Development of Back-calculation Model for Aggregate Gradation Determination Using Fractal Theory

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Abstract. Several studies have shown that fractal theory can be used to analyze the morphology of aggregate materials in designing the gradation. However, the question arises whether a fractal dimension can actually represent a single aggregate gradation. This study, which is a part of a grand research to determine aggregate gradation based on known asphalt mixture specifications, is performed to clarify the aforementioned question. To do so, two steps of methodology were proposed in this study, that is, step 1 is to determine the fractal characteristics using 3 aggregate gradations (i.e. gradations near upper and lower bounds, and middle gradation); and step 2 is to back-calculate aggregate gradation based on fractal characteristics obtained using 2 scenarios, one- and multi-fractal dimension scenarios. The results of this study indicate that the multi-fractal dimension scenario provides a better prediction of aggregate gradation due to the ability of this scenario to better represent the shape of the original aggregate gradation. However, careful consideration must be observed when using more than two fractal dimensions in predicting aggregate gradation as it will increase the difficulty in developing the fractal characteristic equations.

1 Introduction

In developing a design mix formula of asphalt mixture, there is a quality control at each stage of the procedures, starting from preparing materials, testing the materials, determining aggregate gradation and so on. However, it is often encountered that although all parameters at each stage are controlled according to their requirements, the end result partially cannot meet the expectation. One cause of this non-conformance is the aggregate gradation used.

Generally, the selection of aggregate gradation for an asphalt mixture is done by trial and error, and the control of the control of the gradation is only conformity to the specification. This means that it could be more than one possible variation of aggregate gradation that can be selected to make the asphalt mixture, and different variations of the aggregate gradation will give different asphalt mixture properties.

To overcome this problem, this research is proposed to develop a back-calculation model using fractal theory, to obtain aggregate gradation that can produce asphalt mixture with optimal performance. Fractal theory is a manner to quantitatively describe the geometric complexity and the ability to fill the space of a particular object. Fractal geometry has become a keyword to better understand the concept of the dimension of the space of an object in a complex and irregular form. The study and application of the fractal theory of aggregate geometry have been conducted by researchers (1) [1, 2], however, the question that arises then is whether a fractal value can actually represent a unique aggregate gradation. This study is actually expected as an attempt to predict aggregate gradation. The back-calculation model developed in this study is an important part of a major research in developing a model for predicting aggregate gradation (which unknown parameter) based on the specification of the asphalt mixture properties (which is considered as known or given parameters).

2 Fractal Dimension

Aggregate gradation is a pattern with irregular and complex form or arrangement. The arrangement of aggregate gradations is often difficult to analyze conventionally, so the qualitative analysis is often a logical choice for analyzing aggregate gradation morphology [3]. At present, the fractal theory is one of the most effective quantitative approaches in explaining the complexity of aggregate gradation arrangements.

According to Mandelbrot [4], in general, fractal forms are self-similar, meaning that every small part of a fractal can be viewed as small-scale replication of the overall form. Mandelbrot [5] introduced a concept of the fractal to characterize the shape and roughness of an object and an irregular and rock-like surface. The level of roughness characteristics of the surface of the object is then called the fractal dimension.

Fractal dimension is used to describe the morphology of an irregular shape by characterizing the structure in one, two, and three-dimensional spaces using the fractal dimensions D1, D2, and D3 [6]. The fractal dimension is determined by using a geometric rank relationship between each geometric dimension (mass or volume, area and perimeter for three-, two- and one-dimensional spaces, respectively), with the long-term characteristic of an aggregate.

For an aggregate gradation, if the aggregate distribution is analyzed with fractal dimension, Equation 1 will be obtained.

$$P(r) = \frac{r^{3-D} - r_{min}^{3-D}}{r_{max}^{3-D} - r_{min}^{3-D}} \quad (1)$$

in which:

P(r)	=	percentage passing of sieve size
r_{max}, r_{min}	=	maximum and minimum sizes of aggregate

Different gradation curves can be obtained in accordance with the given value *D*, and vice versa. In aforementioned equation, the r_{min} can be ignored because of its small value, therefore, Equation 1 can be re-written as Equation 2.

$$P(r) = \left(\frac{r}{r_{max}}\right)^{3-D} \quad (2)$$

Currently, the main method used to calculate the fractal dimension of aggregate gradation is by calculating the slope according to the dual logarithmic axis ln *P(r)* and ln (r/r_{max}) using the linear regression equation. At this stage, the fractal dimension derived from the slope is used as the basic parameter, as seen in Equation 3.

$$\ln P(r) = (3 - D) \ln (r / r_{max}) + a \quad (3)$$

in which, a is constant.

3 Research Methodology

There are four main steps conducted in this study. They are:

- Determination of aggregate properties and gradation.
  In this study, the kind of the mixture used was asphaltic concrete – wearing course (AC-WC), which composed of dense-graded aggregate. To ensure that the mixture was made of high-quality aggregate material and composed of suitable gradation, aggregate properties measurement and sieve analysis were conducted and then the properties and gradation were evaluated to ensure that they could conform to the specifications of Directorate General of Highway [7].
  Once the suitable aggregate material was selected, three aggregate gradations (named as Gradation A, Gradation B, and Gradation C) were selected. These gradations should be located within the upper and lower bounds of aggregate specification [7]. The criteria of the three gradations selected were as follows:
  a. Gradation A: gradation with all percentage passing is 0.4% lower than that of the upper bound.
  b. Gradation B: middle gradation (i.e. gradation with all percentage passing is average between percentage of passing of upper and lower bounds).
  c. Gradation C: gradation with all percentage passing is 0.4% higher than that of the lower bound.
  The 0.4% increase or decrease of percentage passing of Gradation A and C is only intended to differentiate that both gradations is not upper and lower bounds.
- Determination of Fractal Dimension Based on Aggregate Gradation.
  In this step, three fractal dimensions were determined from three different aggregate gradations (A, B and C) using Equation 3. In this research, two scenarios were proposed in determining fractal dimension, that is, (i) one fractal dimension which represented all sieve sizes; (ii) multi-fractal dimensions in which each fractal dimension reflected one group of sieve size with similar trend/slope.
- Backcalculation of Aggregate Gradation Based on Fractal Dimension.
  Two scenarios in the previous step have their own benefit and drawback, in terms of accurateness and easiness to develop. Using these scenarios, aggregate gradation was back-calculated.
- Evaluation of back-calculated aggregate gradation towards specification.
  Once the aggregate gradation was obtained, an evaluation was conducted to examine whether the back-calculated gradation could conform to the specification [7].

4 Results and Analysis

Table 1 shows the results of aggregate properties measurement and their corresponding requirements as stated in Indonesia General Specification [7]. As shown in the table, all properties of selected aggregate material could fulfill all the requirements.

In Table 2 and Fig. 1, three aggregate gradations were selected. It can be seen that all gradations were within the upper and lower bounds of the specification. The percentage passing of all three gradations in Table 2 was used as a reference to measure the accurateness of back-calculated aggregate gradations developed in this study.

Table 1. Properties of Aggregate

Aggregate properties	Test Method	Requirements*)	Results
Properties of coarse aggregate			
Affinity to asphalt (%)	ASTM D1664-80	> 95	99
Abrasion (%)	ASTM C131-06	< 40	24
Abrasion (%)	ASTM C131-06	< 40	24
Water Absorption (%)	ASTM C127 – 07	< 3	1.341
Specific gravity	ASTM C127 – 07	> 2.5	2.73
Properties of fine aggregate			
Water Absorption (%)	ASTM C127 – 07	< 3	2.229
Specific gravity	ASTM C127 – 07	> 2.5	2.679
Sand Equivalent (%)	ASTM D2419-14	> 50	93.1

Remarks: *) requirement as stated in General Specification version by DHG [7]

Table 2. Several Gradation Used in This Study

Gradation Types	Passing rate at different sieve size (mm), in %									
	19	12.50	9.50	4.75	2.36	1.18	0.60	0.30	0.15	0.075
Upper bound	100	100	90	63	39.1	25.6	19.1	15.5	13	10
Gradation A	100	99.95	89.95	62.95	38.91	25.5	19	15.4	12.9	9.9
Gradation B	100	95	81	53	33.55	22.3	16.05	12.25	9.51	7.01
Gradation C	100	90.4	72.04	43.04	28.42	19.08	13.16	9.23	6.16	4.22
Lower bound	100	90	72	43	28	19	13	9	6	4

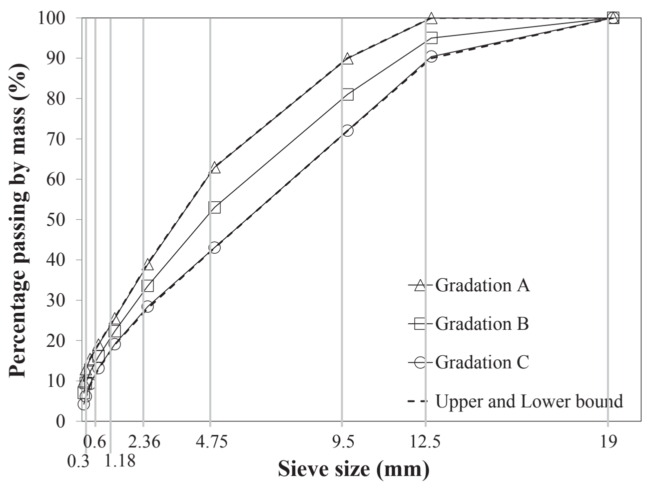


Fig 1. Gradation Curve of Different Gradation Used

Based on these three gradations, fractal dimensions were determined using two scenarios. Figs. 2 and 3 show the fractal characteristic that developed based on one fractal dimension and multi-fractal dimension scenarios, respectively. In this study, two fractal dimensions were used to represent a multi-fractal dimension scenario due to its easiness in developing linear equation from the fractal dimension in future works.

In two fractal dimension scenario, both lines have different fractal dimension D. The fractal dimensions for line 1 and line 2 are called as D1 and D2, respectively. The use of line 1 will separate data of two maximum sieve sizes with the highest deviation against estimated data (see Fig. 2) from the rest. Line 1 only consists of two data, therefore, this line always produces a coefficient of determination equals to unity (see Fig. 3).

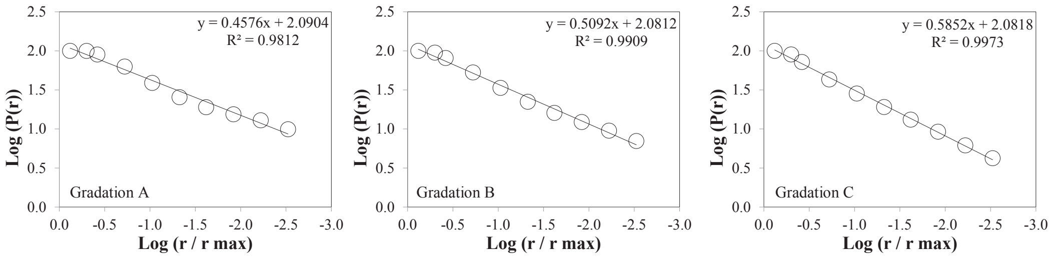


Fig 2. Fractal Characteristic Based on One Fractal Dimension

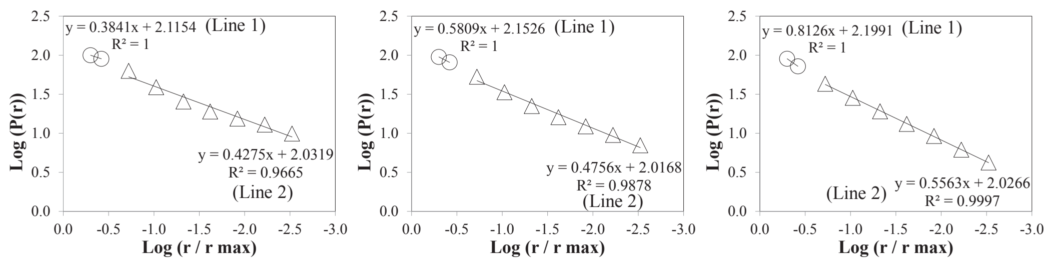
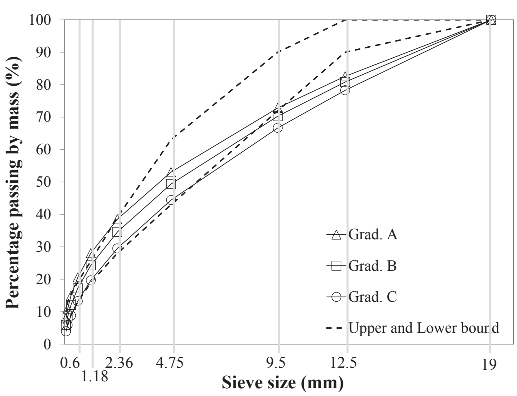


Fig 3. Fractal Characteristic Using Multi-Fractal Dimension

To examine the suitability of the scenario towards the accurateness of aggregate gradation, the linear equation in each scenario was used to predict the percentage passing of each sieve size in the gradation. The results of this back-calculation procedure are depicted in Figs. 4 and 5.



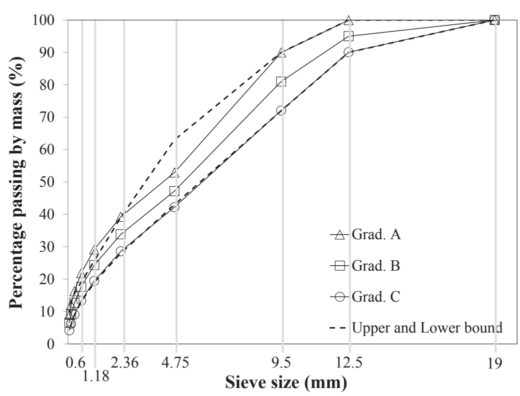


Fig 4. Gradation Curve Based on First Scenario (One Fractal Dimension)

Fig 5. Gradation Curve Based on Second Scenario (Multi-Fractal Dimension)

As shown in Fig. 4, the back-calculated aggregate gradations from one fractal dimension underestimate the original gradation, especially on several sieve size (9.5 mm to 19 mm). Different results are shown in Fig. 5 where the use of multi-fractal dimensions could correct the drawback of the use of one fractal dimension, especially on large sieve sizes. This could be contributed to the ability of multi-fractal dimension scenario to better represent the shape of the original aggregate gradation. However, careful consideration must be observed when using more than two fractal dimensions in predicting aggregate gradation as it will increase the difficulty in developing the fractal characteristic equations.

Once the most suitable scenario in determining aggregate gradation is selected, now it is necessary to determine the procedure in developing linear equations in the scenario. The following procedure might be proposed:

- Fractal dimension is obtained based on the specification of asphalt mixture, especially voids in the mix (VIM). It considers as a fractal dimension for line 2 and called as D2 (see Fig. 3).
- Do sieve analysis of aggregate material.
- Determine percentage passing of nominal maximum size (i.e. one size below the largest size), percentage passing of one- and two-size below nominal maximum sieve size.
- Derive the fractal characteristic equation of line 2 (as seen in Fig. 3) using D2 and percentage passing of two-size below nominal maximum sieve size.
- Derive the fractal characteristic equation of line 1 (as seen in Fig. 3) using percentage passing of nominal maximum and one-size below nominal maximum sieve sizes.
- Predict aggregate gradation using equations of lines 1 and 2.

5 Conclusions

This paper presented a method to predict aggregate gradation using fractal dimension. Two scenarios were proposed in this paper to predict aggregate, i.e. one and multi-fractal dimensions. The results showed that multi-fractal dimension scenario (in this case, two fractal dimension) could produce a better prediction of gradation than that of one fractal dimension scenario due to the ability of this scenario to represent the shape of original aggregate gradation.

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