

Probability calibration of deformation factor for timber roofs in the Polish mountain zones

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Abstract. The resistance parameters of timber material structures decrease with time, depending on the type of load and timber classes. Strength and Modulus of Elasticity reduction effects, referred to as creep-rupture effects, due to long term loading at high stress ratio levels are known for many materials. Timber materials are highly affected by this reduction in strength and deflection with duration of load. Characteristic values of load duration and deformation factors are calibrated by means of using probabilistic methods. The reliability is estimated by means of using representative short- and long-term limit states. Time variant reliability aspects are taken into account using a simple representative limit state with time variant strength and simulation of whole life time load processes. The parameters in these models are fitted by the Maximum Likelihood Methods using the data relevant for Polish structural timber. Based on Polish snow data over 45 years from mountain zones in: Zakopane – Tatra, Świeradów – Karkonosze, Lesko – Bieszczady, the snow load process parameters have been estimated. The reliability is evaluated using representative short – and long –term limit states. The deformation factor k_{def} is obtained using the probabilistic model.

1 Introduction

Timber is a general name which comprises some of the elements of timber materials presented in Fig.1.

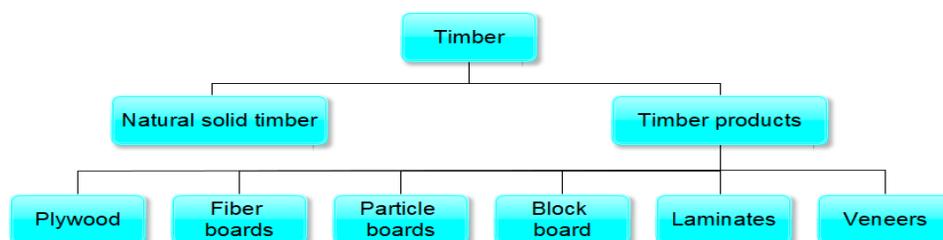


Fig. 1. Timber materials.

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The scope of work is the influence of variable loads (snow) to timber resistance parameters. Characteristic values of timber material parameters have to be factored by the deformation/ creep factor k_{def} / k_c . The final mean values of modulus of elasticity, $E_{mean,fin}$, shear modulus $G_{mean,fin}$, and slip modulus, $K_{ser,fin}$ can be obtained as [1]:
 for serviceability limit state:

$$\frac{E_{mean,fin} / G_{mean,fin} / K_{ser,fin}}{1 + k_{def}} = \frac{E_{mean} / G_{mean} / K_{ser}}{1 + k_{def}}, \quad (1)$$

and for ultimate limit state:

$$\frac{E_{mean,fin} / G_{mean,fin} / K_{ser,fin}}{1 + \psi_2 k_{def}} = \frac{E_{mean} / G_{mean} / K_{ser}}{1 + \psi_2 k_{def}}. \quad (2)$$

2 Deformation factor / Creep factor (k_{def}/k_c)

The k_{def} was derived from creep testing in accordance with EN 1156 [2,3]. The creep factor k_c is defined as the ratio of increase in deflection with time under load to the initial elastic deflections. The creep factor k_c in time t is given as follows:

$$k_c = \frac{(a_t - a_0) - (a_1 - a_0)}{a_1 - a_0}. \quad (3)$$

Where: a_t is the total deflection -in millimeters at time t minutes, a_1 is the deflection in millimeters at 1 minute, a_0 is the deflection in millimeters of unloaded test. The illustration of the creep factor for two load scenarios is presented in Fig. 2.

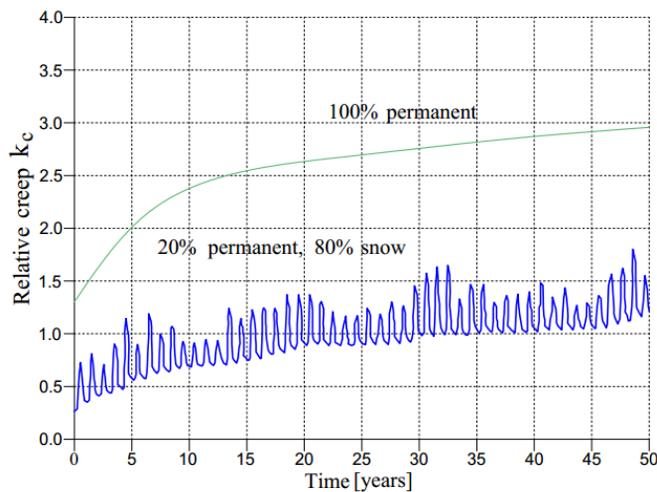


Fig. 2. Relative deflection factor for glulam beam for permanent load and for combination with a random snow load [4, 5].

The strain of wood in time $\varepsilon(t)$ is assumed to consist of the following additive parts: plastic ε_E , normal creep ε_c , mechano-sorptive creep ε_{ms} and free shrinkage strains ε_u [4, 5]:

$$\varepsilon(t) = \varepsilon_E + \varepsilon_c + \varepsilon_{ms} + \varepsilon_u. \quad (4)$$

The deformations are calculated parallel to the grain, a linear strain distribution and Bernoulli hypothesis are assumed.

The **elastic strain** depends on the elastic compliance, which is the function of the current moisture content.

The **normal creep strain**, which is due to the deformation induced by the duration of load is given in a Kelvin series form.

The **mechano-sorptive creep strain**, which is due to additional creep deformation caused by the coupled effect of variable moisture and load duration. In constant stress case this takes the following form:

$$\varepsilon_{ms}(t) = J^\infty \sigma \left[1 - \exp\left(-c \int_0^t du(t)\right) \right]. \quad (5)$$

Where: J^∞ is the limit mechano-sorptive compliance, u is the moisture content and c is the material parameter.

The **shrinkage strain**, the reversible moisture induced strain, is assumed to be affected by the total strain. This strain component does not cause creep, but in cyclic humidity conditions it accounts for the oscillation of the creep curve.

$$\varepsilon_u(t) = \{\alpha_s - b\varepsilon(t)\}[u(t) - u(0)]. \quad (6)$$

Where: α_s is the shrinkage coefficient, b is a material parameter and $\varepsilon(t)$ is the current total strain excluding the shrinkage strain.

The output from the analysis consists of the following distributions at time t :

Moister content distribution history $u(x, y, t)$, strain history $\varepsilon(x, y, t)$, stress history $\sigma(x, y, t)$

Service class 2 will be considered in this paper. Strain history will be dependant on the stress history.

3 Stress history

The effect of sustained stress on timber bending strength and timber deflections has been recognized since at least 1850. Wood developed a time strength curve that was incorporated into the wood design procedure. This curve, often referred to as the “Madison curve”, is still in use.[6]. During the last decades structural reliability methods have been further developed, improved and adopted and are now at a level, where they are being applied in practical engineering problems.

This knowledge has now reached a level where it enables designers to take into account uncertainties in material properties and actions in assessing the load carrying capacity, serviceability and service life of timber structures and connections. Nowadays, most building codes are based on a probabilistic safety approach. The code formats are deterministic with connections to reliability design achieved by failure probability, partial safety factors and characteristic values. Partial safety factors are calibrated for standard cases against probabilistic analysis for similar cases. The condition for calibration that is

the probabilistic analysis and deterministic code should be fulfilled with the same safety requirements. Therefore, the required safety is usually not accomplished by using probabilistic theories in everyday designing process. Timber is a rather complex building material. The timber material characteristics depend on the specific wood species, the geographical location where the wood has been grown and furthermore on the local growing conditions. Timber is an orthotropic material and it consists of high strength grains which are predominantly orientated along the longitudinal axis of a tree. Material characteristics – the ultimate bending stress and the bending stiffness depend on the orientation of the moment axis to the grain direction. Irregularities in regard to grain direction, knots and fissures become highly decisive for load bearing capacity of a timber structural element. It is possible to avoid irregularities with small test specimen only, with so-called clear wood specimens.

The Gerhards [6] damage accumulation model is taken into consideration. The mechanism leading to the reduction of strength of a timber member under sustained load is a creep rupture. This could arise from propagation of voids in the microstructure of the timber at a stress level, lower than the short-term strength. A number of models of creep rupture, involving a damage state variable (similar to that used in the analysis of metal fatigue), have been proposed to assess damage accumulation in wood structural members subject to loading histories, typically modeled as renewal pulse process. The basic model [7-11] has the following form Eq. (7)

$$\frac{d\alpha}{dt} = F[s(t)], \quad (7)$$

where t – time, α - the damage state variable which ranges from 0 (no damage) to 1 (failure), the function $F(.)$ has two constants that must be determined from test data, and $s(t)$ – the ratio of the applied stress to the failure stress under short-term ramp loading.

Damage models are used for mathematical description of the long term strength reduction as a function of stress level and duration of loading. The characteristics of damage models are that α is defined as the degree of damage, i.e. $\alpha = 0$ stands for no damage and $\alpha = 1$ stands for total damage or failure.

The damage accumulation model presented by Gerhard, [6] is

$$\frac{d\alpha}{dt} = \exp(-A + B \frac{\sigma}{f_0}). \quad (8)$$

Where A and B are constant, σ is the stress and f_0 is short term strength of member. Solution of equation (8) is :

$$\frac{\sigma}{f_0} = \frac{A}{B} - \ln \frac{10}{B} \log t = a - b \log t, \quad (9)$$

where;

$$a = \frac{A}{B} + \delta \quad b = \frac{\ln 10}{B}, \quad (10)$$

end δ models the uncertainty related to the formula (7). δ is assumed to be Normal distributed with expected value equal 0 and standard deviation 1.

Assuming constant load and considering f as the residual strength the solution to (9) is as follows:

$$\frac{f}{f_0} = \frac{1}{B} \ln(1 + (1 - \alpha)(\exp B - 1)). \quad (11)$$

This expression is used in simulating the damage due to load duration.

4 Snow loads

A stochastic model of snow load in the mountain zone in Zakopane – Tatra , Świeradów – Karkonosze, Lesko – Bieszczady is established on the basis of meteorological data [12, 13]. Examples of season snow loads in Zakopane , Świeradów, Lesko are shown in Figs 3, 4, 5.

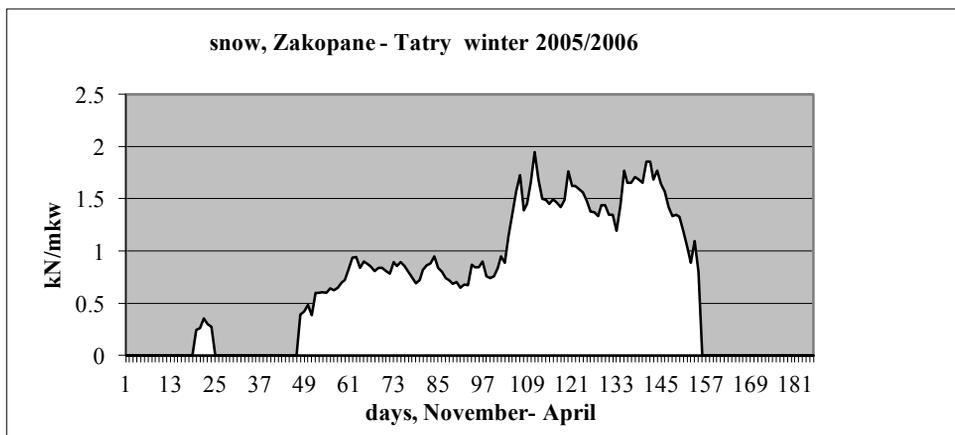


Fig. 3. Snow loads in Zakopane, Poland.

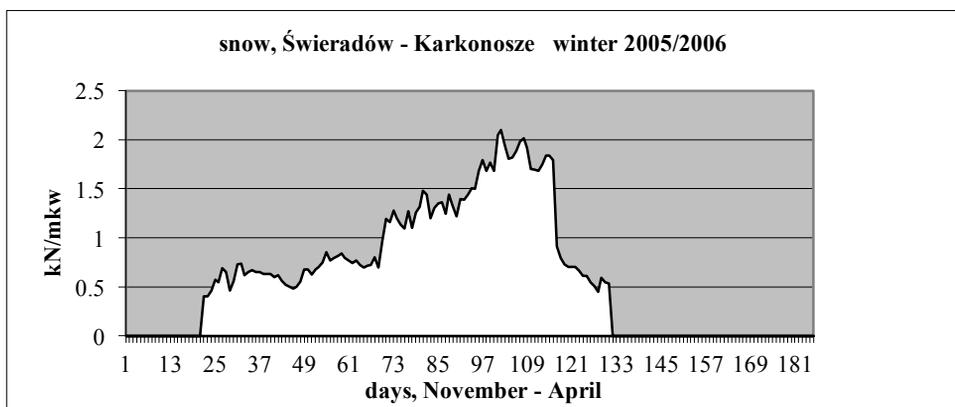


Fig. 4. Snow loads in Świeradów, Poland.

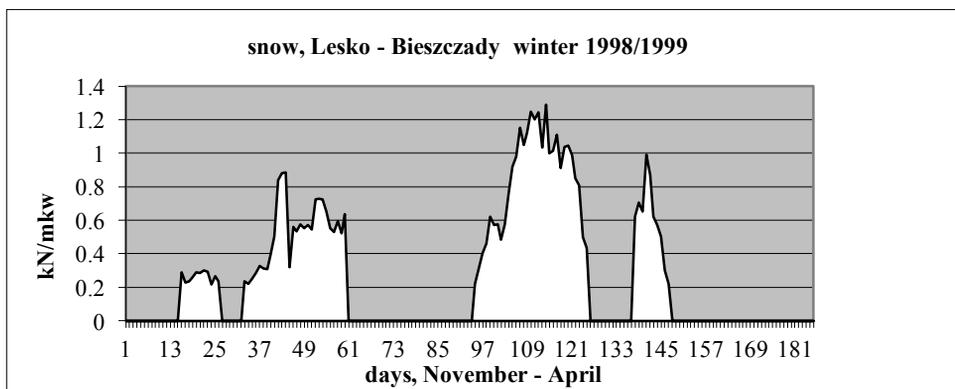


Fig. 5. Snow loads in Lesko, Poland.

A model calibrated against direct measurements of snow load is issued to transform the meteorological data into snow loads model. The load model is illustrated in Fig 6. rectangular and triangular [14].

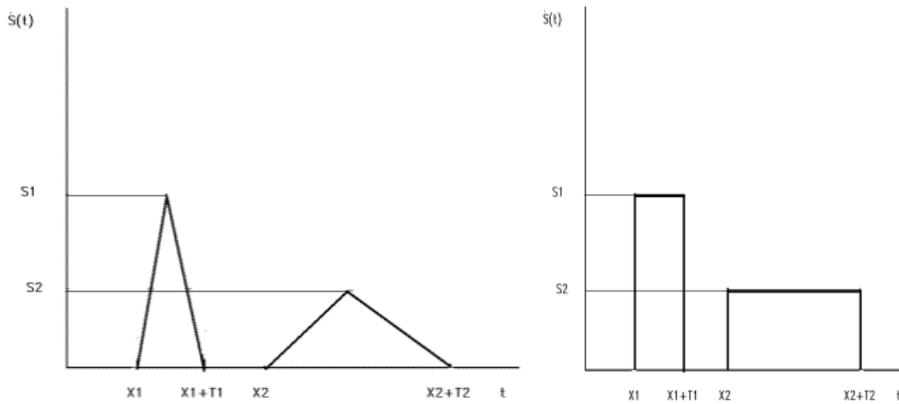


Fig 6. Snow load models – rectangular and triangular.

The snow load on terrain $S(t)$, and duration T of snow packages (snow pulses) are modelled as follows:

- The occurrence of snow packages at times $X1, X2, \dots$ is modeled by a Poisson process.
- The duration between snow packages is exponential distributed with expected value $1/\lambda$, where λ is expected number of snow packages per year.
- The magnitude of the maximum snow load P_m in one snow package (snow pulse) is assumed to be Gumbel [15] distributed with expected value μ_p and standard deviation σ_p .

The Table 1. shows probabilistic parameters of snow packets in Zakopane (Z), Świeradów (S) and Lesko (L).

Table 1. Probabilistic parameters of snow packets

	Zakopane	Świeradów	Lesko
$\mu_p [kN/m^2]$	1.47	1.12	0.80
$\sigma_p [kN/m^2]$	0.49	0.65	0.37
$\mu_{XT} [\text{days}]$	65.58	40.97	53.62
λ	1.43	1.85	1.64

5 Deformation factor estimation

In the code format *EC5* load duration effect and deformation factor are represented by the modification factor k_{mod} and k_{def} . The following design equation for long-term situation with one variable load can be found in the code [16,17].

$$\frac{zf_k k_{mod}}{\gamma_m} - [(1 - \kappa)\gamma_g G_k + \kappa\gamma_Q Q_k] = 0, \quad (12)$$

$$u_{tar} - u_{fin, G_k(1-\kappa)} + u_{fin, Q_k\kappa} = 0,$$

$$u_{fin, G_k(1-\kappa)} = u_{inst, G_k} (1 + k_{def}), \quad (13)$$

$$u_{fin, Q_k\kappa} = u_{inst, Q_k\kappa} (1 + \psi_2 k_{def}).$$

Where: u_{tar} is the limiting design value of the relevant serviceability criterion (EC1990), z is the design variable, f_k is the characteristic value for short-term strength (5% quantile), Q_k is the characteristic values of variable – snow load (2% quantile), G_k is the characteristic value of permanent load (mean value), γ_g is the partial safety factor for permanent load (=1.35), γ_Q is the partial safety factor for variable load (=1.5), γ_c is the partial safety factor for material parameter for glued materials (=1.2) and the coefficient κ which represents proportion between variable and total loads. The corresponding long-term limit state equations is:

$$g = z(1 - \alpha)f_o [(1 - \kappa)G + \kappa Q]. \quad (14)$$

Where α is the damage state variable, f_o is the short term strength, G , Q are the permanent and variable – snow loads.

For short-term situation conditions design and limit state equation the formulas can be described as follows:

$$\frac{zf_k}{\gamma_m} - [(1 - \kappa)\gamma_g G_k + \kappa\gamma_Q Q_k] = 0, \quad (15)$$

$$g = zf_o - [(1 - \kappa)G + \kappa Q]. \quad (16)$$

Where z is the design parameter, f_o – short term strength, G – permanent load, Q variable load.

The reliability indexes are calculated by simulation according to FORM method on the basis of (14), (16) and stochastic model.

By utilizing Monte Carlo simulation for generating random variables the modification factor k_{def} is determined according to the following procedure [16,17].

1. Calculate the short term reliability index β_S for a 50 year reference period using the limit state function (14) and the design equation (1). β_S is calculated as function of κ by simulation (γ_g and γ_Q are fixed).
2. Calculate the long term reliability index β_{L} for a 50 year reference period using the limit state function (12) and the design equation (11) and $k_{def} = 1$. β_{L} is calculated as function of κ by simulation (γ_g and γ_Q are fixed).

Then k_{def} factor can be estimated as follows:

$$k_{def} = \frac{\gamma_m^S(\beta s)}{\gamma_m^L(\beta L)}, \quad (17)$$

where $\gamma_m^S(\beta s)$ is the short term partial safety factor as function of β and $\gamma_m^L(\beta L)$ is the short term partial safety factor as function of β .

The results of calculations of deformation factor for timber materials such as: Solid Timber, Medium Density Fibreboard (MDF), Oriented Strand Board (OSB) and Plywood are presented in Fig. 7.

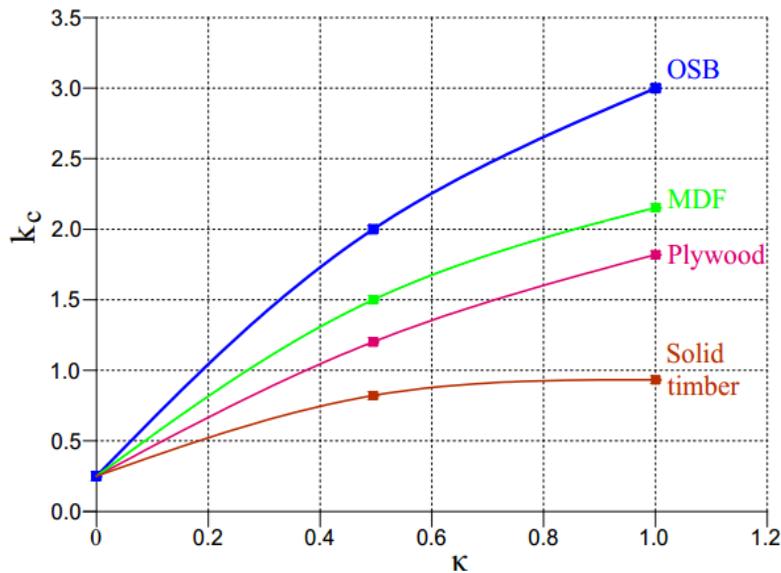


Fig. 7. Deformation/creep factor as function of parameter κ

6 Conclusions

The probabilistic models for the graded timber material properties have been formulated so that they may be readily applied in structural reliability analysis. It has been noted that a significant effect of the time variation of snow impulses-packages on the accumulated damage model has been found. Therefore the observed snow packages are quite different and the triangular and rectangular time variations are included in the present probabilistic calibration of deformation factor. The deformation factor k_{def} value depends significantly on the ratio between permanent and variable (snow) loads. More research is needed on the probabilistic resistance parameters of wood planks and another types of timber boards to describe timber deformation factors.

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