A long-term strength of constructive materials

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Abstract. A long-term strength materials under an axially loading of constructive elements is considered and the estimates of this strength are reduced. The proposed approach is connected with the notion so-called energy of entirety [1]. It is notable that this value can be used instead of known Reiner's invariant [2]. A material (concrete, steel, graph) is considered as a union of its links with statistical disturbed strengths [3]. This conception allows to modify Boltzmann's principle superposition of fraction creep deformations [4] and in addition, implies the identity of non-linear stresses function for the instantaneous and retarding deformations. The degeneration of long-term strength because of vibrational influence take into account and the strengthening of the materials in the course of their formation is considered.

The problems of forecast of a long-term structural safety of buildings necessary are connected with the strength $R(\tau)$ of constructive materials, presenting their basic mechanical characteristic. Here τ is a current temps. A material is a thermodynamic system and in consequence of energetic and mass exchange with environment the generating $R(\tau)$ constraint forces are formed. The accumulated in materials specific energy $W(\tau)$ is called their energy of entirety [1]. These values define the energetic state of the materials and represent their maximal measure of resistance to a destruction [5]. Since the destruction at moment $\tau = t$ is generated by instantaneous deformations the corresponding specific work $A(t)$ is considered as the energetic parameter $W(t)$ of material, named the energy of its entirety. Beside of the force action the magnitude of energy $W(t)$ essentially depends from the corrosion damages [6]. If the destruction of material under constant over of period [$t_0 \le \tau \le t$] normal stress $R(t,t_0)$ take place at moment $\tau = t$, the value $R(t,t_0)$ presents its strength for this period of temps. Therefore the value $R(\infty,t_0)$ is called the long-term strength of this material.

The destruction of a part of the links under an increasing on cross-sections G axially loading $N(\tau)$ results in the redistribution of $N(\tau)$ on cross-sections $G(\tau)$ of the capable to resist links. This generates a non-linear dependence of the deformations on the calculated stresses $\sigma(\tau) = N(\tau)/F$, obtained under the assumption of an equal strength for all links. In fact there is a linear dependence of the deformations of the undamaged links on

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the so-called structural stresses $\sigma_s(\tau) = N(\tau)/F(\tau)$. Here F and $F(\tau)$ are the area of G and G(τ) respectively. Then we obtain the following correlation

$$\sigma_s(\tau) = S(t)\sigma(\tau)\,;\; S(t) = F/F(\tau)\,, \tag{1}$$

where $S(t)$ is called the function of non-linearity of stresses. The constant over of period [t_0, t] stress $\sigma_s(\tau)$ generate the deformation

$$\varepsilon(t,t_0) = \varepsilon_{inst}(t) + \varepsilon_{cr}(t,t_0)\,, \tag{2}$$

where $\varepsilon_{inst}(t) = \sigma_s(t)/E(t)$ is the instantaneous and $\varepsilon_{cr}(t,t_0) = C(t,t_0)\sigma_s(t)$ is the creep,deformation; is the measure of simple creep and is the elasticity module.

An increment $\Delta\sigma_s(t) = \sigma_s(t) - \sigma_s(t_0)$ implies the creep deformation $\Delta\varepsilon_{cr}(t,t_0)$. Since a fraction increment $\Delta\varepsilon_{cr}(t,\tau_i) = C(t,\tau_i)\Delta\sigma_s(\tau_i)$ is independent from a value and duration of the rest increments $\Delta\sigma_s(\tau_j)$; $j \neq i$, we can define $\Delta\varepsilon_{cr}(t,t_0)$ by Boltzmann's principle of superposition. In result with respect to the structural stresses we reduce the linear theological state equation [7], [8].

$$\varepsilon(t,t_0) = \left[\frac{1}{E(t)} + C(t,t)\right]\sigma_s(t) - \int_{t_0}^{t} \sigma_s(\tau)\frac{\partial C(t,\tau)}{\partial \tau}\,d\tau \tag{3}$$

Here $C(t,t)$ is so-called quick creep [9].

The value $\tilde{W}(\tau) = W(t_0) - A(\tau)$ we call a current energetic reserve of entirety. The condition $\tilde{W}(\tau_*) = 0$ means the exhaustion of this reserve, implying the destruction at moment $\tau = \tau_*$. In case $\tau_* = t_0$ we have the shirt-term destruction, when $\tau_* = t > t_0$ the destruction in end of [t_0, t]. Consider the exhaustion of $\tilde{W}(\tau)$ only by force deformations and suppose that $E(\tau) = E(t_0)$; $C(t,\tau) = C(t_0,t_0)$. This means that $W(\tau) = const$; $t_0 \le \tau \le t$. Then for the performed by $R_s(t_0)$ and $R_s(t,t_0)$ specific works we obtain respectively

$$A(t_0) = R_s^2(t_0)\left[1 + C(t_0,t_0)E(t_0)\right]/E(t_0)\,, \tag{4}$$

$$A(t,t_0) = R_s^2(t,t_0)\left[1 + C(t,t_0)E(t_0)\right]/E(t_0) \tag{5}$$

Since from $W(\tau) = const$ it follows $A(t,t_0) = A(t_0)$ we have

$$R_s(t,t_0) = r(t,t_0)R_s(t_0)\,, \tag{6}$$

$$r(t,t_0) = \sqrt{\frac{1 + E(t_0)C(t_0,t_0)}{1 + E(t_0)C(t,t_0)}}\,. \tag{7}$$

In applications the function

$$S(\tau)=1+V\left[\frac{\sigma(\tau)}{R(\tau)}\right]^{m}. \tag{8}$$

is used, where V, m are the empirical parameters [10]. Usually for the concrete is supposed $m=4$ and $C(t,\tau)=C_{\infty}\left[1-\beta e^{-\gamma(t-\tau)}\right]$; $\beta=0,8$. Here C_{∞} is the measure of simple creep as $t\to\infty$, γ is the parameter of open surface. Thus we have $r(\infty,t_0)=\sqrt{\left[1+0,2E(t_0)C_{\infty}\right]/1+E(t_0)C_{\infty}}$. By relations (6) and (8) we obtain the equation

$$V\left[R(\infty,t_0)\right]^{5}+\left[R(t_0)\right]^{4}R(\infty,t_0)-\left(1+V\right)r(\infty,t_0)\left[R(t_0)\right]^{5}=0 \tag{9}$$

with respect to $R(\infty,t_0)$. The positive root of this equation represents the desired estimate of $R(\infty,t_0)$.

In case of linear statement $S(\tau)=1$ and by (6)

$$R(\infty,t_0)=r(\infty,t_0)R(t_0) \tag{10}$$

It is known that the creep of statically loaded material (for example concrete) increase under vibrational influence. This phenvmenon is called the vibrocreep of material concrete. The following correlation

$$C_V(t,t_0)=K_V(\omega,\gamma,\rho)C(t_0,t_0) \tag{11}$$

was established on basis of experiments in [11], [12]. The factor $K_V(\omega,\gamma,\rho)$ is determined by frequency ω, parameter γ and asymmetry $\rho=\frac{\sigma_{\min}}{\sigma_{\max}}$ of stresses. This factor is studied by Davidenkov's invariant – the independence from ω of the dissiped in course of one cycle energy ΔW [13], [14]. Note that to this end the proposed in [1] deformative invariant also can be used. By the relations (6) and (11), taking into account the vibrocreep of materials, we have

$$R_s(t,t_0)=r_V(t,t_0)R_s(t_0) \tag{12}$$

$$r_V(t,t_0)=\sqrt{\left[1+E(t_0)C_V(t_0,t_0)\right]/1+E(t_0)C_V(t_0,t_0)} \tag{13}$$

In the course of formation the energy of entirety $W(\tau)$ of materials increases that results in the increase of the strength $R(\tau)$. Along with $R(\tau)$ the module $E(\tau)$ also increases. Following [15] the equality their functions of the strengthening is supposed

$$\psi_R(\tau)=\psi_E(\tau) \tag{14}$$

The process of strengthening is accompanied decreasing of $C(\tau,\tau)$. On basis of the experiments [11], [16] the following relations are reduced [1].

$$R(\tau)C(t,\tau)=const \tag{15}$$

$$E(\tau)C(\tau,\tau) = const \quad (16)$$

The independence of $I(\tau) = E(\tau)C(\tau,\tau)$ from τ means the expected compensation of the largement of $E(\tau)$ by the reduction of $C(\tau,\tau)$. By solution of described of variation of $E(\tau)$ differential equation we obtain

$$\psi_E(t,t_0) = 1 + \left[\frac{E(t)}{E(t_0)} - 1\right]\left[1 - e^{-\lambda(t-t_0)}\right] \quad (17)$$

Here λ is a physico-chemical parameter of the strengthening. Then according to (6), (14) and (17) we have

$$R_s(t,t_0) = \gamma(t,t_0)R_s(t_0) \quad (18)$$

$$\gamma(t,t_0) = r(t,t_0)\left\{1 + \left[\frac{R(t)}{R(t_0)} - 1\right]\left[1 - e^{-\lambda(t-t_0)}\right]\right\} \quad (19)$$

The factor $\gamma(t,t_0)$ defines the current reserve of strength. Usually for concrete is supposed that $t_0 = 28$ day.

Note that the relation (6) at $R(\tau) = const$ follows from the equality (18).

Cite the concluding remarks.

1. In the above examined problem the application of $W(\tau)$ allows to obtain the estimates of $R(\infty,t_0)$ more simply than by previous approaches.
2. The accounting of strength reserve $\gamma(t,t_0)$ is of interest during of construction of the contemporary buildings.

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