Prediction of Gas Dissolved in Power Transformer Oil by Non-equidistant Multivariable Grey Model

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Abstract.Power transformer is an essential component in the power systems. The concentration of fault characteristic gases dissolved transformer oil is essential to the insulation fault diagnosis. The concentration prediction of the gases is an important supplement for periodical testing. A NMGM(1, 5)model using Non-equidistance Multivariable grey theory for the five characteristic gases dissolved in transformer oil, i. e. hydrogen, methane, ethane, ethylene, acetylene, was constructed. In the built model, the interaction among these gases was comprehensively considered and the disadvantage that only one index extracted from the signal or each index that was dealt with separately was made up, meanwhile, the scope of application is enlarged. Two actual prediction cases were analyzed and the results were compared with those obtained by Non-equidistant GM(1, 1)model. The comparison result indicates the validity and efficiency of the proposed model.

1 Introduction

As test data processing, some information is known and some is unknown, test data processing system is a grey system. Making use of grey system, we can forecast data to unknown information in future. Grey system theory is established in poor information (few sample), which build differential equation model with alone feature by data transform process to fully dig up apparent and latent information in relying on few data and discover knowing orderliness from un-orderliness data and reason the rule in future [1-4]. Fault mechanism of transformer is complex, the relationship between the insulation latent faults and fault feature, some is known, some is unknown, has uncertainty. The insulation fault diagnosis of power transformer is grey system, in recent years, grey model has been predicted the concentration of the gas dissolved in power transformer oil. The Ref.[5] predicts the gas dissolved in power transformer oil by grey GM(1,1) model, the Ref.[6] predicts the gas dissolved in power transformer oil by improved grey GM(1,1) model, the Ref.[7] predicts the gas dissolved in power transformer oil by Multivariable Grey Model (MGM(1,n)) which is extended from GM(1,1) in n variables condition but is not simple combination and is also differ from GM(1,n) which only builds one differential equation with multi-variable one order. But the period of transformer fault detection is non-equidistant, so the GM(1,1) and MGM(1,n) model is not work. The Ref.[8] introduced non-equidistant multi-variable grey model GM(1,M)

which has high fitting precision and high forecasting precision. In this study, we predict concentration of gas dissolved in power transformer oil by non-equidistance multivariable grey model which based on the Ref.[8]. This proposed model not only has high forecasting precision, but also provides the guidance basis to forecasting gas dissolved in power transformer oil, this model and method possesses the vast application foreground in the data processing of domain at other engineering.

2 Non-equidistance Multivariable Grey Model (NMGM (1, n))

This section describes the main structures and operations of the Non-equidistant Multivariable Grey Model (NMGM(1,n)).

Definition 1 Let sequence $X_i^{(0)} = \{X_i^{(0)}(t_1), X_i^{(0)}(t_2), \dots, X_i^{(0)}(t_m)\}$ and $\Delta t_k = t_{(k)} - t_{(k-1)}, k = 2, \dots, m$ if $\Delta t_k \neq \text{const}$ $X_i^{(0)}$ is called non-equidistant sequence.

Definition 2 Let sequence

$$X_i^{(1)} = \{X_i^{(1)}(t_1), X_i^{(1)}(t_2) \dots X_i^{(1)}(t_m)\}, \text{if}$$

$$X_i^{(1)}(t_{k+1}) = X_i^{(1)}(t_k) + X_i^{(0)}(t_{k+1})\Delta t_{k+1}$$

$$k = 1, 2 \dots m - 1 \quad (1)$$

and $X_i^{(1)}(t_1) = X_i^{(0)}(t_1)$ we call $X_i^{(1)}$ is corresponding 1-accumulated generation operator (1-AGO) sequence $X_i^{(0)}$.

NMGM(1,n)model with n-variable 1-order differential equations is as follows:

$$\begin{cases} \frac{dX_1^{(1)}}{dt} = a_{11}X_1^{(1)} + a_{12}X_2^{(1)} + \dots + a_{1n}X_n^{(1)} + b_1 \\ \frac{dX_2^{(1)}}{dt} = a_{21}X_1^{(1)} + a_{22}X_2^{(1)} + \dots + a_{2n}X_n^{(1)} + b_2 \\ \vdots \\ \frac{dX_n^{(1)}}{dt} = a_{n1}X_1^{(1)} + a_{n2}X_2^{(1)} + \dots + a_{nn}X_n^{(1)} + b_n \end{cases} \quad (2)$$

Notes

$$X^{(0)}(t_k) = (X_1^{(0)}(t_k), X_2^{(0)}(t_k), \dots, X_n^{(0)}(t_k))^T,$$

$$X^{(1)}(t_k) = (X_1^{(1)}(t_k), X_2^{(1)}(t_k), \dots, X_n^{(1)}(t_k))^T$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Then, the formula (2) can be expressed as:

$$\frac{dX^{(1)}}{dt} = AX^{(1)} + B \quad (3)$$

the continuous time response function of the formula (3) is

$$X^{(1)}(t) = e^{At}(X^{(1)}(0) + A^{-1}B) - A^{-1}B \quad (4)$$

The prediction accuracy of grey model depends on distinguishing values A and B, while, A and B depend on the background value, the general background value is as follows:

$$Z_i^{(1)}(k+1) = \frac{1}{2}(X_i^{(1)}(t_{k+1}) + X_i^{(1)}(t_k)) \quad (5)$$

The actual background value is as follows:

$$Z_i^{(1)}(k+1) = \int_{t_k}^{t_{k+1}} X_i^{(1)}(t)dt \quad (6)$$

$$(i = 1, 2, \dots, n)$$

Using exponential function vector fit $X_i^{(1)}(t)$ the formula (6) can be expressed as follows:

$$Z_i^{(1)}(k+1) = \frac{X_i^{(1)}(t_{k+1}) - X_i^{(1)}(t_k)}{\ln X_i^{(1)}(t_{k+1}) - \ln X_i^{(1)}(t_k)} \Delta t_{k+1} \quad (7)$$

Notes $a_i = [a_{i1} \ a_{i2} \ \dots \ a_{in} \ b_i]^T$ by least square method, the estimated value $\hat{a}_i$ of parameter a_i is as follows:

$$\hat{a}_i = (L^T L)^{-1} L^T \gamma_i (i = 1, 2, \dots, n) \quad (8)$$

Where

$$L = \begin{bmatrix} -Z_1^{(1)}(2) & -Z_2^{(1)}(2) & \dots & -Z_n^{(1)}(2) & 1 \\ -Z_1^{(1)}(3) & -Z_2^{(1)}(3) & \dots & -Z_n^{(1)}(3) & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ -Z_1^{(1)}(m) & -Z_2^{(1)}(m) & \dots & -Z_n^{(1)}(m) & 1 \end{bmatrix}$$

$$\gamma_i = \begin{bmatrix} X_i^{(0)}(2) \\ X_i^{(0)}(3) \\ \vdots \\ X_i^{(0)}(m) \end{bmatrix}$$

By distinguishing, we obtain the distinguishing values $\hat{A}$ and $\hat{B}$ of parameters A and B are as follows:

$$\hat{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \dots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \dots & \hat{a}_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \dots & \hat{a}_{nn} \end{bmatrix} \quad \hat{B} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \vdots \\ \hat{b}_n \end{bmatrix}$$

The calculating value of NMGM(1,n) is formula(9).

$$\hat{X}^{(1)}(t_k) = e^{\hat{A}(t_k - t_1)}(X^{(1)}(t_1) + \hat{A}^{-1}\hat{B}) - \hat{A}^{-1}\hat{B} \quad (9)$$

$$k = 1, 2, \dots, m$$

By the inverse accumulated generating operation, the fitting data or forecast of the original data is as follows:

$$\begin{cases} \hat{X}_i^{(0)}(1) = X_i^{(0)}(1) \\ \hat{X}_i^{(0)}(k) = (\hat{X}_i^{(1)}(k) - \hat{X}_i^{(1)}(k-1)) \frac{1}{\Delta t_k} \\ i = 1, 2 \dots n, k = 1, 2, \dots, m, m+1, \dots \end{cases}$$

Where, when $k \leq m$, $\hat{X}_i^{(1)}(t_k)$ is fitting value

and when $k > m$, $\hat{X}_i^{(1)}(t_k)$ is forecasting value.

Define absolute error of the I variable is

$$q_i(t_k) = \hat{X}_i^{(0)}(t_k) - X_i^{(0)}(t_k) \quad (10)$$

The relative error of the I variable is

$$e_i(k) = \frac{\hat{X}_i^{(0)}(k) - X_i^{(0)}(k)}{X_i^{(0)}(k)} * 100\% \quad (11)$$

The mean value of relative error of the I variable is

$$\varepsilon_i(avg) = \frac{1}{m} \sum_{k=1}^m |e_i(k)| \quad (12)$$

The mean value of relative error of all data is

$$\varepsilon(avg) = \frac{1}{nm} \sum_{i=1}^n \sum_{k=1}^m |e_i(k)| \quad (13)$$

the ref. [9] introduced the accuracy detecting method, the grades of mean relative error accuracy is as tab.1

Table 1. Reference of accuracy detecting grades

accuracy grades	relative error
first	0.01
second	0.05

third	0.10
forth	0.20

The model is noted as NMGM(1, n) that can be modeled and can be used to forecast or fitting data. Based on the number of variables in actual case, we may obtain NMGM (1,2) model, NMGM (1,3) model, model NMGM(1,4) and so on..

3Concentration prediction of fault gases in power transformer oil by NMGM (1,5)

A NMGM(1 , 5)model using Non-equidistance Multivariable grey theory for the five characteristic gases dissolved in transformer oil, i. e. hydrogen, methane, ethane, ethylene, acetylene, was constructed. The interaction among these gases was comprehensively considered .

3.1 Case I

The DGA data is come from Ref.[6], as table. 2 The data of table. 2 are non-equidistant sequence, so the GM(1,1)model and MGM(1,n) model are not fit . In this case ,we fit the gases using NMGM(1,5) model and non-equidistance GM(1,1) model respectively, The mean value of relative error of all data is of the NMGM(1,5)

model is $\frac{1}{5} \sum_{i=1}^5 \varepsilon_i(avg) = 2.423$, The mean value of

relative error of all data is of the NGM(1,5) model is

$$\frac{1}{5} \sum_{i=1}^5 \varepsilon_i(avg) = 5.7895 .$$

From the results, it is obvious that NMGM(1,5) model can better reflect the relation between hydrogen and methane, ethane, ethylene and acetylene. the proposed model has high fitting precision and high forecasting precision.

3.2 Case II

From August 17th 1984 to March 5th 1985, The 5th transformer of Liu Jia-Xia had collected seven sets of DGA data. As Tab.3 .“Gas ratios” of DGA prediction on march 5th 1985 by non-equidistant multivariable grey model is $C_2H_2 / C_2H_4 = 0.0746$, $CH_4 / H_2 = 1.0224$, $C_2H_4 / C_2H_6 = 1.875$, nodes for “gas ratios” should be 022 , Thus can predict the transformer faults is hyperthermia and superheating fault which is confirmed by the operation department. From the results, it is obvious that NMGM(1,5) model has higher forecasting precision than Non-equidistant GM(1,1) model.

Table 2. Prediction of gas dissolved in power transformer oil uL / L

Gas/time	93-08-1 5	93-09-2 3	93-10-06	93-10-2 7	93-11-0 9	93-11-27	93-12-20	$\varepsilon_i(avg)$
H ₂	35.8	59.7	89.5	116	187	220	292	
NGM(1,1)	35.80	56.27	86.04	124.71	185.46	248.45	280.75	
Relative error	0	-5.74	-3.86	7.51	0.082	12.93	-3.19	5.5517
NMGM(1,5)	35.80	61.1986	87.0367	115.040	199.769	223.393	291.4245	
Relative error	0	2.51	2.75	-0.83	6.83	1.54	-0.2	2.4433
CH ₄	61.6	74.1	129	183	250	294	325	
NGM(1,1)	61.60	64.17	132.49	195.80	236.16	270.41	318.98	
Relative error	0	-13.4	2.71	6.99	-5.54	-8.02	1.85	6.418
NMGM(1,5)	61.60	79.082	128.1611	183.91	261.129	299.670	321.989	
Relative error	0	6.72	-0.65	3.78	4.45	3.33	0.93	3.31
C ₂ H ₆	27.5	30.4	51.8	69.7	104	166	196	
NGM(1,1)	27.50	29.0549	50.3851	70.4767	108.874	170.337	195.94	
Relative error	0	4.42	-2.73	1.11	4.69	2.61	0.0327	2.5988
NMGM(1,5)	27.50	30.6382	53.2135	68.7704	104.138	165.692	197.0469	
Relative error	0	2.382	2.73	-1.33	0.13	-0.19	0.53	1.2153
C ₂ H ₄	95.5	100	191	307	467	514	598	
NGM(1,1)	95.50	99.7308	207.047	319.152	488.333	509.940	566.80	
Relative error	0	-2.692	8.4	3.96	4.57	0.079	-5.2135	4.1541

NMGM(1,5)	95.50	101.138	197.7572	308.405	479.21	515.89	597.010	
Relative error	0	1.1389	3.54	0.046	2.61	0.037	-0.017	1.2315
C ₂ H ₂	0	0	0	1.47	3.61	2.15	3.78	
NGM(1,1)	0	0	0	1.47	2.6704	1.9319	2.8281	
Relative error	0	0	0	0	-26.03	-10.14	-25.18	10.225
NMGM(1,5)	0	0	0	1.47	2.9642	2.1864	3.9278	
Relative error	0	0	0	0	-17.89	1.69	3.91	3.915

Table 3. Prediction of gas dissolved in power transformer oil uL / L

gas/time	H ₂	CH ₄	C ₂ H ₆	C ₂ H ₄	C ₂ H ₂	ε (avg)
1984-08-17	77	101	134	14	10	
1984-09-28	79	108	142	19	8	
1984-10-31	68	88	109	10	9	
1984-11-28	87	90	132	12	12	
1984-12-26	101	128	145	13	10	
1985-01-28	141	141	137	13	11	
1985-03-05	134	137	154	13	11	
NGM(1,1)	135.7730	139.3716	141.9355	11.9618	11.3261	
relative error	1.32	1.73	-7.83	-7.99	2.96	4.366
NMGM(1,5)	133.9954	138.0908	148.5151	12.7542	11.2837	
relative error	-0.0034	0.80	-3.56	-1.89	2.58	1.7667

4 CONCLUSIONS

Dissolved gas analysis (DGA) is the essence to evaluate the state of transformer insulation and analyze transformer insulation faults. With non-equidistance multi-variable grey model the interaction among these gases was comprehensively considered and the disadvantage that only one index extracted from the signal or each index was dealt with separately was made up, meanwhile, the proposed model overcome the deficiency that the MGM(1,n) model don't fit the non-equidistant, The method can be used for model establishing on equal interval, as well as on non-interval. Moreover, non-equidistance multi-variable grey model's fitting precision and prediction is advanced and the scope of application is enlarged.

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