The Moon influence on Vertical Lines Deviations as Geodetic Instruments of Urban Territories Development

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Abstract. The increase in the complexity of facilities entails the requirements increasing for the accuracy of geodetic measurements and the need to revise some existing techniques for accounting of factors' variety that affect the measurements results of urban development. Such factors include the Moon influence on the magnitude and direction of gravity. In order to determine the need for adjusting the methods of accounting for the effects of the moon on the geodetic measurements results, an assessment was made of their maximum influence on the value of the earth's gravity force and the deviation of plumb lines. The results of a computational experiment for estimating the extreme values of the variation in the plumb lines' deviations (up to 1 "), the oscillations of the measured exceedances (up to 0.5 mm per 100 m) and variations in the gravity (up to $5.44 \times 10^{-5} \mu g / s^2$) are sufficient grounds for correcting existing methods of precision leveling and gravimetric measurements.

1 Introduction

The Moon influence on the plumb line and the size of the gravity force is known, but there is some uncertainty in this matter. In the literature one can find conflicting information about the magnitude of this influence. The authoritative Encyclopedia [1] reported that the plumb line under the Moon influence is up to 0.025 ". Other sources claim that the plumb line under the Moon influence is up to several seconds. In some works, the Moon gravitation field is limited to a distance of about 10,000 km, and its influence on the force of gravity at points on the earth's surface is generally denied [3].

In [4] resource, in order to calculate the correction δh , the excess for deviation of a plumb line under the Moon action is given by formula

$$\delta h = 0,84 \cdot 10^{-6} s \sin 2z \cos(A - a),$$

where s is the distance between two points, z is the zenith distance of the Moon, A is the azimuth of the leveling line, and a is the azimuth of the Moon. In the practical manual [6], for the same purposes, the "known" formula is proposed without reference to the source

$$\delta h = 0,68 \cdot 10^{-6} s \sin 2z \cos(A - a).$$

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It should be said that this document is normative, mandatory for application.

Without going into details, we note that both of the above formulas cause doubts. First, the use of constants, as will be shown later, is in the principle incorrect. Secondly, it is necessary to take into account other factors directly influencing the deviation of vertical lines under the Moon influence. The Moon impact on the deviation of vertical lines and the gravity variation can be direct and indirect. By the indirect one we mean not the Moon influence by itself, but the water masses moving under its influence (ebb and flow) and providing thereby an indirect effect on the gravity on the Earth's surface. For example, the article [2] was devoted to the problem of determining the indirect influence. In this paper we consider the problem of determining only the direct influence of the Moon as plumb lines, and on the gravity values. The Earth compression from the poles in our case can be neglected without compromising accuracy. Therefore, we will consider the Earth as an absolutely rigid spherical body and assume that a vertical line at an arbitrary point on the Earth's surface coincides with the radius vector of this point. The Moon impact on such a point is manifested in two ways. First, the point experiences gravity by the Moon; secondly, it is subjected to the action of the centrifugal force caused by the rotation of the Earth-Moon pair around their mass center. To assess the appropriateness of taking into account the Moon influence on the value and direction of the gravity force at points on the Earth's surface, it is necessary to determine the maximum changes in these quantities under the Moon action. With this goal, we determine the changes in the projection g of gravity by the radius vector of the determined point and the projection of f to its normal.

It is obvious that the Moon extreme influence will be manifested in the nearest and most distant from it points of the Earth's surface. (At the same time, this will be the simplest case.) Such points in the plane of the Moon orbit are shown in Fig. 1, where A is the Earth's nearest point to the Moon, B is its point furthest from the Moon, O is the center of the Earth, L is the center of the Moon, C is the center of mass of the Earth-Moon pair.

Fig.1. Points the extreme values of the variations of gravity

At A point, the Moon gravity force $F_1(A)$ and the centrifugal force $F_2(A)$ arising within the point around the center of mass C , rotates are directed in one side of the Moon. Under their influence, the force of gravity decreases. At point B , the gravity force $F_1(B)$ is directed in the same direction, and the centrifugal force $F_2(B)$ is in the opposite direction. The value of the Moon gravitational force F_1 at points A and B is calculated from formula

$$F_1 = \gamma \frac{mM_1}{S^2}, \quad (1)$$

where γ is the gravitational constant, m is the mass of the gravitated body, we take it here and further equal to 1 kg, is the mass of the Moon, and S is the distance from the point under consideration to the Moon center. The centrifugal force F_2 at an arbitrary point on the line is AB

$$F_2 = m\omega^2 \rho, \quad (2)$$

where ω is the angular velocity of rotation, ρ is the distance from this point to the mass center, the point C , located at a distance c from the Earth center along the line Earth-Moon, equal to

$$c = R \frac{M_1}{M_1 + M_2}, \quad (3)$$

where R is the distance between the centers of the Earth and the Moon at the observations time, M_1 is the mass of the Moon, M_2 is the mass of the Earth. Since the distance R varies with time, the distance value c also varies. The revolution period of the Earth-Moon pair around their center of mass is a sidereal month. At point A and point B , the distance ρ to the center of mass C is equal to

$$\rho = r - c,$$
$$\rho = r + c,$$

where r is the Earth radius.

The values of the necessary constants are presented in Table. 1, the calculations results using the above formulas are given in Table. 2, where the disturbing force is considered positive if its direction coincides with the direction of the gravity force, and negative - otherwise. From the table 2 it follows that although the direction of gravity force does not change its magnitude changes appreciably. Consequently, at these variation points, gravity should be taken into account in high-precision gravimetric operations, but they do not influence the leveling results.

Table 1. The values of the constants

Quantities	Value
Gravitational constant γ , $\text{m}^3/(\text{s}^2 \text{ kg})$	$6,672 \cdot 10^{-11}$
The Moon's mass M_1 , kg	$7,35 \cdot 10^{22}$
The Earth's mass M_2 , kg	$5,97 \cdot 10^{24}$
Average radius of the Earth r , m	$6,371 \cdot 10^6$
Average gravity on the Earth G , mgk/s^2	9,813
The distance between the centers of the Earth and the Moon at perigee R , km	356 400- 370400
Distance between the centers of the Earth and the Moon at apogee R , km	404 000 – 406700
Sidereal month, day period	27,322
The angular velocity of rotation around the center of the pair Earth-Moon mass ω , radian /s	$2,662 \cdot 10^{-6}$

From the table 2 it follows that, due to a change in the distance from the Earth to the Moon, the perturbing force can vary by a factor of 1.31, which was implied above, when we talked about the incorrectness of the constants use in the formulas given earlier. In addition, during the Moon passage from perigee to apogee, the sum of the perturbing forces F_1 (B) + F_2 (B) can vary by 1.34 times.

Table 2. The variation of gravity at the points A and B

Quantities	Value	
	At perigee	At apogee
The Earth Radius r , km	6 371	6 371
Distance between the centers of the Earth and the Moon R , km	356 400	406 700
Distance c , km	4 334	4 946
Distance to the Moon $S(A)$, km	350 029	400 329
Force $F_1(A)$, mgk/s^2	-0,000 0400	-0,0000306
Distance ρ (A), km	2 037	1 425
Force $F_2(A)$, mgk/s^2	-0,000 0144	-0,000 0101
Amount $g(A)=F_1(A)+F_2(A)$, mgk/s^2	-0,000 0544	-0,000 0407

Distance to the Moon $S(B)$, km	362 771	413 071
Force $F_1(B)$, mgk/s ²	0,000 0373	0,0000287
Distance $\rho(B)$, km	10 705	11 317
Force $F_2(B)$, mgk/s ²	-0,000 0759	-0,000 0802
Amount $g(B)=F_1(B)+F_2(B)$, mgk/s ²	-0,000 0386	-0,000 0515
Change $\Delta=g(B)-g(A)$, mgk/s ²	0,000 0158	-0,000 0108

1.1 Deviations of vertical lines

A significant spread of the value of the Moon of gravity force at the extreme points serves as a sufficient basis for a more detailed analysis of the Moon effect on the magnitude and direction of the gravity force. With this goal, we first perform an approximate evaluation of the maximum deviations of plumb lines at an arbitrary point D of the section of the earth's sphere:

1) the plane of the Moon orbit (Figure 2), which will be referred to hereinafter as "plane 1"; plane perpendicular to the plane of the Moon orbit and passing through its center (Fig. 3), hereinafter referred to as "plane 2".

Suppose that in Fig. 2 C is the center of the Earth-Moon pair mass, D is an arbitrary point of the Earth's surface lying in the plane 1, r is the Earth radius, u is the angle between the radius vector of point D and L the direction to the Moon center, z is the zenith distance of the Moon at point D , R is the distance between the Earth and the Moon centers, ρ is the distance from the center of the Earth-Moon pair mass to the point under consideration, c is the distance from the center of the Earth to the center of the Earth-Moon pair mass, calculated from formula (3) distance from point D to the Moon center.

Fig. 2. Forces in plane 1

The force f , which affects the deviation of the plumb line at point D , will be determined by the sum of the gravitational force projection and the projection of the centrifugal force on the normal to the radius vector of this point:

$$f = f_1 + f_2, \tag{4}$$

where

$$\begin{aligned} f_1 &= F_1 \sin z, \\ f_2 &= -F_2 \sin(u - v). \end{aligned}$$

If the angle between the changed direction of the plumb line and the direction to the Moon center L is greater than the angle u , then such deviation will be considered positive, otherwise negative, which explains the signs in the last two formulas. The magnitude of the Moon gravity force at the point D is calculated from the formula (1), where the distance S to the center of the Moon mass can be obtained from the triangle ODL by the cosine formula

$$S^2 = r^2 + R^2 - 2rR\cos u.$$

From the same triangle ODL on the basis of the sine theorem follows

$$\sin z = \frac{R}{S} \sin u. \tag{5}$$

Therefore, the projection of the force F_1 on the normal to the radius vector will be equal to

$$f_1 = \gamma \frac{mM_1 R \sin u}{S^3}.$$

The projection on the normal to the radius vector of the centrifugal force F_2 caused at point D by its rotation about the center of the Earth-Moon pair mass is

$$F_2 \sin(u - v) = -m\omega^2 \rho \sin(u - v).$$

Replacing $\rho \sin(v - u)$ product in this formula by the following from the triangle ODC by its equivalent

$$\rho \sin(u - v) = c \sin u,$$

we obtain the relation

$$f_2 = -m\omega^2 c \sin u.$$

Let us now consider the Moon influence on the deviations of the plumb line at the points of the Earth's surface, located on its section by plane 2 (Fig. 3). The force, under which the deviation of the plumb line occurs in this case, is expressed by the same formula (4), where

$$f_1 = F_1 \sin z,$$

$$f_2 = F_2 \sin u.$$

Fig. 3. Forces in plane 2

The quantities F_1 and $\sin z$ can be obtained from the previously given formulas (1) and (5). The rotation of point D (as well as all other points of the Earth body) occurs around the axis passing through point C of the mass center and perpendicular to the plane of the Moon orbit. The value of the centrifugal force is calculated by the formula (2), in which the distance $\rho = CP$ of the point D to C_1C_2 axis is equal to

$$\rho = r \cos u - c.$$

Under these conditions, the projections onto the normal to the radius vector of the F_1 and F_2 forces, acting on the D point will be equal, respectively:

$$f_1 = \gamma \frac{mM_1 R}{S^3} \sin u,$$

$$f_2 = m\omega^2(r \cos u - c) \sin u.$$

1.2 Variations of gravity

The value of the change in the force of gravity g at an arbitrary point D , which belongs to the section of the Earth's surface by the plane 1 (Figure 2), is expressed by the formula

$$g = g_1 + g_2, \quad (6)$$

where

$$g_1 = -F_1 \cos z ;$$

$$g_2 = -F_2 \cos(v-u) .$$

and the values of F_1 F_2 are determined by expressions (1) and (2), respectively. From Fig. 2 there are relations for computing $\cos z$:

$$\cos z = \frac{R \cos u - r}{S} ,$$

and product:

$$\rho \cos(v-u) = r - c \cos u$$

Consequently,

$$g_1 = -\gamma \frac{mM_1(R \cos u - r)}{S^3} , \tag{7}$$

$$g_2 = -m\omega^2(r - c \cos u) .$$

As a result, expressions were obtained for calculating the variation of the gravity force at points that belong to the cross-section of the Earth's sphere by the plane 1.

The gravity variation at point *D* of the section of the Earth's sphere by plane 2 (Figure 3) is also expressed by the formula (6), and the projection by the formula (7). In this case, the projection of the force on the radius vector will be equal to

$$g_2 = -F_2 \cos u ,$$

F_2 value is calculated by the formula (2),

$$\rho = r \cos u - c .$$

Taking into account the last two relations, we find

$$g_2 = -m\omega^2(r \cos u - r) \cos u .$$

2 Materials and Methods

The values of the projections pair f_1 and f_2 and the projections pair g_1 and g_2 calculated for the above formulas and for the body with unit mass, that is, for $m = 1$, are given in Table 3 and Table. 4 respectively.

Table 3. The projection of the force F_1 and F_2 on the normal to the radius vector, 10^{-5} mkg/s²

Angle,°	Plane 1				Plane 2			
	Perigeeux		Apogee		Perigeeux		Apogee	
	f_1	f_2	f_1	f_2	f_1	f_2	f_1	f_2
0	0	0	0	0	0	0	0	0
10	0,71	-0,53	0,54	-0,61	0,71	0,24	0,54	0,16
20	1,39	-1,05	1,06	-1,20	1,39	0,40	1,06	0,25
30	2,02	-1,54	1,54	-1,75	2,02	0,42	1,54	0,20
40	2,58	-1,97	1,97	-2,25	2,58	0,25	1,97	-0,03
50	3,06	-2,35	2,34	-2,68	3,06	-0,13	2,34	-0,46
60	3,44	-2,66	2,62	-3,04	3,44	-0,70	2,62	-1,08
70	3,69	-2,89	2,83	-3,29	3,69	-1,44	2,83	-1,84
80	3,84	-3,02	2,94	-3,45	3,84	-2,26	2,94	-2,68

90	3,86	-3,07	2,96	-3,50	3,86	-3,07	2,96	-3,50
100	3,76	-3,02	2,90	-3,45	3,76	-3,79	2,90	-4,22
110	3,56	-2,89	2,74	-3,29	3,56	-4,34	2,74	-4,75
120	3,26	-2,66	2,51	-3,04	3,26	-4,62	2,51	-4,99
130	2,86	-2,35	2,21	-2,68	2,86	-4,57	2,21	-4,91
140	2,38	-1,97	1,84	-2,25	2,38	-4,19	1,84	-4,47
150	1,84	-1,54	1,42	-1,75	1,84	-3,49	1,42	-3,70
160	1,26	-1,05	0,97	-1,20	1,26	-2,50	0,97	-2,65
170	0,64	-0,53	0,49	-0,61	0,64	-1,31	0,49	-1,38
180	0	0	0	0	0	0	0	0

Table 4. The projection of the force F_1 and F_2 on the radius vector, 10^{-5} mkg/s²

Angle,°	Plane 1				Plane 2			
	Perigueux		Apogee		Perigueux		Apogee	
	g_1	g_2	g_1	g_2	g_1	g_2	g_1	g_2
0	-4,00	-1,44	3,06	-1,01	-4,00	-1,44	-3,06	-1,01
10	-3,94	-1,49	-3,11	-1,06	-3,94	-1,35	-3,11	-0,93
20	-3,74	-1,63	-2,86	-1,22	-3,74	-1,10	-2,86	-0,69
30	-3,43	-1,86	-2,63	-1,48	-3,43	-0,73	-2,63	-0,35
40	-3,01	-2,16	-2,31	-1,83	-3,01	-0,30	-2,31	0,04
50	-2,50	-2,54	-1,92	-2,26	-2,50	0,11	-1,92	0,39
60	-1,91	-2,98	-1,47	-2,76	-1,91	0,41	-1,47	0,62
70	-1,27	-3,46	-0,98	-3,32	-1,27	0,52	-0,98	0,67
80	-0,61	-3,98	-0,47	-3,91	-0,61	0,40	-0,47	0,47
90	0,07	-4,51	0,05	-4,51	0,07	0	0,05	0
100	0,73	-5,05	0,56	-5,12	0,73	-0,67	0,56	-0,47
110	1,36	-5,56	1,04	-5,71	1,36	-1,58	1,04	-0,67
120	1,95	-6,05	1,49	-6,27	1,95	-2,66	1,49	-2,88
130	2,46	-6,49	1,89	-6,77	2,46	-3,84	1,89	-4,11
140	2,91	-6,87	2,23	-7,20	2,91	-5,00	2,23	-5,33
150	3,26	-7,17	2,51	-7,55	3,26	-6,54	2,51	-6,42
160	3,52	-7,40	2,71	-7,81	3,52	-6,87	2,71	-7,28
170	3,67	-7,54	2,83	-7,97	3,67	-7,40	2,83	-7,83
180	3,73	-7,59	2,87	-8,02	3,73	-7,59	2,87	-8,02

The tangent of the deviation Δu of the vertical line can be found from the formula

$$tg\Delta u = \frac{f}{G},$$

where G is the average value of the Earth gravity. Because of the smallness of the integration value, we can take

$$\Delta u = \frac{f}{G}.$$

Then the magnitude of the deviation, expressed in degree, will be equal to

$$\Delta u = \frac{f}{G} 206265''.$$

Variations of the gravity force are expressed by the formula (6). The calculated values of the plumb line deviations and variations in the force of gravity are given in Table 5.

Table 5. Evasion of a steep line and variations of gravity

Angle, °	plumb inclination f , "				Variations g , 10^{-5} mkg/s ²			
	Plane 1		Plane 2		Plane 1		Plane 2	
	Perigeeux	Apogee	Perigeeux	Apogee	Perigeeux	Apogee	Perigeeux	Apogee
0	0	0	0	0	-5,44	-4,07	-5,44	-4,07
10	0,04	-0,01	0,20	0,15	-5,43	-4,17	-5,29	-4,04
20	0,07	-0,03	0,38	0,28	-5,37	-4,08	-4,84	-3,55
30	0,10	-0,04	0,51	0,37	-5,29	-4,11	-4,16	-2,98
40	0,13	-0,06	0,59	0,41	-5,17	-4,14	-3,31	-2,27
50	0,15	-0,07	0,62	0,40	-5,04	-4,18	-2,39	-1,53
60	0,16	-0,09	0,58	0,32	-4,89	-4,23	-1,50	-0,85
70	0,17	-0,10	0,47	0,21	-4,73	-4,30	-0,75	-0,31
80	0,17	-0,11	0,33	0,05	-4,59	-4,38	-0,21	0
90	0,17	-0,11	0,17	-0,11	-4,44	-4,46	0,07	0,05
100	0,16	-0,12	-0,01	-0,28	-4,32	-4,56	0,06	0,09
110	0,14	-0,12	-0,16	-0,42	-4,20	-4,67	-0,22	0,37
120	0,13	-0,11	-0,29	0,52	-4,10	-4,78	-0,71	-1,39
130	0,10	-0,10	-0,36	-0,57	-4,03	-4,88	-1,38	-2,22
140	0,09	-0,09	-0,38	-0,55	-3,96	-4,97	-2,09	-3,10
150	0,06	-0,07	-0,35	-0,48	-3,91	-5,04	-3,28	-3,91
160	0,04	-0,05	-0,26	-0,35	-3,88	-5,10	-3,35	-4,57
170	0,02	-0,03	-0,14	-0,19	-3,87	-5,14	-3,73	-5,00
180	0	0	0	0	-3,86	-5,15	-3,86	-5,15

The calculations results show that the maximum difference in the steepness of the vertical lines can reach 1, and the variations in the force of gravity can range from -3.86 to 5.44 mkg/s². The measured errors excesses between two points located at a distance of 100 m from each other (the maximum distance between the rails when leveling I class), under the Moon influence can reach up to 0.5 mm.

3 Results

Thus, the current methods for accounting for the Moon influence on the magnitude and direction of gravity, including in [5], can not be considered satisfactory, since the magnitude of such influence is significantly underestimated. The above formulas estimate only the Moon extreme impact on the measured values at the points of the Earth's surface belonging to the planes. Another argument in favor of the need to take into account the Moon influence and other factors is the increase in the accuracy of geodetic instruments (levels and gravimeters). Without changing the methods for performing high-precision leveling and gravity measurements and introducing the necessary corresponding corrections, a real increase in the accuracy of these studies can not be achieved. In this case, the ideas of the accuracy achieved, when it is estimated by internal convergence, will be overestimated. To find the actual errors in the results of measuring excess and gravity, formulas are needed for any point on.

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