

Influence of flange on the shear capacity of reinforced concrete beams

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Abstract. In structural engineering (RC, steel, etc.) it is usual to base the shear strength of members on the web only- e.g. in RC the stirrups used are usually called "web reinforcement". Presently all codes, and several researches, base the strength of members on the capacity of the web alone. 93 tests of T-beams failing in shear available from the literature are studied in this work to estimate the influence of flanges on the shear capacity of RC beams. These include 32 ones without web reinforcement and 61 with stirrups. Comparison between test results and theoretical shear capacity show that all available equations conservatively estimate the occurrence of shear failure. In this work an equation for predicting the contribution of the flange to shear capacity in T-beams is presented.

The best available design method obtained from the literature leads to a coefficient of variation (COV) of 17.58% compared to 13.46% for the proposed design method in this work.

1 Introduction

There are significantly different methods to estimate shear capacity of reinforced concrete (RC) beams. The main parameters affecting the occurrence of shear failure are: concrete compressive strength, longitudinal reinforcement ratio, shear span to effective depth ratio, and size effect of the member. Bresler and MacGregor [1] indicate that shear failure is commonly initiated by the occurrence of diagonal cracks developing in the shear span, and that the flexural cracks always come before the occurrence of diagonal cracks in rectangular, I, and T- sections. They noted that the shape of the beam (I and T-sections) influences the shear capacity and the behavior including propagation of diagonal cracking due to the different magnitudes of shearing stress developed in the web. However, not much attention has been given to the behavior of RC beams with T-sections.

Swamy and Qureshi [2] proposed a procedure to estimate the ultimate shear strength of the compression zone of T-beams. This procedure was derived by using the concept of biaxial stress criterion and was based on Mohr's theory of failure. However, this method needs several steps of calculation and is difficult to apply in practice.

Ahmed [3] proposed an empirical equation which is simpler than the procedure of reference 2. This equation includes the flange contribution in predicting the ultimate shear capacity of T-sections.

Existing equations for shear capacity of concrete available in the codes do not take into account the influence of the flange in T-sections [4, 5].

In this work, empirical equations of researchers and design codes for shear capacity of beams are compared

with test results obtained from the literature to find an equation for predicting the contribution of flange to the shear capacity of concrete in RC beams with T-sections.

2 Research significance

Several code and research estimates of shear capacity of RC beams are studied in this work. A data base of 93 tests is used in this work: 32 without stirrups and 61 with stirrups. It is found that including of flange contribution leads to a significantly improved COV for all available 93 tests from the literature. In fact, the proposed equation leads to a COV of 13.46% compared to the range of (17.58% - 42.55%) for the available methods.

3 Experimental investigations

The 93 shear tests have been taken from the literature: Taub and Neville [6]; Swamy and Qureshi [7]; Haddadin et al [8]; Placas and Regan [9]; Palaskas et al [10]; Kotsovos et al [11]; and Thamrin et al [12].

All of these RC T- beams involved slender ones (shear span /effective depth ratio ≥ 2.5) that failed in shear under concentrated loading. Table 1 indicates the range of variables in all 93 tests.

Table 1. Range of variables

Variable	Unit	Range
f'_c	MPa	11.99 – 56.95
a/d	----	2.5 – 10.4
ρ_w	----	0.49% – 5.2%
$\rho_v f_{yt}$	MPa	0 – 2.71
$b_w d$	mm ²	2903 – 76000
b/b_w	----	2 – 7

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where:

- f'_c = cylinder compressive strength of concrete, MPa
- a = shear span, equal to the distance from center of concentrated load to center of support for simply supported members, mm
- d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement, mm
- ρ_w = ratio of A_s to $b_w d$
- A_s = area of non prestressed longitudinal tension reinforcement, mm².
- b_w = web width, mm
- ρ_v = ratio of A_v to $b_w S$
- A_v = area of shear reinforcement within spacing S , mm²
- S = center- to- center spacing of transverse reinforcement, mm
- f_{yt} = specified yield strength of transverse reinforcement, MPa.
- b = flange width, mm

4 Code estimates of RC beam shear capacity

4.1 BS 8110:1997 [13]

$$V_{rBS} = 0.79 \left(\frac{100A_s}{b_w d} \right)^{1/3} \left(\frac{400}{d} \right)^{1/4} \left(\frac{f_{cu}}{25} \right)^{1/3} b_w \cdot d / \gamma_m + \frac{0.95 A_v f_{yt} d}{S} \quad (1)$$

$$0.15 \leq \frac{100A_s}{b_w d} \leq 3.0 \quad (2)$$

$$\frac{400}{d} \geq 1.0 \quad (3)$$

where:

- V_{rBS} = estimated shear strength per reference 13, N
- $\left(\frac{400}{d} \right)$ = size effect, used for enhancement only
- f_{cu} = cube compressive strength of concrete, not to exceed 40 MPa. For comparison in this work, $f_{cu} = f'_c / 0.82$
- $\gamma_m = 1.25$

4.2 Australian Code; AS 3600:2001 [14]

$$V_{rAS} = 0.7 \left[\beta_1 b_w d \left(\frac{A_s f'_c}{b_w d} \right)^{1/3} + \frac{A_v f_{yt} d}{S} \right] \quad (4)$$

$$\beta_1 = 1.1 \left(1.6 - \frac{d}{1000} \right) \geq 1.1 \quad [\text{size effect}] \quad (5)$$

where:

- V_{rAS} = estimated shear strength per reference 14, N

4.3 Canadian code; A23.3:2004 [15]

$$V_{rCAN} = 0.65 \beta \sqrt{f'_c} b_w d_v + \frac{0.85 A_v f_{yt} d_v \cot 35^\circ}{S} \quad (6)$$

$$\beta = 0.18, \text{ with stirrups} \quad (7)$$

$$\beta = \frac{230}{1000 + d_v}, \text{ without stirrups} \quad (8)$$

$$\leq 64 \text{ MPa, upper limit} \quad (9)$$

$$d_v = \begin{cases} 0.9d \\ 0.72h \end{cases} \text{ whichever is greater} \quad (10)$$

where:

- V_{rCAN} = estimated shear strength per reference 15, N
- h = overall thickness or height of member, mm

4.4 Euro code: 2014 [16]

$$V_{rEURO} = [0.12 k (100 \rho_w f'_c)^{1/3}] b_w d + \frac{0.75 A_v f_{yt} d}{S} \quad (11)$$

where:

- V_{rEURO} = estimated shear strength per reference 16, N
- $k = 1.0 + \sqrt{200/d} \leq 2.0$ (12)

4.5 ACI 318M-14 [17]- Simplified method

$$V_{rACI-S} = 0.75 \left[0.17 \sqrt{f'_c} b_w d + \frac{A_v f_{yt} d}{S} \right] \quad (13)$$

where:

- V_{rACI-S} = estimated shear strength per reference 17; based on simplified ACI method, N

4.6 ACI 318M-14 [17]- Detailed method

$$V_{rACI-D} = 0.75 \left[(0.16 \sqrt{f'_c} + 17 \rho_w \frac{V_u d}{M_u}) b_w d + \frac{A_v f_{yt} d}{S} \right] \quad (14)$$

$$\frac{V_u d}{M_u} \leq 1 \quad (15)$$

where:

- V_{rACI-D} = estimated shear strength per reference 17; based on detailed ACI method, N
- V_u = factored shear force at section, N
- M_u = factored moment at section, N.mm

5 Existing research estimates of RC beam shear capacity

5.1 Zsutty approach [18]

$$V_{rZsutty} = 2.17 \left(\rho_w f'_c \frac{d}{a} \right)^{1/3} b_w d + \frac{0.75 A_v f_{yt} d}{S} \quad (16)$$

where:

$V_{rZsutty}$ = estimated shear strength per reference 18, N

5.2 Niwa et al approach [19]

$$V_{rNIWA} = 0.2 (\rho_w f'_c)^{1/3} (d)^{1/4} \left(0.75 + 1.4 \frac{d}{a} \right) b_w d + \frac{0.75 A_v f_{yt} d}{S} \quad (17)$$

where:

V_{rNIWA} = estimated shear strength per reference 19, N

5.3 Sarsam and Al-Musawi approach [20]

$$V_{rS\&A} = 1.8 \left(f'_c \rho_w \frac{V_u d}{M_u} \right)^{0.38} b_w d + \frac{A_v f_{yt} d}{S} \quad (18)$$

$$\frac{V_u d}{M_u} \leq 0.5 \quad (19)$$

where:

$V_{rS\&A}$ = estimated shear strength per reference 20, N

5.4 Kuo et al approach [21]

$$V_{rKUO} = 1.17 \left(\frac{a}{d} \right)^{-0.7} \sqrt{f'_c} b_w C + A_v f_{yt} \left(\frac{d}{S} - 1 \right) \quad (20)$$

$$\frac{C}{d} = \sqrt{2 \rho_w n + (\rho_w n)^2} - \rho_w n \quad (21)$$

$$n = \frac{E_S}{E_C} \quad (22)$$

$$E_C = 4700 \sqrt{f'_c} \quad (23)$$

where:

V_{rKUO} = estimated shear strength per reference 21, N

E_S = modulus of elasticity of reinforcement = 200000 MPa

E_C = modulus of elasticity of concrete, MPa

6 Results and discussion

From the methods used in codes & research proposals, a comparison was made for the ratio of (V_{exp}/V_r), where:

V_{exp} = Shear resistance of tested beam, N

V_r = calculated shear resistance based on different methods of prediction, N

Table 2 gives a comparison of the results of the different methods, based on the ratio of (V_{exp}/V_r).

Table 2. Comparison of the ratio of (V_{exp}/V_r) for all 93 beam tests

Detail	BS [13]	Australian code [14]	Canadian code [15]	Euro code [16]	ACI [17]-simplified	ACI [17]-detailed	Zsutty approach [18]	Niwa approach [19]	Sarsam and Al-Musawi [20]	Kou et al [21]	Proposed method
Equation used	(1)	(4)	(6)	(11)	(13)	(14)	(16)	(17)	(18)	(20)	(24)
Mean	1.808	2.079	2.634	2.433	2.619	2.418	1.903	2.894	1.808	1.832	1.718
Standard deviation	0.34739	0.36546	1.12085	0.94413	0.88921	0.69026	0.66996	1.21593	0.32701	0.32303	0.23112
COV %	19.215	17.580	42.547	38.797	33.957	28.545	35.203	42.021	18.088	17.636	13.464
Max. ratio	2.669	3.066	6.063	5.354	5.341	4.608	4.002	6.194	2.970	2.724	2.348
Min. ratio	1.131	1.184	1.246	1.164	1.148	1.166	0.889	1.241	1.213	1.033	1.130
Range (max/min)	2.360	2.588	4.864	4.190	4.652	3.953	3.113	4.953	2.449	2.637	2.078
*Number <1	0	0	0	0	0	0	1	0	0	0	0

*Number <1 indicates the number of specimens (out of 93) for which $V_{exp} < V_r$.

Regression analysis was performed on all 93 tests. This led to the following equation:

$$V_{r,PROP.} = 1.7 \left(f'_c \rho_w \frac{V_u d}{M_u} \right)^{0.5} \alpha + \frac{\phi_S A_v f_{yt} d}{S} \quad (24)$$

$$\frac{V_u d}{M_u} \leq 0.5 \quad (25)$$

$$\alpha = b_w d + 0.7 (b - b_w) h_f \quad (26)$$

where:

$V_{r,PROP.}$ = estimated shear strength by the proposed equation, N

α = parameter taking into account the effect of flange in T-section.

$\phi_s = 0.8$

As can be seen in Eq. (26), the contribution of flange to shear capacity will be eliminated in the case of beams with rectangular cross section ($h_f = 0$).

The last column in Table 2 indicates the various ratios of (V_{exp}/V_r) for the proposed method - Eq. (24). In this table, the COV values range between (17.58% - 42.55%). By using the proposed equation, the COV value decreases to 13.46%.

Figs. 1-5 indicate the trend of the influence of major parameters on the prediction of shear resistance of 4 methods: BS [13], ACI [17] - detailed method, reference 20, and the proposed method. Fig. 1 is plotted in order to examine the effect of wide variation of f'_c on the shear capacity of T-sections. This Fig. shows that the proposed method reveals little change, despite the change in f'_c between (11.99 – 56.95) MPa. This contrasts with other methods- e.g. the ACI [17] method leads to significantly uneconomic prediction of strength with rising f'_c . Fig. 2 also shows little change in prediction for the proposed method with a/d between (2.5-10.4). Again, the other methods lead to dropping values of the ratios of (V_{exp}/V_r), with the highest in the ACI [17] method.

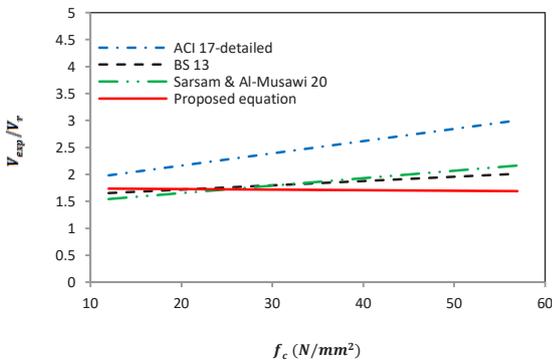


Fig. 1. Influence of f'_c on the ratio V_{exp}/V_r

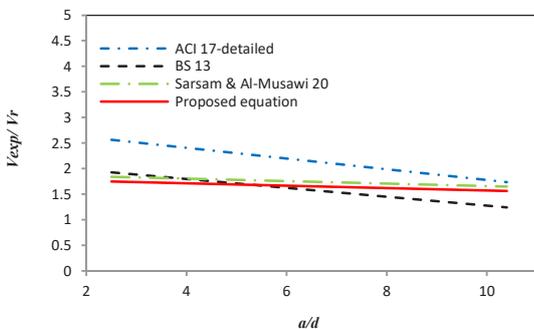


Fig. 2. Influence of a/d on the ratio V_{exp}/V_r

While Fig. 3 illustrates little change in prediction for the proposed method with ρ_w (0.49% - 5.2%), this is not the

case with other methods- especially the ACI [17] method. Similar results are shown in Fig. 4 with significant influence of $\rho_v f_{yt}$ ranging between 0 (without stirrups) – 2.71 MPa. The proposed method shows no trend of $\rho_v f_{yt}$ influence, contrasting with the other methods when the safety factor decreases with rising $\rho_v f_{yt}$. In Fig. 5, the influence of b/b_w ranging between (2-7) on the shear capacity prediction was examined. All the methods show a decrease in the factor of safety with rising b/b_w – especially the ACI [17] method.

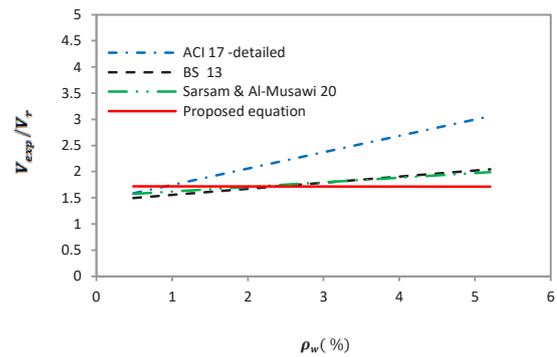


Fig. 3. Influence of ρ_w on the ratio V_{exp}/V_r

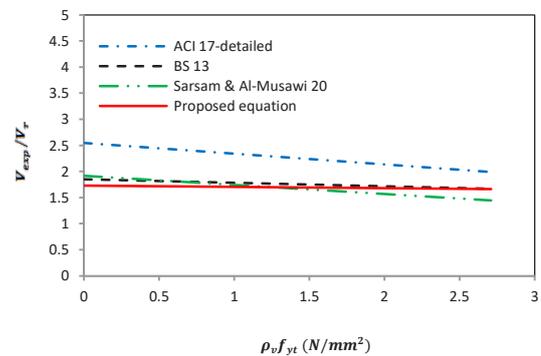


Fig. 4. Influence of $\rho_v f_{yt}$ on the ratio V_{exp}/V_r

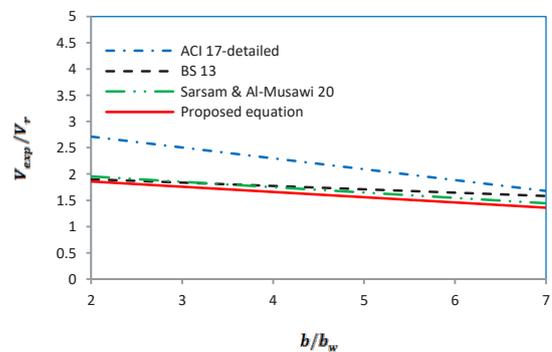


Fig. 5. Influence of b/b_w on the ratio V_{exp}/V_r

7 Conclusions

Based on 93 tests of RC T- beams failing in shear obtained from the literature, the following conclusions are made:

- 1- Regression analysis of all tests indicates that the ratio of (V_{exp} / V_r) drops significantly to 13.46% for the proposed method (which is taking into account the contribution of flange on shear capacity), compared with the COV values of the other methods (17.58% -42.55%).
- 2- The Australian Code [14] Method gives the lowest COV of all existing methods- 17.58%. It is clear from the proposed method that using α -Eq. (26) for flange effect has significantly improved the COV for shear capacity prediction.
- 3- The proposed method by Kuo et al [21] also leads to a significant drop in COV compared to several code methods. In contrast with this method, the proposed method leads to a much simpler prediction of shear strength.
- 4- Of the code methods, the Canadian Code [15] Method gives the highest COV of 42.55%. This is because strength is based only on $(f_c')^{1/2}$. However, with $(f_c')^{1/2}$ in the ACI [17], the COV is lower with the simplified method at 33.96%.
- 5- All the empirical equations used to calculate shear strength conservatively predict the shear capacity of the beams. The proposed method also leads to a conservative prediction for T-beams.
- 6- In contrast with the proposed method, Figs. 1 and 3 show significant safety factor changes, respectively with increasing f_c' and ρ_w by BS [13], ACI [17], and reference 20.
- 7- With the proposed design method- Eq. (24), Figs. 1-4 show little change in safety factor with increasing f_c' , a/d , ρ_w , and $\rho_v f_{yt}$.
- 8- In all design methods, there is a decrease in safety factor with increasing b/b_w – Fig. 5.

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