A fuzzy adaptive sliding mode controller for uncertain nonlinear multi motor systems

Tran Xuan Tinh[1], *Pham Tuan* Thanh[1], *Tran Van* Tuyen[1], *Nguyen Van* Tien[1] and *Dao Phuong* Nam[2,*]

[1]Le Quy Don University (Military Technical Academy), Vietnam
[2]School of Electrical Engineering, Hanoi University of Science and Technology, Vietnam

Abstract. Multi-motor drive systems are nonlinear, multi-input multi-output (MIMO) and strong-coupling complicated system, including the effect of friction and elastic, backlash. They have been widely used in many modern industries. The control law for this dive system much depend on the determining of the tension being hard to obtain this tension in practice based on a load cell or a pressure meter due to the accuracy of sensors or external disturbance. An emerging proposed technique in the control law is the use of adaptive sliding mode control scheme to stabilize closed system. However, the control system would be affected by chattering phenomenon. In order to eliminate this term, fuzzy technique is proposed by adjusting equivalent coefficients. The theory analysis and simulation results point out the good performance of the proposed fuzzy adaptive sliding mode control for the drive system.

1 Introduction

Multi-motor drive systems have been investigated by many researchers in the recent times. The neural network technique based control law have been proposed by Yaoji Me et al. (2013) (see [1-9]). However, it is hard to find the equivalent networks as well as corresponding learning rules. Besides, the model of this system is approximately described as a linear system to use the transfer function to design the control law. Furthermore, the tracking ability or the stabilization of the whole system are not still solved under the effects of neural network based observer. In the multi-motor drive control systems, it is necessary to obtain the belt tension to design the suitable state feedback control law. However, it is hard to measure this belt tension based on sensors, … and the high gain technique based observer is proposed in our work. Besides, the state feedback control design based on sliding mode control technique ensure that it is easy to remove efficient of disturbance and uncertainties. Therefore, an adaptive sliding mode controller is proposed to obtain tracking effectiveness. Moreover, fuzzy technique is considered to eliminate the chattering phenomenon disappearing by sliding mode control. The stability of closed system is obtained and verified by theory analysis, simulations.

2 Problem statements

Due to the effects by backlash and elastic (Fig. 1) and parameters (Table 1), we extend the model in [1] to obtain the following dynamic equation (2, 3) and the corresponding transfer function diagram (Fig. 2):

$$\begin{cases} \dot{\omega}_{r1} = \dfrac{1}{J_{L1}}\left[c_1.f_{11}(\Delta\varphi_1) + b_1\Delta\omega_1 f_{12}(\Delta\varphi_1) - (\mathrm{T}_{L1} + \mathrm{r}_1\mathrm{F})\right] \\ \dot{\omega}_{r2} = \dfrac{1}{J_{L2}}\left[c_2.f_{21}(\Delta\varphi_2) + b_2.\Delta\omega_2 f_{22}(\Delta\varphi_2) - (\mathrm{T}_{L2} + \mathrm{r}_2\mathrm{F})\right]. \\ \dot{F} = C_{12}\left[r_1\omega_{r1} - r_2\omega_{r2}(1 + \dfrac{1}{C_{12}.l}\mathrm{F})\right] \end{cases} \quad (1)$$

Table 1. Parameters of a Multi-Motor System.

$K = E/V$	Transfer function
E	Young's Modulus of belt
V	Expected line velocity
$T = \dfrac{L_0}{AV}$	Time constant of tension variation
L_0, A	Distance between racks, Section area (m^2)
n_{pi}	Number of pole-pairs in the i^{th} Motor
J_1, J_2, J_{L1}, J_{L2}	Inertia moment of Motors and Loads (kgm^2)
T, T_L, φ_r	Motor, Load torque (Nm), Flux of rotor (Wb)
L_r	Self-induction of rotor (H)
c_1, c_2, b_1, b_2	Stiffness and friction coefficient
$\Delta\omega_1, \Delta\omega_2$	The errors of angle speed in presence of backlash, elastic

We denote:

* Corresponding author: nam.daophuong@hust.edu.vn

$$u_{d1} = \frac{1}{J_{L1}}\left[c_1 f_{11}(\Delta\varphi_1) + b_1\Delta\omega_1 f_{12}(\Delta\varphi_1)\right];$$
$$u_{d2} = \frac{1}{J_{L2}}\left[c_2 f_{21}(\Delta\varphi_2) + b_2\Delta\omega_2 f_{22}(\Delta\varphi_2)\right] \quad (2)$$

are components including backlash and elastic to obtain the following equations:

$$\begin{cases} \dot{\omega}_{r1} = -c_1\omega_{r1} - \frac{1}{J_{L1}}\mathrm{T}_{L1} - \frac{1}{J_{L1}}\mathrm{r}_1\mathrm{F} + u_{d1} \\ \dot{\omega}_{r2} = -c_2\omega_{r2} - \frac{1}{J_{L2}}\mathrm{T}_{L2} - \frac{1}{J_{L2}}\mathrm{r}_2\mathrm{F} + u_{d2} \\ \dot{F} = C_{12}r_1\omega_{r1} - C_{12}r_2\omega_{r2} - \frac{1}{l}\omega_{r2}\,\mathrm{F} \end{cases} \quad (3)$$

The model is described by eq. (3) belongs to the class of nonlinear systems as follows:

$$\begin{cases} \dot{x}(\mathrm{t}) = A(\mathrm{x},\mathrm{t})x + B(\mathrm{x},\mathrm{t})u + D(\mathrm{x},\mathrm{t}) \\ \mathrm{y} = x \end{cases}, \quad (4)$$

where

$$A = \begin{bmatrix} k_1 & 0 & k_2 \\ 0 & k_4 & k_5 \\ 0 & 0 & k_7x_2 \end{bmatrix}; B = \begin{bmatrix} k_3 & 0 & 0 \\ 0 & k_6 & 0 \\ 0 & 0 & k_8x_1 + k_9x_2 \end{bmatrix};$$

$$D = \begin{bmatrix} u_{d1} \\ u_{d2} \\ 0 \end{bmatrix}$$

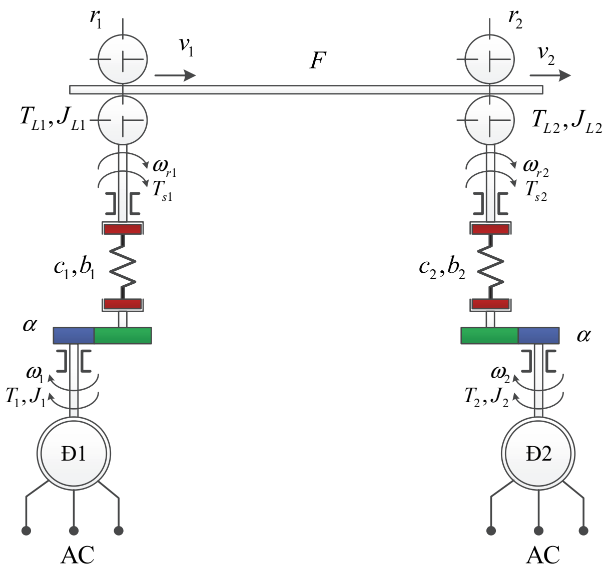

Fig. 1. The Two-Motor Drive System.

Remark 1:
The dynamic equations (1,2, 3) and Figures 1, 2 are described by the effect of friction, backlash, elastic and pointed out the nonlinear property of multi-motor systems.

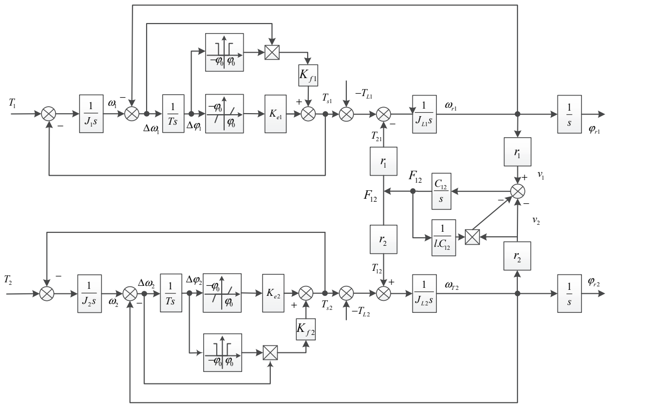

Fig. 2. The corresponding transfer-function diagram of the Two-Motor Drive System.

The control objective is to find the control input vector $u^T = [\mathrm{T}_{L1} \quad \mathrm{T}_{L2} \quad C_{12}]$ to obtain that the desired value are tracked by $x^T = [\omega_{r1} \quad \omega_{r2} \quad F]$ in presence of friction and elastic.

The following assumptions must be satisfied in order to design the control law:

Assumption 1: $x^T = [\omega_{r1} \quad \omega_{r2} \quad F]$ is measurable;

Assumption 2: There are real positive numbers $\omega_{r1\max}, \omega_{r2\max}, T_{L1\max}, T_{L2\max}, u_{d1\max}, u_{d2\max}$ such that $\omega_{r1}, \omega_{r2}, T_{L1}, T_{L2}, u_{d1}, u_{d2}$ are bounded by these values;

3 Fuzzy adaptive sliding mode control design

In this section, the main work is to find a state feedback control law based on the adaptive sliding mode control technique for the class of multi motor systems.

The proposed control law based on the following theorems as follows:

Theorem 1: The adjusting mechanism (5):

$$\begin{cases} \dot{\theta}_a^t = \eta_a\alpha\psi_a(\mathrm{x})\mathrm{s}, \\ \dot{\theta}_b^t = \eta_b\alpha\psi_b(\mathrm{x})\mathrm{s}u_{eq}, \end{cases} \quad (5)$$

with $s = \alpha e + \xi$,

$$u_{eq}(\mathrm{t}) = \hat{B}^{-1}(\mathrm{x})\left(-\hat{A}(\mathrm{x}) + \dot{x}_d - \beta\,\mathrm{sgn}(\mathrm{s})\right) =$$
$$u_{eq} = \hat{B}^T(\mathrm{x},\theta_b^t)\left[\varepsilon_0 I_m + \hat{B}\;(\mathrm{x},\theta_b^t)\hat{B}^T(\mathrm{x},\theta_b^t)\right]^{-1}; \quad (6)$$
$$\left(-\hat{A}(\mathrm{x},\theta_a^t) + \dot{\mathrm{x}}_d - \beta\,\mathrm{sgn}(\mathrm{s})\right)$$

$$A(x) \approx \hat{A}(x,\theta_a^t) = \theta_a\psi_a(\mathrm{x}) = \sum_{i=1}^{p_a}\theta_{ai}\psi_{ai}(\mathrm{x});$$
$$\mathrm{B}(x) \approx \hat{\mathrm{B}}(x,\theta_b^t) = \theta_b\psi_b(\mathrm{x}) = \sum_{i=1}^{p_b}\theta_{bi}\psi_{bi}(\mathrm{x}). \quad (7)$$

ensure some results as follows:
1. All signals of closed loop will be bounded and θ_a, θ_b converge to

$$\theta_a^* = \arg\min_{\theta_{ai}} \left\{ \sup_{x\in\Omega} \left| A(x,t) + D(x,t) - \hat{A}(x,\theta_{ai}) \right| \right\}$$

$$\theta_b^* = \arg\min_{\theta_{bi}} \left\{ \sup_{x\in\Omega} \left| B(x,t) - \hat{A}(x,\theta_{ai}) \right| \right\}$$

as $t \to \infty$

2. If $A^*(x,\theta_a^*) = \hat{A}(x,\theta_a^t); B^*(x,\theta_b^*) = \hat{B}(x,\theta_b^t)$ then the errors converge to zero in finite time;
3. If $A^*(x,\theta_a^*) \neq \hat{A}(x,\theta_a^t); B^*(x,\theta_b^*) \neq \hat{B}(x,\theta_b^t)$ then closed loop system converges to the neighborhood of sliding surface in finite time;

Proof:

The Lyapunov candidate function is selected as follows:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\frac{1}{\eta_a}\tilde{\theta}_a^2 + \frac{1}{2}\frac{1}{\eta_b}\tilde{\theta}_b^2 \quad (8)$$

where $\tilde{\theta}_a = \theta_a - \theta^*_a; \tilde{\theta}_b = \theta_b - \theta^*_b$, we have:

$$\dot{V} = s\dot{s} + \tilde{\theta}_a\dot{\tilde{\theta}}_a + \tilde{\theta}_b\dot{\tilde{\theta}}_b \quad (9)$$

and:

$$\dot{s} = \alpha\dot{e} = \alpha(\dot{x} - \dot{x}_d)$$

$$= \alpha\begin{pmatrix} A(x,t) + B(x,t)u(t) + D(x,t) \\ -\hat{A}(x,t) - \hat{B}(x,t)u_{eq}(t) - \beta\,\mathrm{sgn(s)} \end{pmatrix}$$

$$= \alpha\begin{pmatrix} \left(A(x,t) + D(x,t) - \hat{A}(x,t)\right) \\ +\left(B(x,t) - \hat{B}(x,t)\right)u_{eq}(t) - B(x,t)u_c(t) - \beta\,\mathrm{sgn(s)} \end{pmatrix}$$

However, we have the relation:

$$\varepsilon_a(x,t) + A^*(x,\theta_a^*) - \hat{A}(x,\theta_a) = A(x,t) + D(t) - \hat{A}(x,\theta_a)$$

$$\varepsilon_b(x,t) + B^*(x,\theta_b^*) - \hat{B}(x,\theta_b) = B(x,t) - \hat{B}(x,\theta_b)$$

Therefore, we obtain:

$$\dot{s} = \alpha\begin{bmatrix} \left(\varepsilon_a(x,t) + A^*(x,\theta_a^*) - \hat{A}(x,\theta_a)\right) \\ +\left(\varepsilon_b(x,t) + B^*(x,\theta_b^*) - \hat{B}(x,\theta_b)\right)u_{eq}(t) \\ -B(x,t)u_c(t) - \beta\,\mathrm{sgn(s)} \end{bmatrix} \quad (10)$$

On the other side,

$$A^*(x,\theta_a^*) - \hat{A}(x,\theta_a) = \psi_a(\theta_a^* - \theta_a) = -\psi_a\tilde{\theta}_a$$
$$B^*(x,\theta_b^*) - \hat{B}(x,\theta_b) = \psi_b(\theta_b^* - \theta_b) = -\psi_b\tilde{\theta}_b$$

and we obtain:

$$\dot{s} = \alpha\begin{bmatrix} \left(\varepsilon_a(x,t) - \psi_a\tilde{\theta}_a\right) + \left(\varepsilon_b(x,t) - \psi_b\tilde{\theta}_b\right)u_{eq}(t) \\ -B(x,t)u_c(t) - \beta\,\mathrm{sgn(s)} \end{bmatrix}$$

and:

$$\dot{V} = s\alpha\begin{bmatrix} \left(\varepsilon_a(x,t) - \psi_a\tilde{\theta}_a\right) + \left(\varepsilon_b(x,t) - \psi_b\tilde{\theta}_b\right)u_{eq}(t) \\ -B(x,t)u_c(t) - \beta\,\mathrm{sgn(s)} \end{bmatrix} +$$

$$+\tilde{\theta}_a\eta_a\alpha\psi_a(\mathrm{x})\mathrm{s} + \tilde{\theta}_b\eta_b\alpha\psi_b(\mathrm{x})\mathrm{su}_{eq}$$

$$\dot{V} = s\alpha\left(\varepsilon_a(x,t) + \varepsilon_b(x,t)u_{eq}(t) - B(x,t)u_c(t) - \beta\,\mathrm{sgn(s)}\right) =$$

$$= s\alpha\left(\varepsilon_a(x,t) + \varepsilon_b(x,t)u_{eq}(t) - \bar{\varepsilon}_a - \bar{\varepsilon}_b\left|u_{eq}\right| - \beta\,\mathrm{sgn(s)}\right) \leq$$

$$\leq -\alpha s\beta\,\mathrm{sgn(s)}$$

where $\alpha > 0; \mathrm{s.sgn(s)} \geq 0$ and we obtain $\dot{V} \leq 0 \quad \forall x \in \Omega$.

Theorem 2: The control input $u(t) = u_{eq}(t) - u_c(t)$;

$u_{eq} = \hat{B}^T(\mathrm{x},\theta_b^t)\left[\varepsilon_0 I_m + \hat{B}(\mathrm{x},\theta_b^t)\hat{B}^T(\mathrm{x},\theta_b^t)\right]^{-1}\left(-\hat{A}(\mathrm{x},\theta_a^t) + \dot{\mathrm{x}}_d - \beta\,\mathrm{sgn(s)}\right)$; with $u_c(t) = B^{-1}s\left(\bar{\varepsilon}_b\left|u_{eq}\right| + \bar{\varepsilon}_a\right)$

$s = \alpha e + \xi$ ensure the nonlinear system (6) stability in finite time.

Proof:

We obtain the result:

$$s^T\dot{s} = s^T\alpha\begin{bmatrix} \left(\varepsilon_a(x,t) - \psi_a\tilde{\theta}_a\right) + \left(\varepsilon_b(x,t) - \psi_b\tilde{\theta}_b\right)u_{eq}(t) \\ -B(x,t)u_c(t) - \beta\,\mathrm{sgn(s)} \end{bmatrix}$$

$$\approx s^T\alpha\begin{pmatrix} \varepsilon_a(x,t) + \varepsilon_b(x,t)u_{eq}(t) \\ -\bar{\varepsilon}_a - \bar{\varepsilon}_b\left|u_{eq}\right| - \beta\,\mathrm{sgn(s)} \end{pmatrix}$$

$$\leq -s^T\alpha\beta\,\mathrm{sgn(s)}$$

$$\leq -\|s\|\alpha\beta\,\mathrm{sgn(s)}$$

Therefore, $s = \alpha e + \xi \to 0$ in finite time

Remark 1. it is necessary to ensure that the time of convergence of sliding surface is finite. The fact is described based on the following example:

We consider the system as follows:

$$\begin{cases} \dfrac{dx}{dt} = Ax + Bs \\ \dfrac{ds}{dt} = Cx + Ds \end{cases}$$

where: $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}, C \in \mathbb{R}^{r\times n}, D \in \mathbb{R}^{r\times r}$, s is the sliding surface. Selecting A is Hurwitz matrix and $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is not Hurwitz matrix. We obtain that although s converges to 0 in infinite time, x does not converge to 0.

Remark 2. The proposed control law ensure that the system trajectory converge to sliding surface in finite time.

Remark 3. We almost utilized theorem 1 to design adaptive sliding mode control technique for multi motor systems.

In order to eliminate the effect of chattering phenomenon, fuzzy technique (by Tagaki – Sugeno – Kang) would be proposed to adjust the coefficient α depending on the sliding surfaces s and $\dot{s}$, table 2, 3 and figure 3:

Table 2. Rule Matrix of control.

		s		
		N	Z	P
$\dot{s}$	N	B	M	B
	Z	B	S	B
	P	B	M	B

Table 3. Properties of controller.

AND method	MIN
OR method	MAX
Implication	MIN
Aggregation	MAX
Defuzification	Weighted average

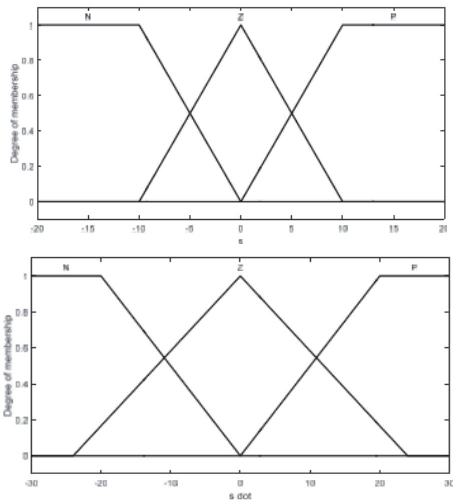

Fig. 3. Fuzzification.

4 Simulation results

In this section, we consider several simulation results to demonstrate the effectiveness of the proposed sliding mode control law based on the two-motor system having parameters as follows:

$$n_{p1} = n_{p2} = 4, J_1 = J_2 = 500Kgm^2, L_{r1} = 0.2H,$$
$$L_{r2} = 0.3H, \omega_{r1d} = \omega_{r2d} = 700v/p, F_d = 250N.$$

Figures 3, 4 show the tracking performance behaviour of velocity based on fuzzy adaptive sliding mode control law in presence of disturbance (figures 4, 5, 8). Figures 6, 7 show the high tracking performance behaviour of velocity based on adaptive sliding mode control law without disturbance.

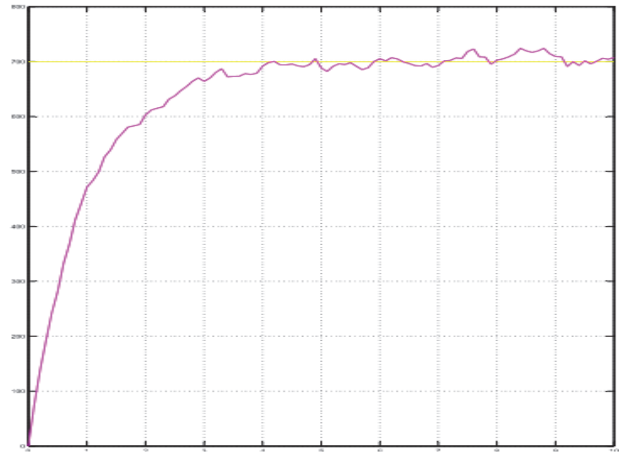

Fig. 4. The behaviour of the first motor's speed in presence of disturbance.

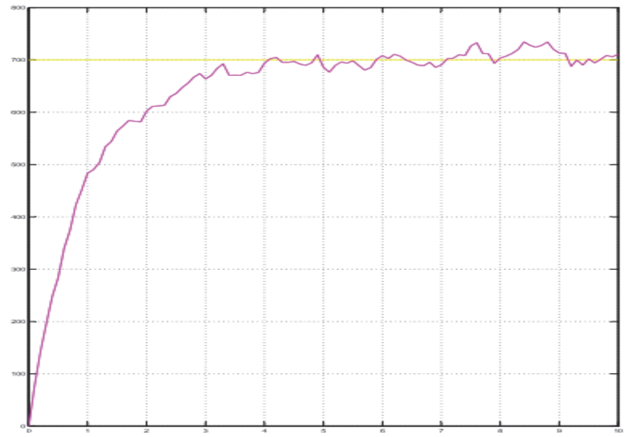

Fig. 5. The behaviour of the second motor's speed in presence of disturbance.

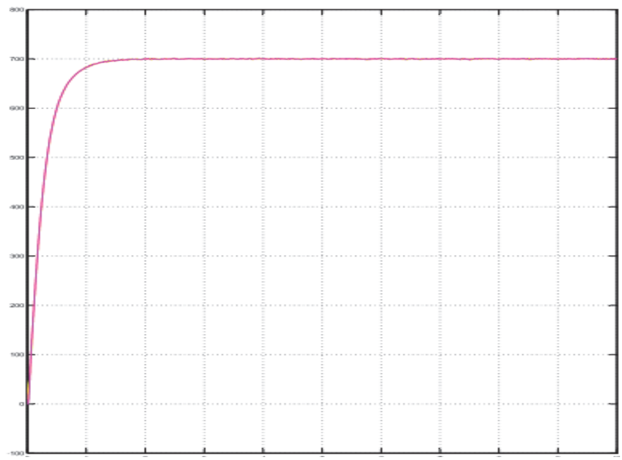

Fig. 6. The behaviour of the first motor's speed without disturbance.

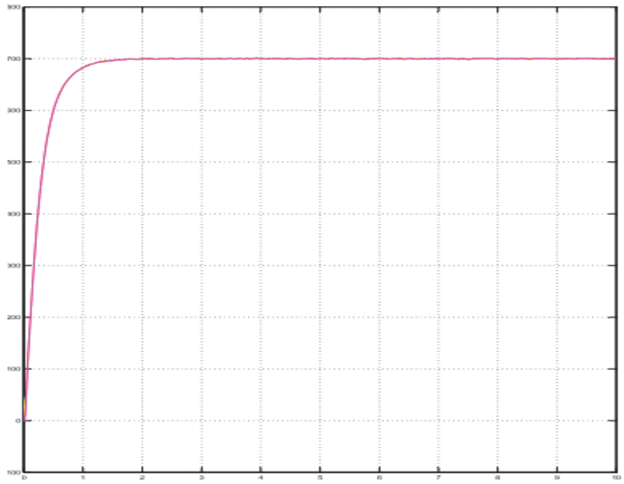

Fig. 7. The behaviour of the second motor's speed without disturbance.

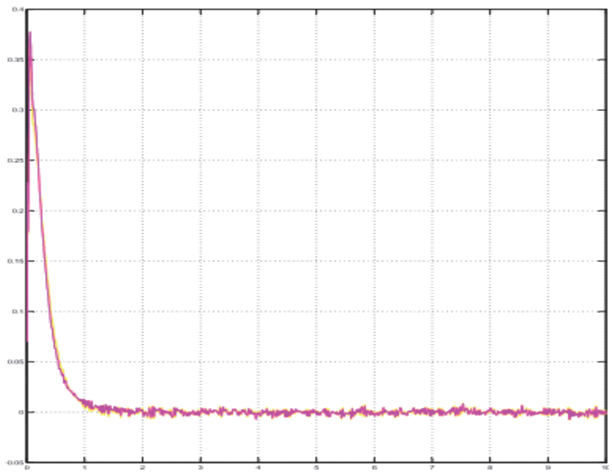

Fig. 8. The speed error between 2 motors.

5 Conclusion

This paper described a fuzzy adaptive sliding mode control law the two-motor system in presence of elastic and backlash, friction. The effectiveness of the proposed control scheme was pointed out by theoretical analysis and simulation results.

References

1. Y Mi et al., Proc. The 2013 International Conference on Electrical Machines and Systems, 2282–2285 (2013)
2. J Zhang et al, *Proc.* The IEEE International Conference on Intelligent Computing and Intelligent Systems, 178–182 (2009)
3. G Liu et al., Proc. The IEEE International Conference on Networking, Sensing and Control, 1476–1479 (2008)
4. L Jinmei et al, Proc. The IEEE International Conference on Industrial Technology, 1–6 (2008)
5. N.T.T. Vu, D. Yu, H.H. Choi, J.-W. Jung, IEEE Trans. Industrial Electronics, **60** (10), 4281–4291, (2013)
6. M. Zhihong, A.P. Paplinski, H.R. Wu, IEEE Trans. Autom. Control, **39**, 2464–2469 (1994)
7. J.J.E. Slotine, W. Li, *Applied Nonlinear Control* (Prentice Hall, New Jersey, 1991)
8. S. Labiod, M.S. Boucherit, T.M. Guerra, Fuzzy Sets Syst., **151**, 59–77 (2005)
9. V. Nekoukar, A. Erfanian, Fuzzy Sets Syst., **179**, 34–49 (2011)