

Mathematical Modeling of the Drying Kinetics of Thinly-Sliced Saba (*Musa Balbasiana*) Using Hot-Air Dryer

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Abstract. Banana is one of the top produced crops in the Philippines, and among its cultivars is the *Musa balbasiana*, commonly known as *saba*. Due to its high moisture content, *saba* is perishable and one of the methods to increase its shelf-life is drying. The shelf life of *saba* can be extended up to six months when dried to 12% moisture content. The research focuses on the effect of banana maturity and chooses a mathematical model which will best fit its drying kinetics. The banana samples, the unripe and ripe *saba*, were bone-dried without pre-treatment using a hot-air tray drier. To produce repeatable data, three trials were done for temperatures 40°C, 50°C and 60°C. Among the three mathematical models used in the study, the treated data for both unripe and ripe *saba* best fit the Laplace Transform Model. Using Lagrange interpolation, the time per trial was computed; unripe *saba* dried at 50°C achieved the ideal moisture content in an average time of 87.5574 minutes for the three trials while that of ripe *saba* dried at 40°C achieved the same moisture content in an average time of 88.8619 minutes for the three trials. With the increase in temperature, the discoloration decreased indicating an enzymatic character of browning.

1 Introduction

Banana, a staple fruit on Filipino tables, is hailed as the Philippines' fourth largest produced crop; the country being able to contribute to 10% of the world supply of banana from the year 1996 up to 2005 [1]. Among the varieties of bananas the Philippines grow, Saba (*Musa balbisiana*), which is usually eaten fresh as dessert when ripe, leads in market demand and production and can be processed as banana flour, ketchup and the popular banana chips [2].

Having a high moisture content, banana is prone to mold growth [3]. Because of its shelf life, it must be consumed a few days after harvest. Post-harvest losses such as spoilage of banana according to researches account up to 40% around the world [4], not only physical but also in the fruit quality [5] and preservation methods such as dehydration is employed. The most common technique for dehydration of food is air-drying because of its easy set-up [6] and a common way to increase their shelf life [7]. Commercially, banana is dried down to 14-15 % final moisture content on a dry basis (12% on a wet basis) which corresponds to an approximate 70% mass loss. At such a level of moisture content, dried banana has a shelf life of at least 6 months [8].

In drying a perishable fruit like banana, wherein a drying medium such as hot air is passed over the exterior of the material, moisture content is related to time.

Mathematical models were constructed to analyze the changes in density and shrinkage of banana during drying by experimentation [9]. Likewise considering shrinkage, Karim and Hawlader, provided a description of the heat and mass transfer properties of banana during drying through the use of a mathematical model [10]. Similar studies use analytical solutions of the diffusion equation by intermittent drying and continuous drying and some of them used the model describing well the drying of bananas and other agricultural products [2].

Mathematical models illustrate the drying kinetics being affected by the temperatures and moisture contents, which in turn affect other superior attributes of banana [2], therefore, analysis of the drying process is of absolute necessity in obtaining quality products. In reference with this, the researchers have come up with the study which involves hot air drying of saba to 12% moisture content which will aid in its preservation and prolonged shelf-life of up to six months.

2 Methodology

2.1. Preparation of saba

Fresh matured banana (ripe and unripe) grown in a farm located in Laguna, were bought from Trabajo market in Sampaloc. The fruit was peeled and cut into thin slices

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of 1-2 mm thickness. No pretreatment was done on the samples.

2.2. Drying Procedure

Immediately after slicing, the banana samples were laid out on the area of the perforated tray and weighed using an analytical balance (+ 0.01 g) as shown in Fig. 3.3, and then placed into the drying chamber which was preheated at 60°C. The tray containing the banana sample was weighed every 15 minute time interval throughout the drying period until three constant weights were recorded. Trials were done in triplicates, this procedure was repeated at 40 and 50°C.

2.2 Data Interpolation and Calculation

Three empirical models that best describes the thin-layer drying kinetics of agricultural products were used in the study: Henderson and Pabis' model shows the effect of moisture loss on the system as time increases, Page model, a modification of the Henderson and Pabis' model, best describes the thin-layer drying of whole bananas, and Laplace transform model provides an easier way to solve differential equations derived from the mass balance around the dryer for the prediction of the system's behavior.

Henderson and Pabis model

The drying model of Henderson and Pabis is shown in equation 1.1:

$$MR = ae^{-kt} \quad (1.1)$$

Equation 1.1 is then linearized as shown in equation 1.2:

$$\ln MR = \ln a - kt \quad (1.2)$$

Modified Page model

The Modified Page model equation is shown in equation 2.1:

$$MR = ae^{-kt^n} \quad (2.1)$$

This equation was linearized as described by equation 2.2:

$$\ln MR = \ln a - kt^n \quad (2.2)$$

Laplace transform model

In deriving the Laplace transform model, a general material balance was formulated stating the rate input minus output in the system equals the rate of accumulation as shown in equation 3.1:

$$M_i - M_o = \tau \frac{dM}{dt} \quad (3.1)$$

To derive the drying rate of banana, Laplace transformation was applied in equation 3.1. The derived equation was represented in equation 3.2:

$$M = M_f - M_f e^{-\frac{t}{\tau}} + M_i e^{-\frac{t}{\tau}} \quad (3.2)$$

2.3 Determination of the moisture ratio

By dividing the difference of the weight of the sample, W, at any time t and the equilibrium weight of saba,

Weq, by Weq, the moisture ratio, MR, is determined as shown in equation 4.1:

$$MR = \frac{W - W_{eq}}{W_i} \quad (4.1)$$

2.4 Determination of the drying time

Lagrange method of interpolation was used to calculate for the time needed for dried *saba* to attain 12% moisture as shown in equation 5.1:

$$y = \frac{(x-a)(x-b)(x-c)}{(d-a)(d-b)(d-c)} f(d) + \frac{(x-a)(x-b)(x-d)}{(c-a)(c-b)(c-d)} f(c) + \frac{(x-a)(x-c)(x-d)}{(b-a)(b-c)(b-d)} f(b) + \frac{(x-b)(x-c)(x-d)}{(a-b)(a-c)(a-d)} f(a) \quad (5.1)$$

3 Results and Discussion

3.1 Analysis of mathematical models

A total error of 0.6798, 0.5033 and 0.9306 for 40°C, 50°C and 60°C respectively shown in Table 1 were obtained by comparing the corrected and experimental moisture ratio values of the most appropriate fit from the three trials conducted for each temperature for unripe *saba*. As observed from Figure 1 the Henderson and Pabis model best represented the drying kinetics of unripe *saba* at 50°C. The overlapping curves for the corrected and experimental values for each temperature signified which is the most appropriate fit. The total error recorded from the other trials varies from 0.7984 to 0.8882, 0.5494 to 0.5731 and 0.9888 to 1.2056, correspondingly for each temperature.

Table 1. Henderson and Pabis model

Banana Maturity	Temperature	Trial	R ²	Total Error	Time (mins) to reach 12% (wet basis)
Ripe	40°C	1	0.97713	0.7984	78.5705
		2	0.97268	0.8882	73.0914
		3	0.98368	0.6798	83.9978
	50°C	1	0.92699	0.5731	99.8809
		2	0.92306	0.5494	102.5936
		3	0.91941	0.5033	107.0004
	60°C	1	0.98930	0.9306	105.5947
		2	0.97131	1.2056	100.5280
		3	0.98605	0.9888	103.7430
Unripe	40°C	1	0.80191	0.5786	118.6259
		2	0.88216	0.8989	106.3204
		3	0.83769	0.9940	135.1477
	50°C	1	0.98020	0.2637	108.2601
		2	0.97884	0.2518	107.1768
		3	0.96316	0.3554	106.6902
	60°C	1	0.91708	1.8606	119.1495
		2	0.99147	0.1461	128.7729
		3	0.88176	1.8972	116.2403

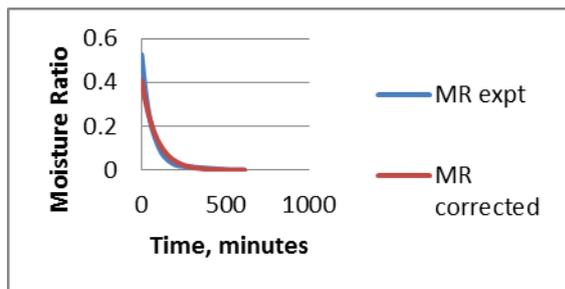


Fig.1. Comparison of Experimental and Corrected Moisture Ratio Values of Unripe *Saba* Using Henderson and Pabis Model (50°C, Trial 3)

Corrected and experimental moisture ratio of unripe *saba* presented a total error of 0.2295 for 40°C, 0.2041 for 50°C and 0.2346 for 60°C for the trial displaying the most appropriate fit as shown in Table 2. Figure 2 shows that Page model best describes the drying kinetics of unripe *saba* at 50°C. The order of reaction and correlation were found to be 0.835 and 0.9245 respectively. The overlapping lines were again observed as to which considerably coincided to choose the most appropriate fit for each temperature. Other trials have total errors varying from 0.2757 to 0.3000, 0.2270 to 0.2945 and 0.2854 to 0.2900 respectively.

Table 2. Modified Page model

Banana Maturity	Temperature	Trial	n	R ²	Total Error	Time (mins) to reach 12% (wet basis)
Ripe	40°C	1	0.795	0.99551	0.2757	84.5922
		2	0.765	0.99566	0.3000	82.7654
		3	0.83	0.98694	0.2295	87.6941
	50°C	1	0.835	0.92449	0.2041	97.1547
		2	0.855	0.96131	0.2270	100.3673
		3	0.895	0.95365	0.2945	105.3460
	60°C	1	1.21	0.99824	0.2346	111.7294
		2	1.26	0.98601	0.2900	107.5572
		3	1.23	0.99553	0.2854	110.4164
Unripe	40°C	1	0.75	0.78706	0.5963	111.5814
		2	0.78	0.87083	0.4912	102.7607
		3	0.87	0.90892	0.7993	131.3016
	50°C	1	1.06	0.98166	0.0809	109.4937
		2	0.95	0.97885	0.1541	106.0543
		3	0.95	0.96337	0.2040	105.7828
	60°C	1	1.4355	0.96144	0.7665	134.6936
		2	1	0.99147	0.1461	128.7354
		3	1.48	0.93651	0.8395	132.8613

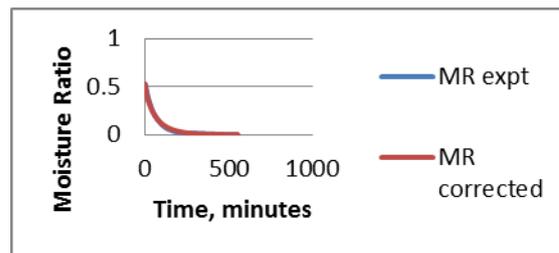


Fig. 2. Comparison of Experimental and Corrected Moisture Ratio Values of Unripe *Saba* Using Modified Page Model (50°C, Trial 1)

Each temperature had three trials and Figure 3 shows the overlapped graphs of corrected and experimental values having the least total error for 60°C as shown in Table 3. The comparison for unripe *saba* at 60°C had the smallest total error equal to 0.1410 and a residence time, τ , of 67.01, while 40°C has a total error of 0.1717 and 50°C, a total error of 0.2138. The other values obtained for the total error ranged from 0.1752 to 0.1876, 0.2280 to 0.2351 and 0.1638 to 0.2214 correspondingly for each temperature.

Table 3. Laplace transform model

Banana Maturity	Temperature	Trial	τ	Total Error	Time (mins) to reach 12% (wet basis)
Ripe	40°C	1	66	0.1752	92.2472
		2	65	0.1876	90.5289
		3	65	0.1717	90.7879
	50°C	1	60	0.2138	89.0309
		2	60	0.2280	88.7680
		3	60	0.2351	88.7867
	60°C	1	67.01	0.1410	109.6246
		2	67.01	0.2214	108.8106
		3	67.01	0.1638	109.2968
Unripe	40°C	1	60	0.6682	85.7605
		2	62.5	0.4523	88.3711
		3	62	0.6657	88.5406
	50°C	1	67.01	0.1169	106.5511
		2	67.01	0.1168	106.3588
		3	67.01	0.1217	106.5505
	60°C	1	72	0.1302	121.8528
		2	74	0.1316	126.0181
		3	74	0.1316	125.4099

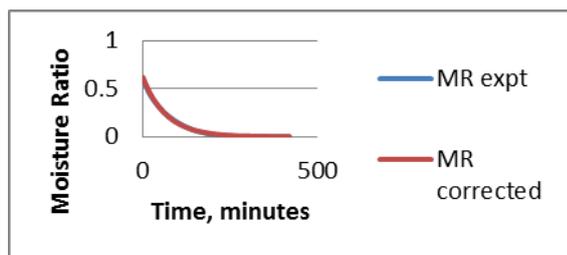


Fig. 3. Comparison of Experimental and Corrected Moisture Ratio Values of Unripe *Saba* Using Laplace Transform Model (60°C, Trial 1)

3.2 Analysis of moisture content and varying temperature

Because of its directly proportional relationship with the drying rate, temperature is one major factor that greatly influence the drying kinetics of *saba*. On the other hand, as the temperature increases, equilibrium moisture decreases, since a greater amount of moisture is removed from the dried banana. This causes a longer drying time to occur. Results from the trials as described by the Laplace Transform Model show that unripe *saba* subjected at 40°C took 90.7879 minutes as compared to 60°C which took 109.6246 minutes and at 50°C, it took 89.0309 minutes.

As for the ripe *saba*, similar results were observed. At 40°C, it took 88.3711 minutes to reach the desired moisture content of 12% wet basis. 50°C and 60°C took 106.3588 and 121.8528 minutes respectively, showing an increasing trend in relation to the temperature.

Table 4 below shows the comparison of the average total error for each temperature for both ripe and unripe *saba*.

Table 4. Average Total Error

Ripe			Unripe		
Henderson and Pabis Model					
40°C	50°C	60°C	40°C	50°C	60°C
0.7888	0.5419	1.0417	0.8238	0.2836	1.3013
Modified Page Model					
40°C	50°C	60°C	40°C	50°C	60°C
0.2684	0.2419	0.2700	0.6289	0.1463	0.5837
Laplace Transform Model					
40°C	50°C	60°C	40°C	50°C	60°C
0.1782	0.22563	0.1754	0.5954	0.1185	0.1311

As evidently shown in Table 4, Laplace Transform model consistently gave the least total error for 40°C, 50°C, and 60°C both for unripe and ripe *saba*. Therefore the Laplace transform, as compared to the Henderson and Pabis Model and Modified Page model is a better fit for the drying kinetics of thinly sliced *saba*.

Table 5 shows the tabulated results of the drying time in minutes to attain the 12% moisture ratio for each temperature for both unripe and ripe *saba* obtained using Laplace Transform model.

Table 5. Drying Time to Attain 12% Moisture Ratio

	Unripe			Ripe		
	40°C	50°C	60°C	40°C	50°C	60°C
Trial 1	92.2472	89.0309	109.6245	85.7605	106.5511	121.8528
Trial 2	90.5289	88.7680	108.8106	88.3711	106.5505	126.0191
Trial 3	90.7879	88.7867	109.2968	88.5406	106.3588	125.4099

For a more convenient way to determine which temperature gave the least drying time for both unripe and ripe *saba* the results are tabulated side by side as presented in the table above. The unripe *saba* dried at 50°C gave the least drying time while for the ripe it was at a temperature of 40°C.

3.3 Observation of the Effect of Banana Maturity

After the drying process, visual inspection was done to monitor the effects of maturity on the dried bananas containing 12% moisture. Temperature affects the physical structure of banana, considering even the slight differences observed in color. The thinly sliced fresh samples of the two banana maturities were white-yellowish, and after drying, a yellow-brownish color developed on these samples. With the increase in temperature, the discoloration decreased indicating an enzymatic character of browning as presented by [11].

Ripe *saba* took a longer time to reach 12% moisture content at temperatures 50 and 60°C with an average time of 106.48 and 122.76 mins respectively compared to the average time of unripe *saba* of 91.34 and 109.25 mins respectively. While at temperature of 40 °C ripe *saba* took only an average time of 87.56 mins compared to 91.19 mins of unripe *saba*. The effect of maturity of *saba* on drying time is that at 40 °C ripe *saba* will be dried faster compared to unripe *saba*, while at temperatures 50 and 60 °C unripe *saba* will be dried faster based on the results obtained.



Fig. 4. Dried unripe banana at 50°C



Fig. 5. Dried ripe banana at 50°C

Theoretically, as temperature increases the drying time should decrease but this study shows that as the temperature increases the drying time also increases. This phenomenon could be explained by the large moisture that can be removed from the *saba* samples because the equilibrium moisture decrease as temperature increases. According to Nguyen and Price (2007), the large difference in the initial moisture content could strongly affect the drying rate and this effect was clearly observed at low temperatures. Mass loss was seen to be insignificant for both ripe and unripe bananas as both showed similar results [12]. However, chemical changes such as the increase in sugar of ripe bananas contributed to its mass loss during drying. This would compensate for the more open structure of ripe bananas which favors an increased drying rate.

Presented in Table 6 is the statistical analysis of the maturity with the corresponding temperature that gave the least drying time using 95% confidence level.

Table 6. Statistical Analysis

Unripe (50°C)			Ripe (40°C)		
Trial 1	Trial 2	Trial 2	Trial 1	Trial 2	Trial 3
85.6705	88.3711	88.5406	89.0309	88.7680	88.7867
Standard Deviation	Average Drying Time	α	Standard Deviation	Average Drying Time	α
1.5585	87.5574	0.05	0.1467	88.8619	0.05
Confidence Interval	Lower Limit	Upper Limit	Confidence Interval	Lower Limit	Upper Limit
1.7635	85.7939	89.3209	0.1660	88.6959	89.0279

4 Conclusions and Recommendation

4.1 Conclusions

Among the three models used in this study, the results obtained using the Henderson and Pabis model and Modified Page model weren't good fit to describe the drying kinetics of *saba* because of a large total error. Based on the total error, the Laplace transform model well describes the drying kinetics of thinly sliced *saba* at all investigated temperatures. Hence, the Laplace transform model can be used to determine mathematical

expression for the drying time. The difference in the moisture that can be removed from *saba* has a greater factor as compared to the temperature in determining the drying time.

4.2 Recommendations

This study can be further improved by using the maturity index instead on basing only on the color of the sample for a more accurate data comparison. The use of different models can also be done to enhance this research. It is also recommended to use a more accurate top load balance. This study recommends that a pre-treatment specifically, water bath, for the sample is done to have same initial moisture content.

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References

1. Postharvest Industry: Banana. Philmech.gov.ph. Retrieved 3 February 2016, from <http://www.philmech.gov.ph/phindustry/banana.htm> (2016)
2. Bezerra, C. V., Amante, E. R., de Oliveira, D. C., Rodrigues, A. M., & da Silva, L. H., Ind. Crops Prod., 241-249 (2013)
3. Saba Banana Production Guide. Retrieved 3 February 2016, from <http://cagayandeoro.da.gov.ph/wp-content/uploads/2013/04/SABA-BANANA-PRODUCTION-GUIDE.pdf>. (2016)
4. Baini, R., Langrish, T.A.G., J. Food Eng., **79**, no. 1, 330–343 (2007)
5. da Silva, W. P., e Silva, C. M., & Gomes, J. P., J. Food Eng., 417-424 (2013)
6. Hofsetz, K., Lopes, C. C., Hubinger, M. D., Mayor, L., & Sereno, A. M., J. Food Eng., **83**, no. 4, 531-540 (2007)
7. Da Silva, W.P., Silva, C.M.D.P.S., Farias, V.S.O., Gomes, J.P., Drying Tech. **30**, no. 2, 164–174 (2012)
8. Robinson, A. A., *Research Design and development of banana dehydration process* (Food Engineering, Sydney, UNSW, Australia, 1980)
9. Talla, A., Puiggali, J.R., Jomaa, W., Jannot, Y., J. Food Eng., **64**, no. 1, 103–109 (2004)
10. Karim, M.A., Hawlader, M.N.A., J. Food Eng., **70**, no. 1, 35–45 (2005)
11. Demirel, D., & Turhan, M., J. Food Eng., **59**, no. 1, 1-11 (2003)
12. Nguyen, M. H., & Price, W. E., J. Food Eng., 200-207 (2007)