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Simulation on Spacecraft Formation Flight and Formation Reconfiguration

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Abstract. Spacecraft formation flight refers to two or more spacecraft according to a certain formation or arrangement of flight, it has important application value for deep spatial exploration, spatial science experiment, ground investigation and military and so. This paper studies the spacecraft formation flight design and formation reconfiguration based on STK. Firstly, the Clohessy-Wiltshire (CW) equation is used to describe the relative motion of the near-circular orbit and deduce the relative orbital dynamics model. Then, based on the dynamic method of the CW equation, the spatial circular formation is designed and the STK is applied to simulate it. Finally, based on the above formation, a simple multi-impulse formation reconfiguration is performed and the simulation test is verified by STK. The simulation results show that the absolute error of orbital elements of the spacecraft is calculated by the relative orbital dynamics model is less than 10^{-5}, and the expected formation can be completed under the condition of two-body environment, and the feasibility of simple multi-pulse formation reconfiguration is proved successfully.

1 Introduction

Spacecraft formation flight technology refers to the multi-spacecraft constructed cluster, the system is closely coordinated with each other, so as to form a unified organic whole, and the performance goes beyond the traditional single spacecraft system [1]. Relative motion dynamics, control and formation design optimization are essential for formation flight technology, so considering the design and reconfiguration attracted the attention of many scholars [2].

Pan[2]emphatically expounds the development of spacecraft formation flight and its key technology and introduced that spacecraft formation flight play an important role for GPS and relative navigation technology, it laid a solid foundation for the research work of many scholars. Abbott[3] present a method for finding a minimum-power dipole solution for a given set of desired forces, which does not rely on arbitrary parameterizations as have prior approaches, and the structure enables further analysis of numerical conditioning and convergence. Wu [4]designed a linear quadratic control algorithm based on HILL equation dynamics model to verify the feasibility of that control algorithm and its simulation realization method, and laid a certain foundation for formation reconfiguration. Seo[5] investigates Collision avoidance strategies for multiple UAVs (Unmanned Aerial Vehicles) based on geometry and Numerical simulations are performed to demonstrate the performance of the proposed strategies. Jiang[6]studied two-point boundary value problems of relative motion, the algebraic method was used to study the geometric properties of the relative orbit and provided a reliable mathematical basis for the formation design. Meng [1] further studied the theory and method involved in the two-body orbit formation of relative motion dynamics and formation design, for the relative navigation, control and formation optimization design provides a reliable theoretical basis.

Aiming at the multi-purpose and multi-tasking of spacecraft formation, design the formation becomes the key and foundation of the task. Therefore, the spatial circular formation is designed based on the C-W equation dynamic method and by applying simple multi-impulse to achieve the formation reconfiguration, and the simulation test is verified and analyzed based on STK [6].

2 Design Spatial Circular Formation Based on Dynamic Equation

2.1 Spatial circular formation

The spatial circular formation refers to the around spacecraft with reference spacecraft as the center and the relative motion trace is circle orbit. In addition, the phase difference between the three spacecraft is 120° and uniform distribution on the spatial circle.

The orbital elements of the reference spacecraft as follows:

Table 1. The orbital elements of the reference spacecraft.

	a /km	e	i /°	Ω /°	ω /°	M /°
Reference Spacecraft	7000	0	60	10	20	30

2.2 Formation design based on C-W equation

Model design flow chart as follows:

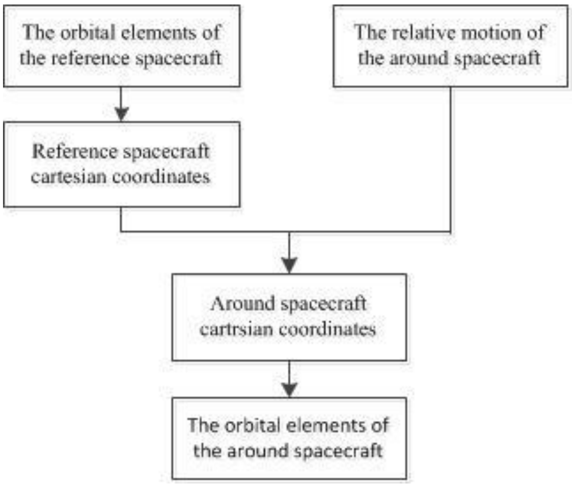

Figure 1. Model design flow.

Assuming that the earth is a homogeneous sphere and regardless of any perturbation factor, $[x, y, z]^T$ is the relative position vector of the spacecraft relative to the reference spacecraft, by solving the CW equation[1]:

$$\begin{cases} \ddot{x} = 2n\dot{y} + 3n^2 x \\ \ddot{y} = -2n\dot{x} \\ \ddot{z} = -n^2 z \end{cases} \tag{1}$$

The analytical form of equation solution can be obtained:

$$\begin{cases} x(t) = \frac{\dot{x}_0}{n}\sin(nt) + \left(-3x_0 - \frac{2\dot{y}_0}{n}\right)\cos(nt) + 2\left(2x_0 + \frac{\dot{y}_0}{n}\right) \\ y(t) = 2\left(3x_0 + \frac{2\dot{y}_0}{n}\right)\sin(nt) + \frac{2\dot{x}_0}{n}\cos(nt) - 3\left(2nx_0 + \dot{y}_0\right)t \\ \qquad + (-\frac{2\dot{x}_0}{n} + y_0) \\ z(t) = \frac{\dot{z}_0}{n}\sin(nt) + z_0\cos(nt) \end{cases} \tag{2}$$

$x_0, \mathrm{y}_0, \mathrm{z}_0$ is the initial state of the relative motion of a given reference frame.

If the formula $\dot{y}_0 = -2nx_0$ can be satisfied, the drift will be eliminated so that relative motion formation is stable [7-8]. $n = \sqrt{\mu/a_{ref}}$ is the average orbital angular velocity of reference spacecraft and μ is geocentric gravitational constant. Further processing the stable relative motion solution, get the simpler form:

$$\begin{cases} x = -A\cos(nt + \theta) \\ y = 2A\sin(nt + \theta) + L \\ z = -B\cos(nt + \varphi) \end{cases} \tag{3}$$

among them:

$$A = -\sqrt{x_0^2 + (\dot{x}_0 / n)^2}, \begin{cases} \cos\theta = -x_0 / A \\ \sin\theta = \dot{x}_0 / (nA) \end{cases}$$
$$B = -\sqrt{z_0^2 + (\dot{z}_0 / n)^2} \tag{4}$$
$$\begin{cases} \cos\varphi = -z_0 / A \\ \sin\varphi = \dot{z}_0 / (nA) \end{cases}; L = -\frac{2\dot{x}_0}{n} + y_0$$

A, θ, B, φ, L are the amplitude and phase of the elliptical orbital plane, the amplitude and phase along the normal direction of the orbital surface, and the position of the ellipse center in the reference frame. On this basis, considering that the center is located at the origin of reference frame, it is necessary to meet the constraints $L = -\frac{2\dot{x}_0}{n} + y_0 = 0$.

From the analysis, it can be:

$$\begin{cases} \cos\theta = -x_0 / A \\ \sin\theta = \dot{x}_0 / (nA) \\ y_0 = \frac{2\dot{x}_0}{n} \\ \dot{y}_0 = -2nx_0 \\ \cos\varphi = -z_0 / B \\ \sin\varphi = \dot{z}_0 / (nB) \end{cases} \tag{5}$$

The relative motion state of around spacecraft relative to reference spacecraft $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$ is obtained. The formation design ultimately get the initial motion state of each spacecraft in the formation, the Cartesian coordinate state of the geocentric inertia system, or the orbital elements. Therefore, the formation design of the spacecraft formation based on the C-W equation is as follows:

(1). According to the orbital elements of the reference spacecraft calculate the position vector R_c and velocity vector V_c of the reference spacecraft in the geocentric inertial system.

(2).According to the expected radius, the around spacecraft in the orbital coordinate system relative to the reference spacecraft position r , speed v is calculated.

(3).Can be converted by the conversion matrix $Q = Q_z[-\Omega]Q_x[-i]Q_z[-u]$ ($Q_j(\theta)$ is the elementary transformation matrix that turns θ around the j axis),the around spacecraft in the orbit coordinate system relative to the reference spacecraft position r , the speed v through the coordinate conversion to the inertial coordinate system;

(4).The position R_h and velocity V_h of the around spacecraft in the geocentric inertial system can be obtained by adding the sum of the position r, vector v in step (3) and the position R_c, vector V_c of the reference spacecraft.

(5).According to the position R_h and velocity V_h of the around spacecraft, the orbital elements of the spacecraft can be calculated.

2.3 Simulation results and analysis

Based on the above mathematical model, can calculate the orbital elements of the three around spacecraft, and then according to the following six graphs can be analyzed: The semi-major axis a of ellipse of the around spacecraft is less than 50 meters. Eccentricity e difference is less than 10^{-10}; Inclination i difference is less than 0.1°. The right ascension of ascending node Ω is less than 0.1°. True anomaly ω are 0°, 120° and 240° respectively. The Argument of latitude M are 50°, 290° and 170° respectively.

The orbital elements of the around spacecraft are shown in Table 2:

Combined with orbital elements of the above four spacecraft, STK is used to make the simulation and the following simulation results are obtained.

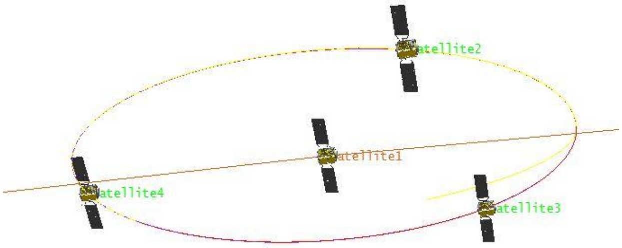

Figure 2. The spatial circular formation obtained by STK.

When the results of the mathematical model are calculated and visual simulation is made by STK, four spacecraft formation is the spatial circular formation, which that three around spacecraft was 120° phase difference. It can be seen that the C-W equation is used to design formation, and the correctness of the mathematical model and simulation experiment is proved.

3 Formation reconfiguration based on simple multi-impulse

3.1 Carpe formation

Carpe formation refers to the around spacecraft take reference spacecraft as center, one of the around spacecraft and reference stars are coplanar, and the around one do an elliptical motion. The relative motion of the other two around spacecraft is perpendicular to the orbit plane of reference spacecraft, and make a linear oscillatory motion, phase difference between the three spacecraft is 90°.

3.2 The condition of formation reconstruction

The initial phases of the three spacecraft in the spatial circle are: 0° , 120° , -120° . It is known that the motion equations of the three around spacecraft relative to the reference spacecraft are:

$$\begin{cases} x = -500\cos(nt+\theta) \\ y = 1000\sin(nt+\theta) \\ z = -500\sqrt{3}\cos(nt+\theta) \end{cases} \tag{6}$$

According to the orbital elements of the reference spacecraft in the spatial circle, the orbital elements of Sat_1', Sat_2' and Sat_3'in the CarPe formation can be calculated. The calculation results are shown in the table 3.

In order to reconstruct the CarPe formation, the three around spacecraft are maneuvered by adding a simple multi-impulse so that it can be realized.

Table 2. The orbital elements of the reference spacecraft and three around spacecraft.

	a /km	e	i /°	Ω /°	ω /°	M /°
ref	7000	0	60	10	20	30
Sat_1	7000.000072	7.14286×10^{-5}	59.9946	10.0053	49.9974	0
Sat_2	7000.000232	7.14286×10^{-5}	60.0067	10.0028	289.999	120
Sat_3	7000.000232	7.14286×10^{-5}	59.9988	9.992	170.004	240

Table 3. The orbital elements of the reference spacecraft and three around spacecraft.

	a/km	e	i/°	Ω/°	ω/°	M/°
ref	7000	0	60	10	20	30

Sat_1’	7000	7.14286×10^{-5}	60	10	20	7.105×10^{-15}
Sat_2’	7000	0	60.0026	10.0036	20.0006	30
Sat_3’	7000	0	59.9974	9.99638	19.9994	30

3.3 Design elliptical formation on orbit plane

An impulse is applied to the Sat_1, which is maneuvered to perform elliptical flight relative to the reference spacecraft on the spacecraft's orbital plane. Since this maneuver is a coplanar orbit maneuver, it is necessary to apply impulse to the normal direction to eliminate the relative motion of the vertical direction. ΔV_1 is impulse which applied to the spacecraft, the relative motion equation as follows:

$$\begin{cases} x=-500\cos f \\ y=1000\sin f \\ z=-500\sqrt{3}\cos f+\dfrac{\Delta V_1}{n}\sin(f-f_{o1}) \end{cases} \tag{7}$$

To eliminate the relative motion of the vertical direction of orbital plane (that is the Z direction), according to the above equation:

$$\begin{cases} \dfrac{\Delta V_1}{n}\sin f_{o1}+500\sqrt{3}=0 \\ \dfrac{\Delta V_1}{n}\cos f_{o1}=0 \end{cases} \tag{8}$$

The available f_{o1} solutions are $f_{o1}=k\pi+\frac{1}{2}\pi, k=0,1,2,\dots..n$, then $\Delta V_1=500\sqrt{3}n\approx0.93358m/s$.

From the above parameters:

$$\begin{cases} \Delta V_1=nB_c=n\left[B_f^2+B_0^2-2B_fB_0\cdot\cos(\eta_f-\eta_0)\right]^{1/2} \\ M_1=-\eta_0-\eta_c+\dfrac{\pi}{2} \end{cases} \tag{9}$$

$$\begin{cases} \mathrm{n}\,\eta_c=\dfrac{B_f\sin(\eta_f-\eta_0)}{B_c} \\ \cos\eta_c=\dfrac{B_f\cos(\eta_f-\eta_0)-B_0}{B_c} \end{cases} \tag{10}$$

$$n=\sqrt{\mu/a^3} \quad t_1=M_1/57.3n \tag{11}$$

The time of the impulse t_1 can be obtained.

3.4 Design oscillation on the vertical direction of spacecraft orbital plane

First, it is necessary to eliminate the shifting ΔL between the formation center of the spatial circle and the formation center of CarPe. By applying a radial control impulse M_s once:

$$\begin{cases} \Delta V_s=-\dfrac{1}{2}n\Delta L \\ M_s=-\eta_0+\dfrac{\pi}{2} \\ t_s=M_s/57.3n \end{cases} \tag{12}$$

Substituting the formation parameters, the amount of the impulse along the normal direction and the time of the application can be applied.

And then three times along the trace control impulse would be applied:

$$\begin{cases} \Delta V_{T1}=\dfrac{n}{8}A_c=\dfrac{n}{8}\left[A_f^2+A_0^2-2A_fA_0\cdot\cos(\theta_f-\theta_0)\right]^{1/2} \\ M_{01T}=-\theta_0-\theta_c \end{cases}$$

$$\sin\theta_c=\frac{A_f\sin(\theta_f-\theta_0)}{A_c} \qquad \cos\theta_c=\frac{A_f\cos(\theta_f-\theta_0)-A_0}{A_c} \tag{13}$$

$$\Delta V_{T2}=-2\Delta V_{T1}=-2\Delta V_{T3} \quad n=\sqrt{\mu/a^3} \quad t=M_T/57.3n$$

$$M_{02T}=M_{01T}+\pi$$

$$M_{03T}=M_{02T}+\pi$$

The initial formation parameters of Sat_2’and Sat_2 are obtained from the previous contents:

$$\begin{cases} A_0=-\dfrac{1}{2} \quad B_0=\dfrac{\sqrt{3}}{2} \quad \theta_0=120^\circ \quad \eta_0=120^\circ \\ A_f=0 \quad B_f=1 \quad \theta_f=0^\circ \quad \eta_f=0 \end{cases} \tag{14}$$

Substituting the formation parameters to obtain the amount of impulse along the trace and the time of the application.

Finally, a control impulse M_W is applied once along normal direction of the orbital plane:

$$\begin{cases} \Delta V_W=nB_c=n\left[B_f^2+B_0^2-2B_fB_0\cdot\cos(\eta_f-\eta_0)\right]^{1/2} \\ M_W=-\eta_0-\eta_c+\dfrac{\pi}{2} \end{cases}$$

$$\sin\eta_c=\frac{B_f\sin(\eta_f-\eta_0)}{B_c} \qquad \cos\eta_c=\frac{B_f\cos(\eta_f-\eta_0)-B_0}{B_c} \tag{15}$$

$$n=\sqrt{\mu/a^3} \quad t=M_W/57.3n$$

Substituting the formation parameters, the amount of the impulse along the normal direction and the time of the application are obtained. Calculating the amount of the applied impulse and the time to get the following two tables:

Table 4. Impulse amount during reconfiguration (m / s).

	Frist trace direction	Second trace direction	Third trace direction	Normal direction	Radial direction
Sat_1	0	0	0	0.9336	0
Sat_2	0.04716	-0.09433	0.047163	1.426070	-0.1617
Sat_3	0.0876	-0.1752	0.0876	1.4261	0.1617

Table 5. Apply the time interval during reconfiguration (s).

	Frist trace direction	Second trace direction	Third trace direction	Normal direction	Radial direction
Sat_1	0	0	0	4371.3872	0
Sat_2	1457.1291	2914.2581	2914.2581	1633.4966	5342.8065
Sat_3	1457.1291	2914.2581	2914.2581	1280.7615	3399.9678

3.5 Simulation results and analysis

According to the above results that the spatial circular formation can complete the formation reconfiguration successfully, and the simulation process of the reconfiguration as follows:

(1) The state before reconfiguration:

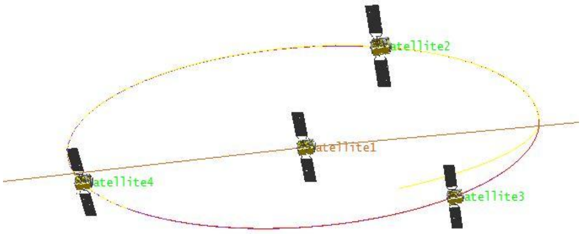

Figure 3. The state before reconfiguration.

(2) During the process of orbital maneuver, the relative motion between the spacecraft is:

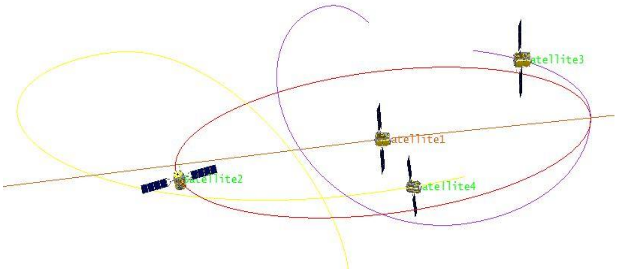

Figure 4. State of motion during reconfiguration(1).

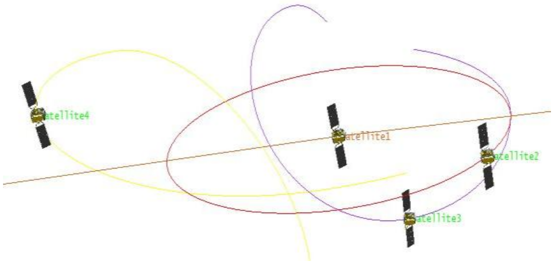

Figure 5. State of motion during reconfiguration(2).

(3) Maneuver is completed, Carpe formation as shown:

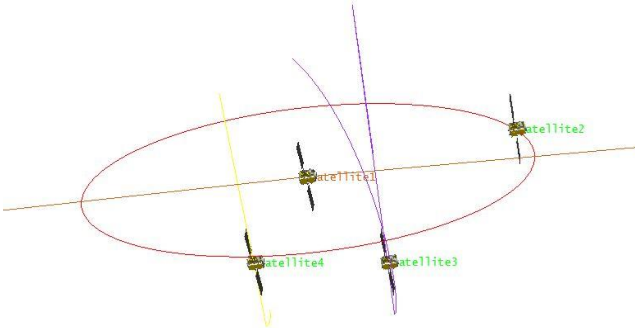

Figure 6. The state after reconfiguration.

According to the goal of the formation reconfiguration, taking advantage of the results which are calculated by the mathematical model to carrying out visualization simulation by STK, the formation of the four spacecraft is changed from the spatial circle formation by applying multiple impulses (Table 4, Table 5),gradually become the CarPe formation, the above analysis process can successfully verify the correctness of the mathematical model and the simple multi-impulse formation reconfiguration.

4 Conclusion

Based on the characteristics of spacecraft formation flight, this paper designs the spatial circular formation, and takes advantage of the simple multi-impulse to complete the formation reconfiguration task, which provides a more effective solution for the complicated and changeable tasks in spatial. The whole formation system cooperates with each other to realize the multi-purpose and multi-tasking of the spacecraft formation network [9-10], which can be applied to the formation design of the specific task.

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