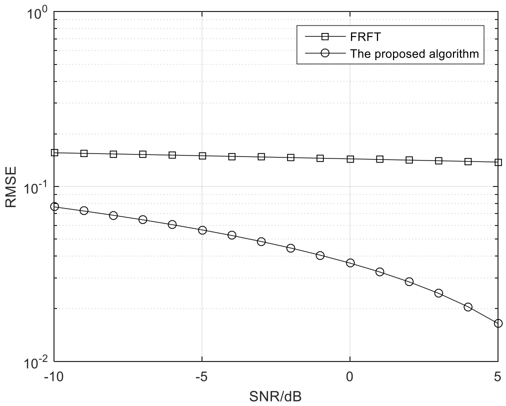

**Fig. 7.** RMSE of channel Amplitude

It is clear that the RMSE of the algorithm decrease with the increase of SNR and the proposed algorithm has a low RMSE compared with FRFT, which show the proposed algorithm is superior to FRFT on the condition of underwater acoustic channel. The simulation results are consistent with the theoretical analysis.

## 5 Conclusion

Aiming at the problem that traditional parameters estimation algorithm has poor performance in underwater acoustic multipath channel for Chirp signal, this paper proposed an algorithm based on FRFT and FFT. Simulation results show that the proposed algorithm can improve RMSE of initial frequency and channel delay about 1dB under the similar complexity of FRFT algorithm. It has great application prospects.

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