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Multiaxial fatigue life estimation in low-cycle fatigue regime including the mean stress effect

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Abstract. The goal of the present paper is to discuss the reliability of a strain-based multiaxial Low-Cycle Fatigue (LCF) criterion in estimating the fatigue lifetime of metallic structural components subjected to multiaxial sinusoidal loading with zero and non-zero mean value. Since it is well-known that a tensile mean normal stress reduces the fatigue life of structural components, three different models available in the literature are implemented in the present criterion in order to take into account the above mean stress effect. In particular, such a criterion is formulated in terms of strains by employing the displacement components acting on the critical plane and, then, by defining an equivalent strain related to such a plane. The Morrow model, the Smith-Watson-Topper model and the Manson-Halford model are applied to define such an equivalent strain. The effectiveness of the new formulations is evaluated through comparison with some experimental data reported in the literature, related to biaxial fatigue tests performed on metallic specimens under in- and out-of-phase loadings characterised by non-zero mean stress values.

1 Introduction

Engineering metallic components and structures are frequently subjected to complex cyclic loading during their service life, which produces a multiaxial stress field generally characterised by tensor components with mean values different from zero.

In order to investigate the effect of mean stress and mean strain on fatigue life, two different tests are usually performed: stress-controlled cyclic test with constant mean stresses, or strain-controlled cyclic test with constant mean strains [1,2].

In High-Cycle Fatigue (HCF) regime, since cyclic behaviour of material can be considered as linear-elastic, the above tests are essentially equivalent, and one or the other can be indifferently used to examine the effect of mean stress or mean strain on the fatigue life.

As is well-known, a tensile mean normal stress/strain superimposed upon an alternating normal stress/strain strongly reduces the fatigue resistance of structural components, while a compressive mean normal stress/strain has a beneficial effect [1,3]. This can be explained by noting that a tensile mean normal stress increases the crack opening and accelerates the fatigue damage accumulation, whereas a compressive normal stress reduces the crack propagation through the friction between the two surfaces of the crack [1,4].

Stress-based methods are usually employed in HCF regime, since the material behaviour can be considered as linear-elastic and, consequently, stresses can be easily estimated using elastic analysis. In the literature, several models based on the S-N approach have been developed to properly evaluate the mean stress effect: for instance, the Soderberg relationship, the Goodman relationship, and the Gerber parabola [5-8].

In Low-Cycle Fatigue (LCF) regime, instead, the cyclic response of material is elasto-plastic. Consequently, stress-controlled cyclic test and strain-controlled cyclic test may produce quite different results.

In particular, in the case of *stress-controlled cyclic tests* with constant mean stress, the occurrence of a plastic deformation causes a ratcheting strain (also named cyclic creep strain) [1,2]. The ratcheting strain progressively increases cycle by cycle, and the ratcheting strain accumulation intensifies the fatigue damage, resulting in shorter fatigue life [9,10].

It follows that, in stress-controlled condition, a fatigue failure model should be able to take into account both mean stress and ratcheting strain effects (an interesting review concerning the constitutive models for the ratcheting behaviour of metals is reported in Ref. [9]). However, the analysis of the ratcheting strain phenomenon is complex, since the ratcheting behaviour depends on many factors including the value of mean stress and stress amplitude, the loading history, the loading rate and the microstructural characteristics. Due to such variables, it is very difficult to find fatigue damage models concerning with the mean stress effect and containing a unique parameter able to take into account the ratcheting strain. An interesting energy-based model has been proposed by Xia and co-workers in order to properly evaluate the mean stress and the ratcheting strain effect on fatigue life [1,2].

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In LCF regime under *strain-controlled cyclic tests*, the occurrence of plastic deformations usually results in a mean stress which may relax fully or partially during the loading cycle (i.e. mean stress relaxation). The mean stress usually quickly decreases in the early fatigue life, and then reaches a constant value.

Fatigue damage models may be developed only considering the mean stress effect, since the effect of the applied mean strain is negligible. In particular, it has experimentally been observed that the mean strain does not contribute to the crack opening mechanism, when the mean stress relaxes to a stable value (about one thousand cycles) [11]. Mean stress relaxation depends on the magnitude of the applied strain amplitude [11-13], that is, the rate of mean stress relaxation is greater for higher values of strain amplitude.

The evaluation of the mean stress influence on fatigue life is a complex problem. Different models are available in the literature in order to take into account the above mean stress: the most popular ones are those by Smith [14], Morrow [15,16], Smith-Watson-Topper (SWT) [17], Walker [18] and Manson-Halford (MH) [19]. Note that the above models are currently used, but nowadays none of them is considered better than the others.

In the present paper, three different models are implemented in the critical plane-based multiaxial fatigue criterion for LCF regime recently proposed by Carpinteri et al. [20, 21], in order to take into account the above mean stress effect. Such a criterion is formulated in terms of strains by employing the displacement components acting on the critical plane and defining an equivalent strain as the parameter to quantify the fatigue damage. The Morrow model, the SWT model and the MH model are applied to define such a strain.

The effectiveness of the new formulations is evaluated through comparison with experimental data [22] related to biaxial fatigue tests performed on metallic specimens under both proportional and non-proportional loadings characterised by non-zero mean values.

2 Mean stress effect on multiaxial fatigue life estimation

The present critical plane-based multiaxial fatigue criterion, recently proposed by some of the present authors [20,21], is formulated in terms of strains in order to estimate the fatigue lifetime under LCF regime. Such a criterion aims to reduce the multiaxial strain state to an equivalent uniaxial one.

Figure 1 summarises how to use the above-mentioned criterion to estimate fatigue lifetime of structural components in LCF regime. In more detail, by starting from the strain state at a material point P, the averaged directions of the principal strain axes may be computed on the basis of their instantaneous directions by means of the averaged values of the principal Euler angles. The normal $\mathbf{w}$ of the critical plane (see Figure 1) is linked to the above directions through the off-angle δ, defined as follows:

$$\delta = \frac{3}{2}\left[1-\left(\frac{1}{2\left(1+\nu_{eff}\right)}\cdot\frac{\gamma_a}{\varepsilon_a}\right)^2\right]45° \qquad (1)$$

being ν_{eff} the effective Poisson ratio. Moreover, ε_a and γ_a are determined through the tensile and torsional Manson-Coffin equations, respectively:

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \qquad (2a)$$

$$\gamma_a = \frac{\tau'_f}{G}\left(2N_f\right)^{b_0} + \gamma'_f\left(2N_f\right)^{c_0} \qquad (2b)$$

where N_f is the number of loading cycles to failure, E is the Young modulus, G is the shear modulus and σ'_f, ε'_f, b, c, τ'_f, γ'_f, b_0 and c_0 are material constants to be determined by running appropriate axial fatigue tests.

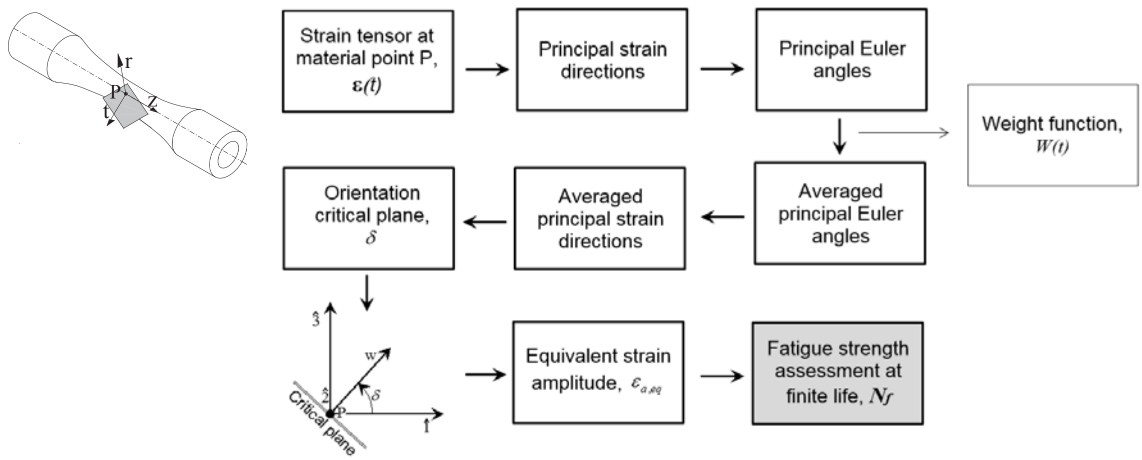

Fig 1. Graphical representation of the low-cycle multiaxial fatigue criterion.

Let us consider a local frame $Puvw$, attached to the critical plane, and the corresponding strain tensor. Details about the definition of such a frame may be found in Ref. [20].

An equivalent strain amplitude, $\varepsilon_{eq,a}$, is computed through a quadratic combination of the normal displacement amplitude, $\eta_{N,a}$ (being $\eta_{N,a}=\varepsilon_{w,a}$), and the shear displacement amplitude, $\eta_{C,a}$ (being $\eta_{C,a}=\sqrt{(\gamma_{uw,a}/2)^2+(\gamma_{vw,a}/2)^2}$), acting on the critical plane:

$$\varepsilon_{eq,a}=\sqrt{(\eta_{N,a})^2+\left(2(1+\nu_{eff})\cdot\frac{\varepsilon_a}{\gamma_a}\right)^2(\eta_{C,a})^2} \qquad (3)$$

All terms in Eq. (3) depend on the number of loading cycles to failure, N_f. Therefore, by equaling Eq. (3) to Eq. (2a), the value of N_f can be worked out by means of an iterative procedure.

The original formulation of the criterion can be used to predict the fatigue life at zero mean stress. Now three different models are implemented in such a formulation in order to include the effect of non-zero mean stress:

(i) The Morrow model [15-16]. Such a model introduces the mean stress correction into the elastic term of the tensile Manson-Coffin equation (Eq.(2a)):

$$\varepsilon_{a,M}=\frac{\sigma'_f}{E}\left(1-\frac{\sigma_{z,m}}{\sigma'_f}\right)(2N_f)^b+\varepsilon'_f(2N_f)^c \qquad (4)$$

where $\sigma_{z,m}$ is the value of the mean axial stress at point P, measured during the strain-controlled cyclic test.

Consequently, Eqs (1) and (3) are rewritten by assuming $\varepsilon_a=\varepsilon_{a,M}$;

(ii) The Smith-Watson-Topper (SWT) model [17]. Such a model assumes that the fatigue life depends on the product of the maximum normal stress and the normal strain amplitude and, consequently, the tensile Manson-Coffin equation (Eq. (2a)) turns out to be:

$$\varepsilon_{a,SWT}=\frac{1}{\sigma_{z,max}}\left[\frac{(\sigma'_f)^2}{E}(2N_f)^{2b}+\sigma'_f\varepsilon'_f(2N_f)^{c+b}\right] \qquad (5)$$

where $\sigma_{z,max}$ is the maximum value of the axial stress at point P, measured during the strain-controlled cyclic test.

Consequently, Eqs (1) and (3) are rewritten by assuming $\varepsilon_a=\varepsilon_{a,SWT}$.

(iii) The Manson-Halford (MH) model [19]. Such a model is an extension of the Morrow model by assuming that also the plastic term of the tensile Manson-Coffin equation (Eq. (2a)) is affected by the presence of the mean stress:

$$\varepsilon_{a,MH}=\frac{\sigma'_f}{E}\left(1-\frac{\sigma_{z,m}}{\sigma'_f}\right)(2N_f)^b+\varepsilon'_f\left(1-\frac{\sigma_{z,m}}{\sigma'_f}\right)^{\frac{c}{b}}(2N_f)^c \qquad (6)$$

Consequently, Eqs (1) and (3) are rewritten by assuming $\varepsilon_a=\varepsilon_{a,MH}$.

3 Criterion validation and discussion

In the present paper, the different relationships reported in Eqs (4)-(6) are implemented in the original formulation of the criterion in order to verify whether they are able to estimate the lifetime of some experimental tests available in the literature [22].

The examined experimental data are related to 27 specimens made of Inconel 718 [22, 23], whose static and fatigue properties are listed in Table 1. The above specimens are subjected to synchronous sinusoidal in- and out-of-phase loading with zero and non-zero mean value. In particular, 19 specimens are subjected to in-phase loading (i.e. phase angle, Φ, equal to 0°), 2 specimens to out-of-phase loading with $\Phi=45°$, and the remaining 6 specimens are characterised by a phase angle equal to 90°.

The value of the effective Poisson ratio ν_{eff} is here obtained from a best fitting procedure by considering only the experimental data related to loading histories with zero-mean strain value [22]. The following error index, I, has been optimised by varying the value of ν_{eff} from 0.1 to 0.5:

$$I=\frac{\varepsilon_{eq,a}-\varepsilon_a}{\varepsilon_a} \qquad (7)$$

being ε_a defined by Eq. (2a). At the end of such a best fitting procedure, the value of ν_{eff} is taken equal to 0.34.

The accuracy of the proposed fatigue lifetime estimation has been evaluated by means of the root mean square error method [24]. In more detail, the value of the root mean square logarithmic error is computed as follows:

$$E_{RMS}=\sqrt{\frac{\sum_{i=1}^{n}\log^2\left(N_{f,\exp}/N_{f,cal}\right)}{n}} \qquad (8)$$

Table 1. Static and fatigue properties of Inconel 718 [22, 23].

E (MPa)	σ'_f (MPa)	b (-)	ε'_f (-)	c (-)	G (MPa)	τ'_f (MPa)	b_0 (-)	γ'_f (-)	c_0 (-)
208500	3950	-0.151	1.5	-0.761	77800	2146	-0.148	18.0	-0.922

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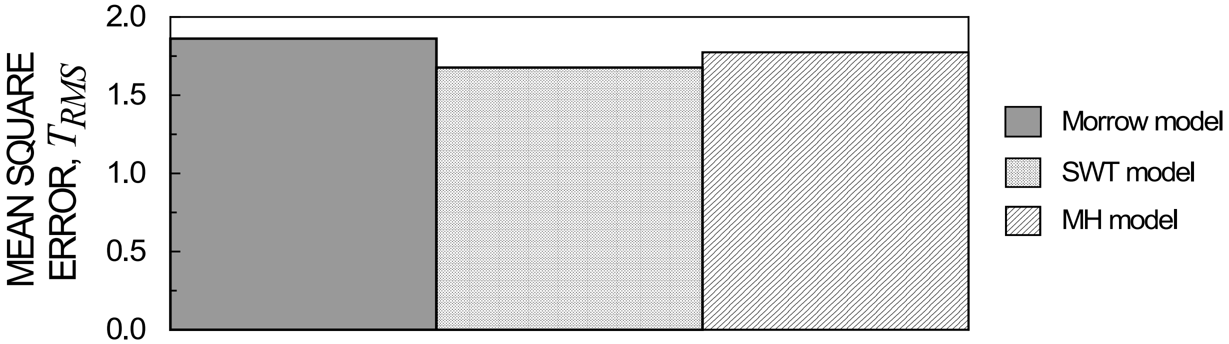

Fig. 2. Mean square error related to Inconel 718, determined by applying the present criterion together with the Morrow model or the SWT model or the MH model.

where n is the total number of data, $N_{f,exp}$ is the experimental multiaxial fatigue life, and $N_{f,cal}$ is the theoretical multiaxial fatigue life determined by considering the different approaches presented in Eqs (4)-(6).

Finally, the mean square error T_{RMS} is given by

$$T_{RMS} = 10^{E_{RMS}} \quad (9)$$

Figure 2 shows the mean square error computed for all the values of fatigue life, in accordance with the three different models presented in Section 2 in order to take into account the effect of the mean stress. The analysis of the results clearly proves that the present criterion together with the three models produces evaluations within the scatter band 2, being the value of the T_{RMS} lower than 2. Moreover, it can be noticed that the lifetime estimated through the SWT model (Eq. (5)) is slightly more accurate than the other two ones, being T_{RMS} equal to 1.68 whereas the mean square error values related to the MH model and the Morrow model are equal to 1.77 and 1.86, respectively.

As a consequence of such a remark, the comparison between experimental data and theoretical estimations in terms of fatigue life is limited to the results determined by implementing Eq. (5) in the present criterion (see Figure 3).

In Figure 3, the solid line indicates $N_{f,cal} = N_{f,exp}$, the dashed lines correspond to $N_{f,cal} / N_{f,exp}$ equal to 0.50 and 2 (scatter band 2), and the dot-dashed lines correspond to $N_{f,cal} / N_{f,exp}$ equal to 0.33 and 3 (scatter band 3).

Figure 3 demonstrates that 78% of the estimated results fall within the scatter band of 2, whereas all of them fall within the scatter band of 3, and this holds true independent of both the value of the mean stress and the degree of non-proportionality.

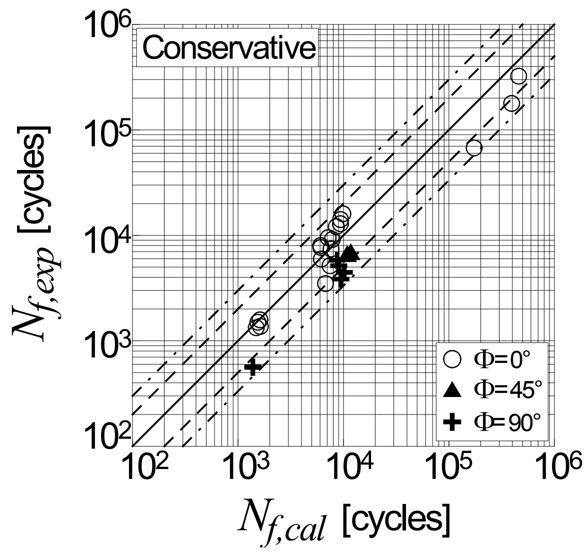

Fig. 3. Comparison of the experimental fatigue lifetime, $N_{f,exp}$, [22] and the theoretical one, $N_{f,cal}$, estimated through the present criterion together with the SWT model (Eq. (5)).

4 Conclusions

In the present paper, the reliability of a strain-based multiaxial LCF criterion in estimating the fatigue lifetime of metallic structural components subjected to sinusoidal loading with zero and non-zero mean stress value has been discussed.

The effect of the non-zero mean stress has been taken into account by implementing three models available in the literature: the Morrow model, the Smith-Watson-Topper model, and the Manson-Halford model. Since such a criterion is formulated in terms of strains, the above-mentioned models are implemented in the definition of both the critical plane orientation and the equivalent strain amplitude, which is the parameter quantifying the fatigue lifetime.

The effectiveness of the new formulations is evaluated through some experimental data reported in the literature, related to biaxial fatigue tests performed on metallic specimens under in- and out-of-phase loadings characterised by non-zero mean values. The accuracy of the proposed formulations has been evaluated by means of the root mean square error method, which proves that all the implemented models provide values of the mean square error lower than 2. According to the mean square error value, the SWT model results to be slightly more accurate than the other two models.

By comparing the experimental data with the theoretical estimations determined by means of the SWT model in terms of fatigue life, it has been observed that the agreement is satisfactory, and this holds true for both compressive and tensile non-zero mean stress values.

Therefore, the implementation of the SWT model in the present criterion seems to be a promising tool to assess the fatigue lifetime of metallic components in LCF regime with non-zero mean stress, although different materials need to be processed in order to validate the robustness of the criterion for practical applications.

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