Prescribed Performance Control for Two-axis Optronic Stabilized Platform

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Abstract. Aiming at improving the tracking and stabilizing performance of two-axis optronic stabilized platform with Stribeck friction and uncertain velocity disturbance, a prescribed performance control strategy with unknown initial errors is designed. By designing a new performance function, the limit of traditional prescribed control that the initial error has to be known accurately is broken through. The strategy possesses strong robustness against unknown disturbance, and the state error is restrained to a predefined arbitrary small residual. It is guaranteed that the closed-loop system is uniformly ultimately bounded. The simulation results demonstrate the effectiveness of proposed strategy.

1 Introduction

Optronic stabilized platform(OSP) is a kind of high-precision servo tracking system, which is usually mounted on a mobile carrier for stable tracking of moving targets. Optical tracking system is widely used in robot self-balance system[1], unmanned aerial vehicle[2], airborne aiming system [3] and non-strapdown seeker servo system[4].The control object of the optical tracking system is the optical tracking platform. The main factors that affect the tracking performance are the carrier speed interference and the friction torque between the shaft systems[5][6].Two difficulties when dealing with the improvement of the system performance. One is the interference suppression in order to ensure tracking stability and the other is the system response and accuracy. How to effectively suppress interference in order to ensure the stability of tracking, and how to ensure that the system quickly and accurately track the target are two important aspects to improve system performance.

To realize the system performance with fast dynamic response and high-precision control of OSP, researches have tried varieties of approaches. For classical control structure, the proportional-integral-derivative control has been used to many systems[7], however, the robustness can’t be guaranteed. To enhance the robustness of the system, $H_\infty$ control is introduced to eliminate the sensitivity for disturbance, however, the robustness conflicts with its performance sharply[8]. Furthermore, sliding mode control is also used to deal with the non-linearity of OSP systems [9], it enables all system states with arbitrary values to converge to user-specified surface, while its chattering problem is an unavoidable drawback. Observer-based control is also used to eliminate disturbance and gets better performance [10]. Disturbance observer achieves high-precision estimation of interference, while the parameters have to be adjusted intricately. In reference [11], the adaptive RBFNN was designed to estimate and compensate for disturbances to improve the control performance of OSP. However, its control performance was easily affected by the selected upper bound of residual approximation error, and the problem of calculating delay have not been solved yet.

A common characteristic of the aforementioned researches is that the transient and steady performance cannot be prescribed, methods mentioned can only guarantee the convergence of error to a residual set. In this case, a prescribed performance control(PPC) is proposed in[12], based on PPC, it’s easy to design a prescribed performance bound to acquire desired transient and steady performance[13]. So far, there has not been application of PPC to OSP.

In this paper, PPC is utilized to improve the performance of OSP in the presence of unknown model perturbation and external disturbance. A new performance function is designed to break through the limitation of knowing the initial errors accurately. The designed scheme does not reside to the information of disturbance, which simplify the controller design and increase the practicality of the algorithm.

The paper is organised as follows:

In Section 2, the OSP model and preliminaries are presented. In Section 3, prescribed performance controller is designed along with the stability analysis. In Section 4, simulations are carried out to demonstrate the effectiveness.
2 Problem formulation

2.1 Constitution of two-axis optronic stabilized platform

Fig.1 shows the schematic diagram of two-axis stabilized platform. We can see that stabilized platform consists of two gimbals, which are pitch gimbal and yaw gimbal. From Fig.1 we can see the relationships between two gimbals: gyroscopes measuring the angular rate of pitch and yaw gimbal, angel sensors measuring the angle of pitch and yaw gimbal, moment sensors measuring the moment of pitch and yaw motor.

The system is driven by two servo motors, the sensor such as imaging sensor is placed in the inner frame.

![Figure 1. Schematic diagram of the two-axis optronic stabilized platform](image)

2.2 Dynamic model of the plant

![Figure 2. Stabilized-loop block diagram](image)

Fig.2 shows the pitch control block of stabilized platform. Where the block within the red imaginary line stands for the servo motor; \( C(s) \) is the controller of stabilized loop; \( k_{PWM} \) is the power amplifier coefficient; \( T_i(\dot{\theta}) \) is the moment of friction; \( \dot{\theta}_p \) is the angular rate of stabilized platform in the inertial space; \( \dot{\theta}_g \) is the perturbation of angular rate; \( T_f(\dot{\theta}) \) is the friction moment; \( k_g \) is the simplified transfer function of rate gyroscope, cascade coefficient \( 1/k_g \) makes it a unit feedback system; \( T_e \) is the moment output of servo motor.

Combined with the dynamic equation of stable platform and the dynamic equation of motor, mathematical model is acquired as follows [14]:

\[
\begin{align*}
\dot{x}_1 &= \frac{x_1}{J_L} \frac{F_f(x_1 + \dot{\theta}_p)}{J_L} \\
\dot{x}_2 &= -\left(\frac{C_s C_e}{L_a} x_1 - \frac{R_s}{L_a} x_2 + \frac{C_m k_{PWM}}{L_a} u - \frac{C_s C_e}{L_a} \dot{\theta}_g\right)
\end{align*}
\]

Where \( x_1, x_2 \) are state variables, they represent \( \dot{\theta}_i \) and \( T_e \) respectively. \( u \) is the input of controller.

The friction moment is modified as Striebeck friction model as Fig.3[15].

\[
F_f(v) = [F_c + (F_c - F_e) e^{-\frac{(v-v_s)^2}{\sigma^2}}] \text{sgn}(v) + B v
\]

Where \( F_c \), \( F_e \) are coulomb friction and static friction force respectively.

![Figure 3. Friction force with Striebeck effect](image)

2.3 Control problems for the stabilized platform

There are some troublesome characteristics in the stabilized platform:
1) It is hard to establish accurate mathematical model of the stabilized platform due to its coefficient uncertainty and perturbation;
2) The strong nonlinearity of friction model at velocity zero point may cause dead-area phenomenon of velocity tracking;
3) When there exists unknown external velocity disturbances and internal friction disturbances, it is hard to guarantee the transient and steady state performance.

3 Controller design

3.1 Prescribed performance

In this subsection, some preliminaries of prescribed performance are summarized. And a new performance function is designed.

**Definition 1.** [16] A smooth function \( h(t) : \mathbb{R}^+ \rightarrow \mathbb{R}^+ \) will be called a performance function if it satisfies:

1) \( h(t) \) is positive and decreasing;
2) \( \lim_{t \rightarrow +\infty} h(t) = h_\infty > 0 \).
According to Definition 1, a novel performance function is formulated as:
\[
h_i(t) = \text{csch}(\kappa_i t + \theta_i) + \hat{h}_{ix}
\] (3)

With \( \kappa_i \) and \( \theta_i \) chosen positive constants appropriately, where \( i = 1,2,\ldots,N \). Then \( h_i(t) \) satisfies:
1. \( h_i(t) = \text{csch}(\kappa_i t + \theta_i) + \hat{h}_{ix} = \frac{2}{e^{\kappa_i t} - 1} + \hat{h}_{ix} > \hat{h}_{ix} \)
2. \( \lim_{t \to +\infty} h_i(0) \to +\infty \)

Consider tracking error \( e_i(t) \), prescribed performance is achieved if the following inequality is satisfied:
\[
-\delta_i h_i(t) < e_i(t) < \delta_i h_i(t)
\] (4)

Where \( \delta_i, \delta_i \) are positive constants associated with \( e_i(t), \quad i = 1,2,\ldots,N \).

The performance bound is constrained by (4), \( e_i(t) \) is shown in Fig. 4. When \( \theta_i \) is chosen adequately small, and \( \delta_i, \delta_i \) are positive, we can easily get \(-\delta_i h_i(0) \to -\infty \) and \( \delta_i h_i(0) \to +\infty \). So, for a bounded \( e_i(0) \) and an adequately small \( \theta_i \), the following inequality is acquired:
\[
-\delta_i h_i(0) < e_i(0) < \delta_i h_i(0)
\] (5)

Appropriately, with the aforesaid proposed performance function (3), there is no need to get accurate initial error \( e_i(0) \).

**Remark1**

1. Provided that \(-\delta_i h_i(0) < e_i(0) < \delta_i h_i(0)\), \( \delta_i h_{ix} \), \( \delta_i h_{ix} \) determine the maximum allowable size of the error and the steady state. The parameter \( \kappa_i \) domains the decreasing rate of \( h_i(t) \), which will domain the decreasing rate bound of \( e_i(t) \) when the overshoot is no more than \( \delta_i h_i(0) \), \( \delta_i h_i(0) \). All parameters mentioned above can be arbitrary regulated.
2. The performance function applied in [17,18] is imposed by \( \rho(t) \), the tracking error \( e(t) \) is given as follows:
\[
\begin{align*}
-\delta_i h_i(t) < e(t) < 1, & \quad \text{if } e(0) > 0 \\
-1 < e(t) < \delta_i h_i(t), & \quad \text{if } e(0) < 0
\end{align*}
\] (6)

where \( 0 \leq \delta_i \leq 1 \), \( 0 \leq \delta_i \leq 1 \). Appropriately, the behavior bounds are different with different initial value of the error. We can also see that the controller design and stabilized analysis are supposed to differ because of different initial value. Additionally, acquisition of the \( e(0) \) adds the difficulty of algorithm applying.

Considering that it is unable to devise controller directly based on (4), an error transfer function \( \mu(e_i) \) is introduced to transfer the “constrained” system into equivalent “unconstrained” system.
\[
e_i(t) = \mu_i(e_i) h_i(t)
\] (7)

With
\[
\mu(e_i) = \frac{-\delta_i e_i - \delta_i e_i}{e_i + e_i}
\] (8)

Where \( e_i, i = 1,2,\ldots,N \) donates transfer error. \( \mu(e_i) \) satisfies:
1. \( \mu(e_i) \) is smooth and strictly increasing;
2. \( \lim_{t \to +\infty} \mu_i(e_i) = -\delta_i, \lim_{e_i \to +\infty} \mu_i(e_i) = \delta_i \).

Based on the properties of \( \mu(e_i) \), original error is transferred into
\[
-\delta_i < \mu_i(t) < \delta_i
\] (9)

If we multiply the above formulas, considering \( h_i(t) \) is positive, we can get
\[
-\delta_i h_i(t) < e_i(t) < \delta_i h_i(t)
\] (10)

Appropriately, (4) is equivalent illustrated by Eq. (7).

Furthermore, since \( \mu_i(e_i) \) is strictly increasing as well as \( h_i(t) \geq \hat{h}_{ix} > 0 \), we can get the inverse transformation (7).
\[
e_i(t) = \mu_i^{-1}(e_i) = \frac{1}{2} \ln \left( \frac{e_i / h_i + \delta_i}{\delta_i - e_i / h_i} \right)
\] (11)

And
\[
\dot{e}_i(t) = \tau_i (\dot{e}_i - \dot{h}_i / h_i)
\] (12)

With
\[
\tau_i = \frac{1}{2} \left( \frac{1}{e_i / h_i + \delta_i} - \frac{1}{e_i / h_i - \delta_i} \right) > 0
\] (13)

\[
\dot{h}_i(t) = \kappa_i - \kappa_i [\coth(\kappa_i t + \theta_i)]
\]

### 3.2 Design of prescribed performance controller
**Assumption 1.** The desired trajectory \( x_d(t) \) is limited and unknown, and \( \dot{x}_d(t) \) is limited. The following control scheme is defined

\[
\mu_2(t) = -\frac{\ddot{x}_1}{h_2(t)}
\]

\[
a_2 = -k_2 \epsilon_1
\]

\[
\mu(t) = \frac{\ddot{x}_2}{h_1(t)}
\]

\[
a_2 = -k_2 \epsilon_2
\]

\[
u = a_3
\]

with

\[
\begin{align*}
\ddot{x}_1 &= \ddot{\theta} - \dot{\theta} = x_1 - x_d \\
\ddot{x}_2 &= \ddot{T}_c = T_c - T_d = x_2 - a_2 \\
\epsilon_1 &= \ln\left(\frac{\mu(t) + \delta_1}{\delta_1 - \mu(t)}\right) \\
\epsilon_2 &= \ln\left(\frac{\mu(t) + \delta_2}{\delta_2 - \mu(t)}\right) \\
h_1(t) &= \text{csch}(k_1 t + \theta_1) + h_1(x) \\
h_2(t) &= \text{csch}(k_2 t + \theta_2) + h_2(x)
\end{align*}
\]

where \( k_1 > 0 \), \( k_2 > 0 \) are design constants of proportion controlling coefficient, \( u \) is the controller input.

Through inspection of Eqs. (14)-(15), it is easy to find that the control law doesn’t need any derivatives of \( x_i \) and \( a_i \), so the controller design and implementation are greatly simplified.

### 3.3 Stabilized analysis

**Assumption 2.** There is a bounded constant \( B_f \), satisfying

\[
\frac{\partial F_f(x)}{\partial x_i} \geq B_f
\]

**Assumption 3.** The velocity disturbance \( \Delta \) is limited.

**Theorem 1.** Consider the closed-loop system consisting of stabilized platform with controller Eq.(14), all the signals involved are uniformly ultimately bounded.

Choose the following Lyapunov function as

\[
V = \dot{V}_1 + \dot{V}_2
\]

\[
\dot{V}_1 = \epsilon_1 \dot{\epsilon}_1 + \epsilon_2 \dot{\epsilon}_2
\]

\[
\dot{V}_2 = \epsilon_1 \dot{\epsilon}_1 + \epsilon_2 \dot{\epsilon}_2
\]

where

\[
\dot{\mu}(t) = \frac{\ddot{x}_i \cdot h_i(t) - \ddot{x}_1 \hat{h}_1(t)}{h_1^2(t)}
\]

Combining with Eq.(1), we can get

\[
\dot{\mu}(t) = \frac{x_i / J_L - F_f(x_i + \Delta) - \ddot{x}_d - \hat{h}_1(t) \cdot \mu(t)}{h_1(t)}
\]

Furthermore, with (18) we have

\[
\dot{\dot{V}}_1 = \frac{2 \epsilon_1(t) \left( -k_1 \epsilon_1 + h_1(x) \cdot \mu(t) \right) / J_L}{h_1(t)(1 - \mu_1^2(t))}
\]

According to Assumption 3, and within the metabolic range of \( x_1 \), \( F_f(x_i + \Delta) \) is limited. \( \ddot{x}_d, \hat{h}_1(t), \mu_2(t), \mu_1(t) \) are limited too.

Define a limited constant \( F_c \), it is obvious that

\[
\left[ h_2(t) \cdot \mu_1(t) / J_L - F_f(x_i + \Delta) - \ddot{x}_d - \hat{h}_1(t) \cdot \mu_2(t) \right] \leq F_c
\]

Moreover,

\[
\dot{V}_1 \leq \frac{2 \epsilon_1(t) \left( \frac{\ddot{x}_1}{h_1(t)(1 - \mu_1^2(t))} - 2 \epsilon_2(t) \right)}{h_1(t)(1 - \mu_1^2(t))}
\]

Define

\[
\Omega_{e_1} = \left\{ e_1 \mid |e_1| \leq \frac{F_c}{k_1} \right\}
\]

It is obvious that \( \dot{V}_1 \) will be negative if \( e_1 \notin \Omega_{e_1} \).

Thus, \( e_1 \) is bounded.

Similarly, combined with Eq. (1) and Eq. (14),the following equation is acquired.

\[
\dot{\mu}_2(t) = \frac{c_a C_a}{L_a} x_i - \frac{R_a}{L_a} x_2 + \frac{C_{PWM}}{L_a} u
\]

Combining with Eq. (14) and Eq. (23),We can get

\[
\dot{V}_2 = \frac{2 \epsilon_2(t) \left( \frac{x_i / J_L - \ddot{x}_d - \hat{h}_1(t) \cdot \mu_2(t)}{h_1(t)} \right)}{h_1(t)(1 - \mu_1^2(t))}
\]
According to Eq. (21), $x_1$ is limited. Based on the Singular value theory, $x_2$ is limited. Considering that $\Delta$, $\hat{h}_1(t)$ are limited, based on Eq. (12), $\hat{e}_1$ is limited.

Define a limited constant $F_2$, it is obvious that

$$\frac{C}{L_e} x_1 - \frac{R}{L_o} x_2 - \frac{C}{L_a} \hat{e}_1 - \Delta \hat{h}_1(t) \cdot \mu_e(t) \leq F_1$$  \hspace{1cm} (25)

Moreover,

$$\dot{V}_1 \leq - \frac{2|\hat{e}_2(t)| \cdot F_2}{h_2(t)(1 - \mu_e^2(t))} \cdot 2C_m k_{PWM} \cdot k_e \hat{e}_2^2(t)$$

$$= \frac{2|\hat{e}_2(t)| (F_2 - |\hat{e}_2(t)| C_m k_{PWM} \cdot k_e / L_a)}{h_2(t)(1 - \mu_e^2(t))}$$  \hspace{1cm} (26)

Define $\Omega_{e_2} = \left\{ \hat{e}_2 \left| \hat{e}_2(t) \leq \frac{L_e F_2}{C_m k_{PWM} \cdot k_e} \right. \right\}$  \hspace{1cm} (27)

It is obvious that $\dot{V}_2$ will be negative if $\hat{e}_2 \notin \Omega_{e_2}$.

Thus, $e_2$ is bounded.

Thus, all the signals involved are uniformly ultimately bounded. This is the end of proof.

Remark 2. By Theorem 1, the bound of $e_1$ and $e_2$ are achieved. So we can conclude that the velocity tracking error and the moment tracking error can converge to the prescribed performance bound.

### 4 Simulation results

In this section, simulations are carried out to verify the effectiveness of the proposed control scheme for stabilized platform. The plant is stabilized platform of seeker.

The simulation step size is chosen as $T = 0.001s$.

The parameters of the control system are given in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$j_L$</th>
<th>$C_m$</th>
<th>$L_e$</th>
<th>$R_a$</th>
<th>$k_{PWM}$</th>
<th>$C_e$</th>
<th>$F_C$</th>
<th>$F_S$</th>
<th>$B_f$</th>
<th>$V_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>0.0012</td>
<td>0.625</td>
<td>0.062</td>
<td>5.1</td>
<td>3.75</td>
<td>0.75</td>
<td>0.2</td>
<td>0.3</td>
<td>0.017</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Choosing step signal as the reference signal, Fig. 5 shows the response of angular velocity tracking with the velocity disturbance chosen as $\sin(3t - 0.5)(\text{rad} / s)$.

From Fig. 5, we can see that the proposed method responds quickly, and it has smaller overshoot and smaller tracking error than PID method.

### Figure 5. Angular velocity tracking response

<table>
<thead>
<tr>
<th>(a) Output error of moment with parameter perturbation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b) Output error velocity with parameter perturbation</td>
</tr>
</tbody>
</table>

![Output error of moment with parameter perturbation](image)

![Output error velocity with parameter perturbation](image)

**Figure 6.** Response of system state variety with step signal

To verify the robustness of the proposed method, 50% perturbation is applied to coefficient $\frac{1}{J_e} \frac{C}{L_a} \frac{R}{L_o}$.

From Fig. 6, we can see that the error of states converges to zero with acceptable accuracy. Under the condition of intense parameter perturbation and angular velocity disturbance, the steady deviation of the response is no more than 3%.

### Figure 7. Response of system state variety of sine signal

![Response of system state variety of sine signal](image)
To verify the control effect of the plant’s angular velocity crossing the zero point frequently, sine signal is chosen as the reference signal, with the disturbance chosen as $\cos(3\pi t)$ (rad/s), similarly, 50% parameter perturbation is introduced.

We can see from Fig.7 that the stabilized platform is capable of achieving prescribed bounds on the transient and steady state output error performance, through observation of Fig.7, it is easy to find that the system achieves more than 95% isolation of internal and external disturbance.

Remark 3. The proposed controller doesn’t reside to the construction of friction model or special analytical friction expressions, it has promoting value in engineering practice with its easy design structure and relaxed application condition.

5 Conclusion
In this paper, a prescribed performance controller, which allows transient performance and steady value of moment and angular velocity tracking errors to achieve prescribed bound, is addressed for stabilized platform with Striebeck friction. The designed performance function takes satisfying effect without acquiring the initial error. It’s verified that the plant using the proposed method responds rapidly. And it achieves high-precision tracking with parameter perturbation and unknown disturbance. Besides, the phenomenon of speed dead-area is eliminated by the proposed method. Finally, because of its simple design structure and its loose applying condition, it has promoting value in engineering practice.

References