Magnetic field computation and simulation of the coil systems using Comsol software

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Abstract. The article considers the calibration system computation with the inhomogeneity of magnetic field less than 0.1 %. Method of calculation is described. The numerical simulation using finite-element analyze was made for such system as: Helmholtz coils, improved Helmholtz coils, four and six coil system. Authors made the calculation of the magnetic field homogeneity toward axial direction. Based on procure results, authors analyzed magnetic field homogeneity and compared different types of coil systems.

1 Introduction

The measuring of magnetic flux density plays the key role in such brunches of since and technology like: task of orientation, navigation, stabilization, flaw detection, nondestructive testing, magnetic tomographic measurement, shielding action for quantum computer, etc.

Magnetometer gains currency for magnetic field measuring [1]. The stormy developing of robotic engineering complexes of surface-mounted, underwater and space system appropriation requires magnetometers evolution that will provide opportunity for weak magnetic field measuring. The theme of magnetic field measuring was reviewed by following scientists [2,3].

Permanent field source of magnetic flux density is used for magnetometers calibration and graduation. The inaccuracy of magnetic field source must be three times less than the inaccuracy of the calibrated sensor. For instance, for the measuring device with inaccuracy near of 1 %, it is necessary to provide homogeneity magnetic field with inaccuracy 0.1-0.3 %.

Some of the most ordinary ways for homogeneity magnetic field creation is using of Helmholtz coil. There are a lot of works devoted to the calculation of Helmholtz coils magnetic field [4-8]. One of the others way for uniform magnetic field generation is to use spheroidal coils[9]. The system with bigger number of coils is used to create magnetic field with higher [10], such as: Improved Helmholtz coils, four and six coils system. In this article we made the comparison of magnetic homogeneity in axial direction for four types of coils with similar radius.

2 Methods of magnetic field’s axial component computation for the coil system

According to Biot-Savart law [11], flux density on a distance “z” from the center of coordinate frame created by circular coil can be calculated by the following formula:

\[ B(z) = \frac{H_0 N I R^2}{2} \left( \frac{2}{R^2 + (z - h)^2} \right)^{\frac{1}{2}} \]

where \( H_0 \) is vacuum permeability equals to \( 4\pi \times 10^{-7} \) H/m; \( N \) is the number of windings in coil; \( I \) is current value in the conductor, A; \( R \) is coil radius, m; \( h \) is distance between coil center and central point of the coordinate system, m.

Helmholtz coils is formed by two circular coils with the similar radius and equal distance from the coordinate frame (Figure 1).

![Figure 1. Helmholtz coil](image-url)
The magnetic flux density in the axial direction for the Helmholtz coil is calculated by:

\[
B(z) = \frac{\mu_0 N R^2}{2} \left[ \left( \frac{z + h}{2} \right)^2 + R^2 \right]^\frac{3}{2} + \left( \frac{z - h}{2} \right)^2 + R^2 \right]^\frac{3}{2}, \quad (2)
\]

To calculate the ‘h’ it is necessary and appropriate to solve criterion (3). The second member of Taylor series in the condition of \( z = 0 \) must be equal to zero:

\[
B^{(2)}(0) = 0. \quad (3)
\]

Considering equations (2) and (3) we can have the following formula:

\[
\frac{\mu_0 N R^2}{2} \left[ \frac{30k^2}{\left( R^2 + h^2 \right)^{\frac{7}{2}}} - \frac{6}{\left( R^2 + h^2 \right)^{\frac{5}{2}}} \right] = 0. \quad (4)
\]

Solution of the equation (4) demonstrates that ‘h’ equals to ‘R/2’. As it can be seen from the above the final formula for the calculation of magnetic flux density in the axial direction for the Helmholtz coil could be written as:

\[
B(z) = \frac{\mu_0 N R^2}{2} \left[ \left( \frac{z + R}{2} \right)^2 + R^2 \right]^\frac{3}{2} + \left( \frac{z - R}{2} \right)^2 + R^2 \right]^\frac{3}{2}, \quad (5)
\]

Improved Helmholtz coils (Figure 2) consists of three coils with equal distance between each other and the same radius of the coil. In paper [7] authors made a research for three coil system. The Magnetic flux density for that coil system is defined by the 6th equal.

\[
B(z) = \frac{\mu_0 N R^2}{2} \left[ k \left( \frac{z^2 + R^2}{2} \right)^\frac{3}{2} + \left( \frac{z + h}{2} \right)^2 + R^2 \right]^\frac{3}{2} + \left( \frac{z - h}{2} \right)^2 + R^2 \right]^\frac{3}{2} + \left( \frac{z - h}{2} \right)^2 + R^2 \right]^\frac{3}{2} \right], \quad (6)
\]

where ‘k’ is the coefficient that defines the number of windings in the central coil.

Homogeneity magnetic field created by the improved Helmholtz coils must fulfill the following condition:

\[
B^{(2)}(0) = 0, \quad B^{(4)}(0) = 0. \quad (7)
\]

The set of equations (7) is also appropriate for the three coils system of Maxwell.

Computation of the magnetic flux density in the axial direction for the multicoil system (Figure 3) is performed by the following equation:

\[
B(z) = \frac{\mu_0 N R^2}{2} \left[ k_1 \left( \frac{z + h_1}{2} \right)^2 + R^2 \right]^\frac{3}{2} + k_1 \left( \frac{z - h_1}{2} \right)^2 + R^2 \right]^\frac{3}{2} + \left( \frac{z - h_1}{2} \right)^2 + R^2 \right]^\frac{3}{2} \right] + \ldots + k_n \left( \frac{z + h_n}{2} \right)^2 + R^2 \right]^\frac{3}{2} + k_n \left( \frac{z - h_n}{2} \right)^2 + R^2 \right]^\frac{3}{2} \right], \quad (8)
\]

Where \( k_i \) is the coefficient that defines the number of windings in \( i \)th coil; \( h_i \) is the distance between the coil center and the starting of coordinate system.

**Figure 2. Improved Helmholtz coils**

**Figure 3. Multicoil system**

In the case when coils number is even-numbered the first member of the equation set is equal to zero.

Homogeneity magnetic field created by the system with \( n/2 \) pairs of coils must fulfill the constraint condition(9):

\[
\begin{align*}
B^{(2)}(0) &= 0 \\
B^{(4)}(0) &= 0
\end{align*}
\]

\[
\begin{align*}
B^{(2)}(0) &= 0 \\
B^{(4)}(0) &= 0
\end{align*}
\]

\[
\begin{align*}
B^{(2)}(0) &= 0 \\
B^{(4)}(0) &= 0
\end{align*}
\]
Each of equations allows the defining of the magnetic flux density value on the coordinate system center possible. Due to the value of magnetic flux density in the coordinate frame center and equations that are defining the changing character of magnetic field in the axial direction, we receive expression for relative inhomogeneity of magnetic flux density in the direction ‘z’ for each analyzed types of coil system:

$$\delta_z = \frac{B(z) - B(0)}{B(0)} \cdot 100\%.$$ \hspace{1cm} (10)

It is important to take into account that the calculation is carried out for coils with the cross section aiming to zero. Real constructions have some volume, the finite element analysis was carried out in to take into account of the cubic content.

3 Finite element analysis

Authors used the program Comsol to conduct the finite element analysis. Every coil system was provided with its own task. The geometry of Helmholtz coils is presented on the figure 4. The size of coils section was chosen according to the number of windings in coil.

Figure 4. The geometry of Helmholtz coils in Comsol

The calculation was carried in module “Magnetic field”. For the windings parameters setting the authors used Multi-turn coil section. The following parameters were set: the number of windings, the current in a conductor and the movement direction. The last parameter was set through the reference edge. It should be noted that the current movement direction in both coils match.

The block with manually setting mesh was added to the model in order to receive precise measuring data avoiding fluctuation. It helped to receive data with interval lesser than 1mm. The set mesh for the improved Helmholtz coils is presented on the figure 5.

Figure 5. Finite element mesh for the improved Helmholtz coils

The stationary type of the task solution was chosen for each of the models as well as iterative solver.

The calculated coils parameters values are presented in the table 1.

Table 1. Coils parameters

<table>
<thead>
<tr>
<th>i</th>
<th>Parameter value</th>
<th>h,m</th>
<th>N</th>
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<tr>
<td></td>
<td>Helmholtz coils</td>
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<td>-</td>
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<td>Improved Helmholtz coils</td>
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</table>

4 Calculation results

The figure below presents results of the magnetic field calculation of the six coils system in the axial direction with the DC value of 1 mA.
The figure 7 shows the heterogeneity for four coil system in two-dimensional space. The maximum value of the magnetic flux density heterogeneity is observed near coils surface.

The figure 8 presents the heterogeneity of the magnetic field in the axial direction for the considered types of coil systems.

The changing of one of the parameters presented in the table 1 leads to the magnetic field inhomogeneity change.

The figure 9 shows that the variation of the magnetic field inhomogeneity parameter in the case of the fourth coils pairs depends on the value of ‘h’.

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Conclusion

The conducted comparison showed the needed heterogeneity of the magnetic field less than 0.1 % is provided by the four and six coils system. The increase of the coils number allows to enlarge the homogeneity of the magnetic field in the axial direction. Thus, four and six coils systems could be used to create a homogeneous magnetic field.

References

10. V. Ogay, Mater. Sci. Eng.,6,1-6(1014)