MATEC Web of Conferences **169**, 01037 (2018) https://doi.org/10.1051/matecconf/201816901037
IMETI 2017

Research on two dimensional Wiener stochastic degradation model based on the wear model

Hui Zhang[a], Jun Yao and Yan-Lin Zhao

School of Reliability and System Engineering, Beijing University of Aeronautics and Astronautics, Beijing, China

Abstract. One dimensional Wiener degradation process is often used to describe the degradation of product performance. However one dimensional Wiener degradation process doesn't sufficiently consider the relevance of multiple degradation factors and wear process, which lead to inaccurate results. To overcome these problems, a new two dimensional Wiener stochastic degradation model is proposed, which applys to the products on wear process and stochastic degradation process. Combining wear model and two dimensional Wiener stochastic degradation model, a new reliability analytical form is obtained by constructing the Fokker-Planck equation. Then using the relation among wear volume, degradation characteristic lifetime and drift parameter, parameters of two dimensional Wiener degradation model on the basic of wear model can be estimated. Compared with the existing approaches, the proposed method can effectively improve accuracy. Finally, a case study is illustrated the application and advantages of the proposed method.

1 Introduction

Products subjected to the action of the external environment, the material properties of the product will gradually change. And because of the randomness of the external environment and the material properties, the degradation of the product is also random. So stochastic process[1] is usually used to describe the degradation process of product performance. Wiener process[2-5] is the most often used method to describe the degradation of product performance. Tang[6] et al estimated failure time distribution by a Wiener degradation model based on intermediate data. Wang[7] used degradation data of Wiener processes with random effects to study reliability level of the bridge. Peng[8] et al proposed Bayesian method to assess the reliability of the products with Wiener process degradation. Nicolai[9] et al proposed an application to coatings on steel structures by comparing models for measurable deterioration with Wiener processes. Balka[10] et al reviewed and implemented the cure models based on first hitting times of Wiener processes. Si[11] et al estimated remaining useful life based on a nonlinear diffusion degradation process with monitoring degraded signal. Most of the studies on degradation of product performance are based on one dimensional Wiener process degradation, without taking into account the multiple degradation and the physical mechanism of degradation.

Wear is one of the common failure types of components. When the wear occurs, the features of the product can be reduced or failed, reliability and safety will be lost if products are continued to put into

[a] Corresponding author : zhanghui1133018@163.com

use. Many scholars have put forward relevant theories and models about the wear[12,13], which predict the lifetime of the products by calculating the wear volume and wear rate. Hu[14] et al proposed finite element modeling (FEM) approaches with Lagrangian increment method for 3D metal turning of hardened steel H13 by common ceramic tool and ultrafine-grained tool respectively have been investigated by simulation of DEFORM-3D software and turning test. Lin[15] combined the finite element numerical simulation analysis method with the Archard wear calculation model and temperature effect, predicted the most serious wear parts, and proposed a formula for calculating the total wear rate. Gao[16] et al estimated die service life of foging spiral gear based on modified Archard method. Kim[17] et al estimated die service life against plastic deformation and wear during hot forging processes with finite element modeling. Most researches on the wear only consider physical mechanism of wear, predict wear rate and the wear volume by finite element simulation, without considering the impact of the external environment randomness on products degradation.

In this paper, on the basis of the wear model, considering the influence of the randomness of the external environment on the products degradation, combined with the theory of random degradation, a model of two dimensional Wiener stochastic degradation model is proposed based on the wear model. Using 1000h reliability test data of the steam turbine pump as an example, the rationality and accuracy of the method proposed in this paper is verified by comparing with calculation results of other methods.

2 The performance degradation model based on two dimensional Wiener process

2.1 Binary linear Wiener process model

Assuming that product performance degradation process in $\{X(t)=(X_1(t),X_2(t))^T, t\geq 0\}$ [18], namely, the degradations of products are binary. At the time t, it follows:

$$X(t) \sim N(t\mu, t\Sigma) \tag{1}$$

where

$$\mu=(\mu_1\ \mu_2)^T, \Sigma=\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{2}$$

ρ is the correlation coefficient. Assume $\rho\neq 0$, which means the two marginal degradation process $\{X_1(t), t\geq 0\}$ and $\{X_2(t), t\geq 0\}$ are related.

2.2 Reliability Assessment

Initial degradation $X_1(t)=X_2(t)=0$, When one of $\{X_1(t), t\geq 0\}$ and $\{X_2(t), t\geq 0\}$ exceeds its failure threshold, product fails. The lifetime of the product can be represented as:

$$T=\inf\{t: X_1(t)>l_1 \text{ or } X_2(t)>l_2\} \tag{3}$$

Then the reliability function $R(t)$ is derived:

$$R(t)=P\{X_1(s)<l_1, X_2(s)<l_2, 0\leq s\leq t\} \tag{4}$$

When Domine and Pieper[19] studied the first hitting time of binary Wiener process, the Fokker-Planck equation was constructed (Kolmogrov forward equation) based on the initial performance of $X_1(0)=x_{0,1}$ and $X_2(0)=x_{0,2}$, the analytic form of the reliability fuction $R(t)$ was obtained by solving the equation:

$$R(t)=\int_0^{\alpha}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{2r}{\alpha\sigma_2^2(1-\rho^2)t}\sin(\frac{n\pi}{\alpha}\varphi_0)\cdot \exp\left[\frac{\sigma_2\mu_1-\sigma_1\mu_2\rho}{\sigma_1\sigma_2^2(1-\rho^2)}r\cos\varphi-\frac{\mu_2}{\sigma_2^2\sqrt{1-\rho^2}}r\sin\varphi\right]\cdot \exp\left[\frac{(\mu_1\rho\sigma_1\sigma_2-\mu_2\sigma_1^2)x_{0,2}+(\mu_2\rho\sigma_1\sigma_2-\mu_1\sigma_2^2)x_{0,1}}{(1-\rho^2)\sigma_1^2\sigma_2^2}\right]\cdot \exp\left[-\frac{\sigma_1^2\mu_1^2-2\mu_1\mu_2\sigma_1\sigma_2\rho+\sigma_2^2\mu_1^2}{2(1-\rho^2)\sigma_1^2\sigma_2^2}t-\frac{r^2+r_0^2}{2(1-\rho^2)\sigma_2^2t}\right]\cdot \sin\left(\frac{n\pi}{\alpha}\right)I_{\frac{n\pi}{\alpha}}\left(\frac{rr_0}{(1-\rho^2)\sigma_2^2t}\right)drd\varphi \tag{5}$$

Where $\alpha=\arctan\left(-\frac{\sqrt{1-\rho^2}}{\rho}\right)+\pi$; r_0, φ_0 are the solution of the equation below:

$$\begin{cases} r_0\cos\varphi_0=\frac{\sigma_2}{\sigma_1}(l_1-x_{0,1})-\rho(l_2-x_{0,2}) \\ r_0\sin\varphi_0=\sqrt{1-\rho^2}(l_2-x_{0,2}) \end{cases} \tag{6}$$

$I_v(z)$ is the modified Bessel function.

$$I_v(z)=\sum_{k=0}^{\infty}\frac{\left(\frac{z}{2}\right)^{2k+v}}{k!\Gamma(k+v+1)} \tag{7}$$

When $x_{0,1}=x_{0,2}=0$, the reliability function $R(t)$, lifetime function $F(t)=1-R(t)$, and density function $f(t)=dF(t)/dt=-dR(t)/dt$ will be obtained. If $x_{0,1}$ and $x_{0,2}$ in formula (5) are regarded as the performance degradation when product run to a certain time, then formula (5) is actually the reliability of the continue running time of product, and $F(t)=1-R(t)$ is the distribution of the remaining lifetime.

2.3 The performance degradation data model

Assuming a total of N products of a degradation test, at the time of $t_{i1},t_{i2},\cdots t_{im_i}$, measure the degradation X and Y and get measured data:

$$\begin{matrix} X_{11}, & X_{12}, & \cdots & X_{1m_1} \\ X_{21}, & X_{22}, & \cdots & X_{2m_2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1}, & X_{n2}, & \cdots & X_{nm_n} \end{matrix} \quad \text{and} \quad \begin{matrix} Y_{11}, & Y_{12}, & \cdots & Y_{1m_1} \\ Y_{21}, & Y_{22}, & \cdots & Y_{2m_2} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}, & Y_{n2}, & \cdots & Y_{nm_n} \end{matrix} \tag{8}$$

Note $\Delta X_{ij} = X_{ij} - X_{i,j-1}$, $\Delta Y_{ij} = Y_{ij} - Y_{i,j-1}$, $i = 1,2,\cdots,n, j = 1,2,\cdots m_i$ as the degradation increment of sample i during moment $t_{i,j-1}$ to $t_{i,j}$, thus the following data is derived:

$$\begin{matrix} \Delta X_{11}, & \Delta X_{12}, & \cdots & \Delta X_{1m_1} \\ \Delta X_{21}, & \Delta X_{22}, & \cdots & \Delta X_{2m_2} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta X_{n1}, & \Delta X_{n2}, & \cdots & \Delta X_{nm_n} \end{matrix} \text{ and } \begin{matrix} \Delta Y_{11}, & \Delta Y_{12}, & \cdots & \Delta Y_{1m_1} \\ \Delta Y_{21}, & \Delta Y_{22}, & \cdots & \Delta Y_{2m_2} \\ \vdots & \vdots & \ddots & \vdots \\ \Delta Y_{n1}, & \Delta Y_{n2}, & \cdots & \Delta Y_{nm_n} \end{matrix} \tag{9}$$

3 Method for estimating parameter μ and σ^2

3.1 Maximum likelihood estimation

ΔX_{ij} , ΔY_{ij} are degradation increment of sample i from $t_{i,j-1}$ to $t_{i,j}$, note $\Delta t_{ij} = t_{ij} - t_{i,j-1}$, $(\Delta X_{ij}, \Delta Y_{ij})^T$ are independent and obey two-dimensional normal distribution $N(\mu\Delta t_{ij}, \Delta t_{ij}\Sigma)$.

Using the maximum likelihood estimation(MLE) method to get the estimated values of $\mu_1, \sigma_1{}^2, \mu_2, \sigma_2{}^2$:

$$\hat{\mu}_1 = \frac{\sum_{i=1}^{n} X_{im_i}}{\sum_{i=1}^{n} t_{im_i}}, \quad \hat{\sigma}_1^2 = \frac{1}{\sum_{i=1}^{n} m_i}\left[\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{\left(\Delta X_{ij}\right)^2}{\Delta t_{ij}} - \frac{\left(\sum_{i=1}^{n} X_{im_i}\right)^2}{\sum_{i=1}^{n} t_{im_i}}\right] \tag{10}$$

$$\hat{\mu}_2 = \frac{\sum_{i=1}^{n} Y_{im_i}}{\sum_{i=1}^{n} t_{im_i}}, \quad \hat{\sigma}_2^2 = \frac{1}{\sum_{i=1}^{n} m_i}\left[\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{\left(\Delta Y_{ij}\right)^2}{\Delta t_{ij}} - \frac{\left(\sum_{i=1}^{n} Y_{im_i}\right)^2}{\sum_{i=1}^{n} t_{im_i}}\right] \tag{11}$$

Estimation of the correlation coefficient:

$$\hat{\rho} = \frac{1}{M\hat{\sigma}_1\hat{\sigma}_2}\left[\sum_{i=1}^{n}\sum_{j=1}^{m_i}\frac{\left(\Delta X_{ij} - \hat{\mu}_1\Delta t_{ij}\right)\left(\Delta Y_{ij} - \hat{\mu}_2\Delta t_{ij}\right)}{\Delta t_{ij}}\right] \tag{12}$$

In this paper, in order to measure the two degradations at the same time, the equal interval measurement is adopted, which is so called stable measurement, i.e. $\Delta t_{ij} = \Delta t$, so $\left(\Delta X_{ij}, \Delta Y_{ij}\right)^T$ are independent and identically distributed bivariate normal samples,

$$\left(\Delta X_{ij}, \Delta Y_{ij}\right)^T \sim N\left(\mu\Delta t, \Sigma\Delta t\right) \tag{13}$$

the estimations of $\mu_1, \sigma_1{}^2, \mu_2, \sigma_2{}^2$ by the properties of the multivariate normal distribution is derived as follows:

$$\hat{\mu}_1=\frac{\sum_{i=1}^{n}X_{im_i}}{\sum_{i=1}^{n}t_{im_i}},\ \hat{\sigma}_1^2=\frac{1}{\sum_{i=1}^{n}m_i}\left[\frac{1}{\Delta t}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\Delta X_{ij}\right)^2-\frac{\left(\sum_{i=1}^{n}X_{im_i}\right)^2}{\sum_{i=1}^{n}t_{im_i}}\right] \tag{14}$$

$$\hat{\mu}_2=\frac{\sum_{i=1}^{n}Y_{im_i}}{\sum_{i=1}^{n}t_{im_i}},\ \hat{\sigma}_2^2=\frac{1}{\sum_{i=1}^{n}m_i}\left[\frac{1}{\Delta t}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\Delta Y_{ij}\right)^2-\frac{\left(\sum_{i=1}^{n}Y_{im_i}\right)^2}{\sum_{i=1}^{n}t_{im_i}}\right] \tag{15}$$

Estimation of the correlation coefficient:

$$\hat{\rho}=\frac{1}{\Delta t\hat{\sigma}_1\hat{\sigma}_2\sum_{i=1}^{n}m_i}\sum_{i=1}^{n}\sum_{j=1}^{m_i}\left(\Delta X_{ij}-\hat{\mu}_1\Delta t\right)\left(\Delta Y_{ij}-\hat{\mu}_2\Delta t\right) \tag{16}$$

3.2 Parameter estimation of two dimensional Wiener stochastic degradation model based on the wear model

3.2.1 Archard Wear Model

The Archard wear model is a classic wear calculation model by Professor J.F.Archard[20]. The model is verified by experiment and it has a good practicability. The model is as follows:

$$V=K\frac{F_nS}{H}\propto a+b\gamma \tag{17}$$

Where V and S are the wear volume and relative slip distance, respectively, F_n is the normal force, K is the wear coefficient, H is the Brinell hardness. Whereas a and b are unknown parameters, γ is a variable.

Archard[21]wear model is a linear model, it can be seen that wear volume is a linear function of time t, considering the randomness in the process of wear, the Archard wear model is combine with the Markov model to form a new wear model:

$$\Delta\propto\alpha_0+\alpha_1 t+\varepsilon \tag{18}$$

Where Δ is the wear volume, α_0 and α_1 are determined coefficient, ε is a random diffusion, and $\varepsilon\sim N\left(0,\sigma^2\right)$, t is the characteristic lifetime.

We use T to represent the lifetime of the product, so its expectation is $\bar{T}=E\left(T\right)=\frac{l_1-\alpha_0-\varepsilon}{\alpha_1}$. By measuring the degradation of products at $T_1,T_2,\cdots T_n$, $X_1,X_2,\cdots X_n$ are obtained. So the point estimations of α_0,α_1 are:

$$\hat{\alpha}_0=\bar{X}-\frac{\bar{X}\sum\left(T_i-\bar{T}\right)\left(X_i-\bar{X}\right)}{\sum\left(T_i-\bar{T}\right)^2},\ \hat{\alpha}_1=\frac{\bar{X}\sum\left(T_i-\bar{T}\right)\left(X_i-\bar{X}\right)}{\sum\left(T_i-\bar{T}\right)^2} \tag{19}$$

3.2.2 Method of parameter estimation of two dimensional Wiener degradation model based on the wear model

In the Wiener process, the distribution of product lifetime T is inverse Gaussian distribution, and the characteristic function of T can be calculated because of the stability of Wiener process and the characteristic of independent increment, which is $E\left[\exp(-\theta T)\right]=\exp\left\{-l\left\{\sqrt{\mu_2+2\theta}-\mu\right\}\right\}$, and its expectation is: $E\left[T\right]=\dfrac{l}{\mu}$.It can be seen that the expectation of lifetime T is only related to the drift parameter. When one of the two degradations exceeds its failure threshold, product fails. In this case, the working time is the lifetime of the product. By combining the two expectations, it follows:

$$E\left(T_1\right)=\frac{l_1}{\mu_1}=\overline{T}_1\approx\frac{l_1-\alpha_{11}-\varepsilon}{\alpha_{12}}\quad E\left(T_2\right)=\frac{l_2}{\mu_2}=\overline{T}_2\approx\frac{l_2-\alpha_{21}-\varepsilon}{\alpha_{22}} \tag{20}$$

Estimations of μ_1,μ_2

$$\hat{\mu}_2=\frac{l_2\alpha_{22}}{l_2-\alpha_{21}-\varepsilon},\ \hat{\mu}_2=\frac{l_2\alpha_{22}}{l_2-\alpha_{21}-\varepsilon} \tag{21}$$

Bring them into the formula (21), the reliability function $R\left(t\right)$ and distribution function $F\left(t\right)$ of the model proposed in this paper are obtained:

$$\begin{aligned}
R(t)=&\int_0^{\alpha}\int_0^{\infty}\sum_{n=1}^{\infty}\frac{2r}{\alpha\sigma_2^2(1-\rho^2)t}\sin(\frac{n\pi}{\alpha}\varphi_0)\\
&\cdot\exp\left[\frac{\sigma_2 l_1\beta_{12}\left(l_2-\beta_{21}-\varepsilon\right)-\sigma_1 l_2\beta_{22}\left(l_1-\beta_{11}-\varepsilon\right)\rho}{\sigma_1\sigma_2^2\left(1-\rho^2\right)\left(l_1-\beta_{11}-\varepsilon\right)\left(l_2-\beta_{21}-\varepsilon\right)}r\cos\varphi-\frac{l_2\beta_{22}}{\sigma_2^2\sqrt{1-\rho^2}\left(l_2-\beta_{21}-\varepsilon\right)}r\sin\varphi\right]\\
&\cdot\exp\left[\frac{(l_1\beta_{12}\left(l_2-\beta_{21}-\varepsilon\right)\rho\sigma_1\sigma_2-l_2\beta_{22}\left(l_1-\beta_{11}-\varepsilon\right)\sigma_1^2)x_{0,2}+(\beta_{22}\left(l_1-\beta_{11}-\varepsilon\right)\rho\sigma_1\sigma_2-_1\beta_{12}\left(l_2-\beta_{21}-\varepsilon\right)\sigma_2^2)x_{0,1}}{(1-\rho^2)\sigma_1^2\sigma_2^2\left(l_1-\beta_{11}-\varepsilon\right)\left(l_2-\beta_{21}-\varepsilon\right)}\right]\\
&\cdot\exp\left[-\frac{\sigma_1^2 l_2^2\beta_{22}^2\left(l_1-\beta_{11}-\varepsilon\right)^2-2\left(l_1-\beta_{11}-\varepsilon\right)\left(l_2-\beta_{21}-\varepsilon\right)\sigma_1\sigma_2\rho+\sigma_2^2 l_1^2\beta_{12}^2\left(l_2-\beta_{21}-\varepsilon\right)^2}{2(1-\rho^2)\sigma_1^2\sigma_2^2\left(l_1-\beta_{11}-\varepsilon\right)^2\left(l_2-\beta_{21}-\varepsilon\right)^2}t-\frac{r^2+r_0^2}{2(1-\rho^2)\sigma_2^2 t}\right]\\
&\cdot\sin\left(\frac{n\pi}{\alpha}\right)I_{\frac{n\pi}{\alpha}}\left(\frac{rr_0}{(1-\rho^2)\sigma_2^2 t}\right)drd\varphi
\end{aligned} \tag{22}$$

$$F\left(t\right)=1-R\left(t\right) \tag{23}$$

4 The example of reliability assessment of steam turbine pump

Steam turbine is the power device of ship, and steam turbine pump is the key part of the steam turbine as the output device of lubricating oil. Once a fault occurs, it will affect the whole ship power system. Now a 1000h reliability test data of the steam turbine pump is used as an example to verify the rationality and accuracy of the method proposed in this paper. The 1000h reliability test of steam turbine pump can be divided into two stages. Firstly, individual diagnostic test for 60 hours. Then in the remaining 940 hours, according to the test pattern for continuous operation test, it is a total of 47 cycles with 20h for each cycle. Each cycle is interspersed with automatic variable conditions, parameter fluctuations, exhaust safety valve and other test items.

Steam turbine pump belongs to a complex mechanical system with continuous, variable condition, and reciprocating motion. Wear, fatigue and fracture of the parts are the main reasons for the lifetime and failure of the steam turbine pump. Therefore, both domestic and foreign reseachers have proposed concepts of life evaluation of key parts. In order to master the wear of the key parts of the test process completely and accurately, reliability test program requires precise measurements of the thickness and size of the key parts before and after test. In this paper the retarder rotor support bearing clearance and the support bearing temperature are selected as key parameters to assess the steam turbine pump lifetime, because these data can reflect the working state of the steam turbine pump. Reliability and lifetime assessment of key parts is an important part of describing the reliability level and service life of the whole machine. At the same time, it also provides reference for the determination of maintenance cycle and maintenance level.

In accordance with the requirements of the test program, the steam turbine pump must be dismantled for inspection when total running time reaches 100, 200, 300, 400, 500, 600, 700, 800, 900 and 1000 hours. Table1 is the data of retarder rotor support bearing clearance and the support bearing temperature. According to the design requirements of the bearing, when the support bearing temperature is higher than 80℃, it is considered that the bearing fails, namely, the failure threshold of the support bearing temperature is 80℃. When the bearing clearance is greater than 0.3mm, it is considered failure, that is to say, the failure threshold of the bearing clearance is 0.3mm. And the reliability index of the steam turbine pump is that the reliability is 0.7 when the working time reach 10000h.

Table 1. Data of retarder rotor support bearing clearance and the support bearing temperature.

Time(h)	100	200	300	400	500	600	700	800
Support bearing temperature (℃)	62.81	63.64	61.67	63.45	61.41	59.89	63.08	63.77
Reducer rotor support bearing clearance (mm)	0.10	0.11	0.11	0.11	0.11	0.11	0.12	0.12

4.1 Reliability assessment of steam turbine pump

We use three methods to assess the reliability of steam turbine pump, respectively, one dimensional Wiener degradation model based on the maximum likelihood method, two dimensional Wiener degradation model based on the maximum likelihood method and the method we proposed where μ_1 and σ_1^2 are parameters to support bearing temperature, μ_2 and σ_2^2 are parameters to reducer rotor support bearing clearance and ρ is the correlation coefficient between the two.

4.1.1 One dimensional Wiener degradation model based on MLE

The parameters of the support bearing temperature and the reducer rotor support bearing clearance are estimated by MLE, results are shown in Table 2

Table 2. Parameters estimator of one dimensional Wiener degradation model based on MLE.

Parameters	μ_1	σ_1^2	μ_2	σ_2^2
Estimator	0.0070	0.0091	2.714×10^{-5}	9.928×10^{-8}

Bringing estimated results of parameter into one dimensional Wiener degradation model, the reliability of support bearing temperature is 0.8370 in 10000h, and the reliability of reducer rotor support bearing clearance is 0.8032 in 10000h. Because when one of the two parameters reaches its failure threshold, the steam turbine pump fails. So choose the result which reach its failure threshold first as reliability assessment result of steam turbine pump. In this example, the assessment of reducer rotor support bearing clearance is taken as result of steam turbine pump, namely, the reliability assessment of steam turbine pump is 0.8032 when the working time reach 10000h. The reliability curve is shown in Figure 1.

 https://doi.org/10.1051/matecconf/201816901037

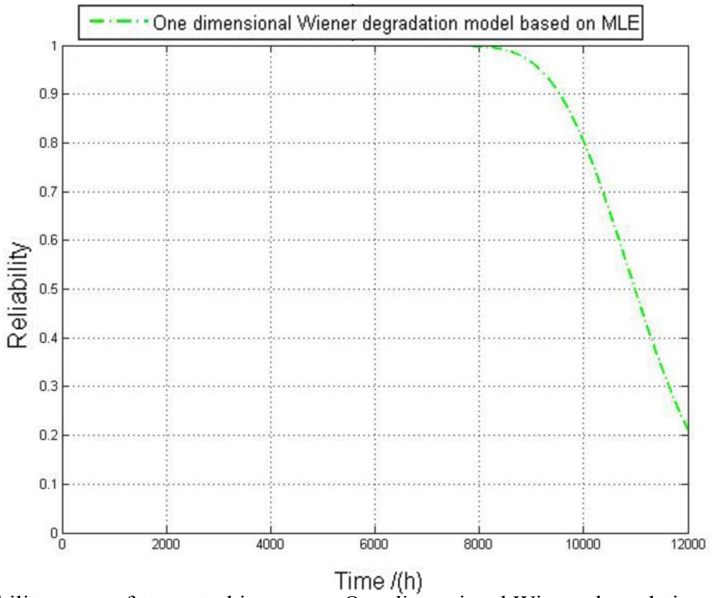

Figure 1. The reliability curve of steam turbine pump- One dimensional Wiener degradation model based on MLE.

4.1.2 Two dimensional Wiener degradation model based on MLE

Table 3. Parameters estimator of two dimensional Wiener degradation model based on MLE.

Parameter	μ_1	σ_1^2	μ_2	σ_2^2	ρ
Estimator	0.0797	0.6043	1.500×10^{-4}	2.000×10^{-6}	0.7997

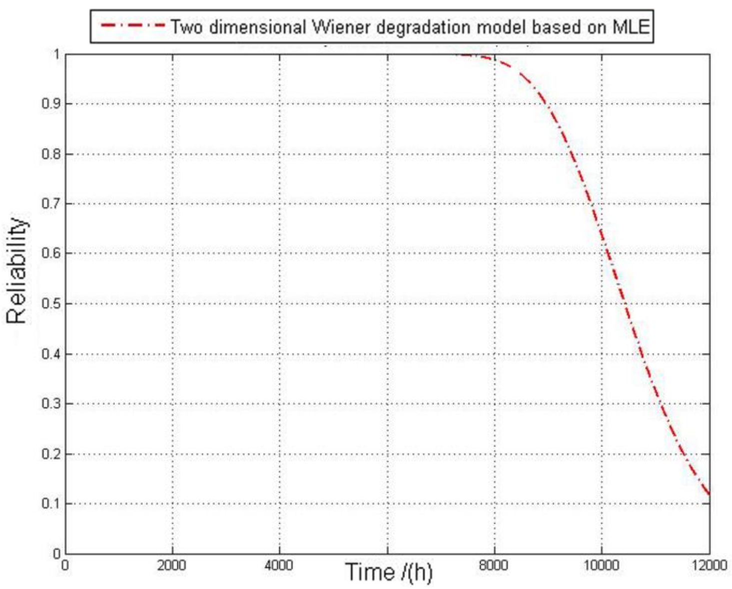

Figure 2. The reliability curve of steam turbine pump- Two dimensional Wiener degradation model based on MLE.

Stable measurement is used to measure two parameters. Considering the correlation between these two parameters, the results of μ_1 σ_1^2 μ_2 σ_2^2 and ρ are derived by bringing the data of two parameters into formula (10)- (12). Results are shown in Table 3

Bringing the parameter estimator into formula (5), the reliability of steam turbine pump is obtained, it is 0.6376 at 10000h. The reliability curve is shown in Figure 2.

4.1.3 Two dimensional Wiener degradation model based on wear model

Bringing the data of two parameters into formula (12) (21), the results of μ_1 σ_1^2 μ_2 σ_2^2 and ρ are obtained. Results are shown in Table 4

Table 4. Parameters estimator of two dimensional Wiener degradation model based on wear model.

Parameter	μ_1	σ_1^2	μ_2	σ_2^2	ρ
Estimator	0.0823	0.6043	1.610×10^{-4}	2.000×10^{-6}	0.8956

Bringing the parameter estimator into formula (22), the reliability of steam turbine pump at 10000h is obtained, which is 0.6920. The reliability curve is shown in Figure 3.

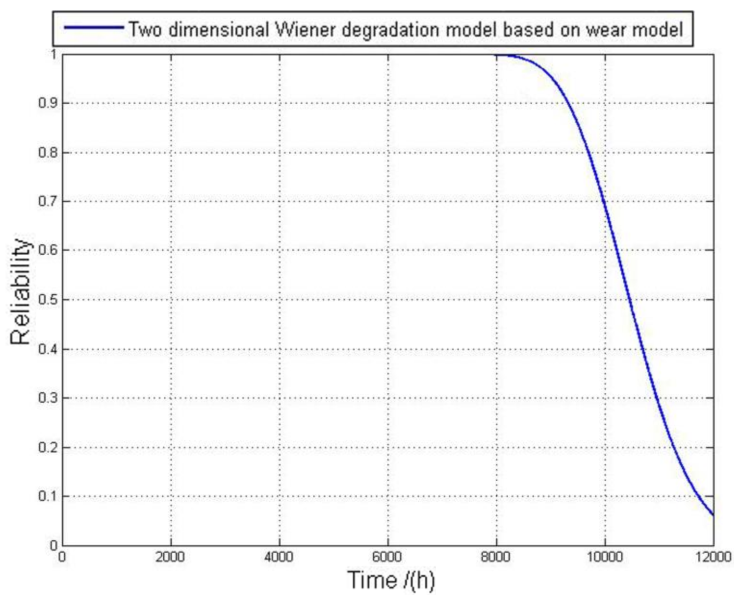

Figure 3. The reliability curve of steam turbine pump- Two dimensional Wiener degradation model based on wear model.

4.2 Comparison and analysis

Taking parameters estimator of three methods into their reliability functions respectively, the reliabilities at 10000h, are obtained, then calculate the relative error based on index. As shown in the Table 5, the reliability curves of the three methods are shown in the Figure 4.

Table 5. Relative errors of the three method.

Method	R	Relative Error
One dimensional Wiener degradation model based on MLE	0.8032	14.74%
Two dimensional Wiener degradation model based on MLE	0.6376	8.91%
Two dimensional Wiener degradation model based on wear model	0.6920	1.14%

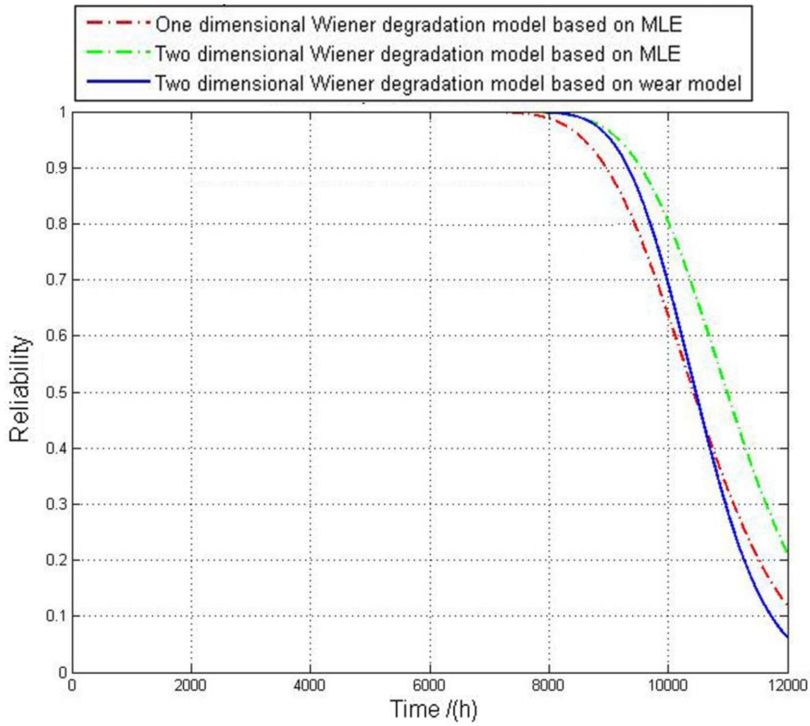

Figure 4. The reliability curves of the three methods.

Comparing and analyzing the reliability calculation results and reliability curves of the three methods, it shows: the relative error of one dimensional Wiener degradation model based on MLE is the maximum which is 14.74%, followed by two dimensional Wiener degradation model based on MLE which is 8.91%. The minimum is the method proposed in this paper, which is 1.14%. It can be seen that the method proposed in this paper is the closest to the real index. From the Fig.4, the reliability falling speed becomes faster over time. As time goes by, support bearing temperature showed a fluctuating upward trend, which will increase the wear of metal components. Then reliability will decrease. Therefore, the method proposed in this paper is more in line with the actual situation.

5 Conclusion

(1) In practical engineering, more than one parameter affects the life of the product. When assess the reliability of the product, only considering one parameter to estimate the whole product is not comprehensive. The evaluation results of the multivariate degradation process are more in line with the actual situation than one dimensional degradation process.

(2) In this paper, from the point of view of two performance parameters, combined with the theory of random degradation, a model of two dimensional Wiener stochastic degradation model is proposed based on the wear model for the products which have a wear process and uncertain stochastic degradation processes. The two dimensional Wiener degradation model is optimized by the method proposed in this paper in a certain extent.

(3) From the result of steam turbine pump reliability assessment, compared to the results of one dimensional Wiener degradation model based on MLE and two dimensional Wiener degradation model based on MLE, the method proposed in this paper has the minimum relative error which is 1.14% on the reliability 0.7 at 1000h. Thus the method proposed in this paper is more in line with the actual situation and more reliable.

References

1. J. Liu, J. Xie, H. Zuo, et al., Acta Aeronaut, Astronaut, Sin, **36**(2), 564-574 (2015)
2. S. Mishra and O.A. Vanli, J. Nondestruct. Eval., **35**(1), 1-13 (2016)
3. W. Kahle and A. Lehmann, *The Wiener Process as a Degradation Model: Modeling and Parameter Estimation* (Birkhäuser Boston, 2010)
4. F. Guérin, M. Barreau, A. Demri, et al., IFAC, **43**(3), 74-79 (2010)
5. P. Sander and R. Badoux, *Bayesian methods in reliability* (Kluwer Academic Publishers, 1991)
6. J. Tang, T.S. Su, NavRes Log, **55**(3), 265 (2008)
7. X. Wang, J. Multivariate Anal, **101**(2), 340 (2010)
8. B.H. Peng, J.L. Zhou and Z.Q. Pan, Syst. Eng. Theory Pract, **30**(3), 543 (2010)
9. R.P. Nicolai, R. Dekker and J.M. Noorwijk, Reliab Eng Syst Safe, **92**(12), 1635-1640 (2007)
10. J. Balka, A.F. Desmond and P.D. McNicholas, Lifetime Data Anal, **15**(12), 147-176 (2009)
11. X.S. Si, W. Wang, C.H. Hu, et al., Ieee T Reliab, **61**(1), 50-67 (2012)
12. F. Antonio Dorini and R. Sampaio, J. Appl Mech, **79**(5), 051008-051014 (2012)
13. Silva CRÁD, Pintaude G. Tribol Int, **41**(6), 473-481(2008)
14. H.J. Hu and W.J. Huang, Int J Adv Manuf Tech, **69**(1-4), 31-39 (2013)
15. G.Y. Lin, D. Feng, X.Y. Zheng, et al., J. Cent South Univ, **40**(5), 1245-1251 (2009)
16. Z.S. Gao, X.Z. Deng, F.X. Chen, et al., China Mech. Eng., **25**(2), 26-229 (2014)
17. D.H. Kim, H.C. Lee, B.M. Kim, et al., J. Mater Process Tech, **166**(3), 372-380 (2005)
18. G. Jin, *Reliability Technology based on Degradation* (Beijing, China, 2014)
19. M. Domine and V. Pieper, Probab Eng Inform Sc, **7**(4), 545-555 (1993)
20. J.F. Archard, J. Appl Phys, **24**(8), 981-988 (1953)
21. X. Shen, L. Cao and R. Li, MACE, 325-329 (2010)