Calibration testing of discs using photoelasticity method

Ludmila Frishter[*]

Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

Abstract. Using of the photoelasticity method for determining the stresses under the action of specified forced deformations and, in particular, temperature deformations, which do not satisfy the compatibility conditions, is relevant in the study of composite structures. Photoelasticity method, which is a continual method, and the method of "defrosting" forced deformations, as its subsection, allow obtaining a stress-strain state in the composite area with forced deformations on models made of optically - responsive material. The method of defrosting forced deformations, using the procedure of preliminary freezing of model elements with subsequent defrosting of the entire model, is an effective, versatile and promising method for simulation of stresses as a result of specified forced deformations. In the study of composite structures by the method of photoelasticity and defrosting "defrosting" of forced deformations, a model composed of elements with previously created forced deformations is created. To determine the created forced deformations, calibration tests are performed, in which the following actual loads are determined: forced deformations, pressure, optical and mechanical characteristics of the model material: modulus of elasticity, material fringe value. Novelty of the calibration test method is the definition of “frozen” forced deformations in the blank of the photoelasticity model using a single composite disk model without testing additional models - beams [1, 2]. Taken into account the material parameters of model: Young's modulus, the price of a polymeric material strip, which are different for batches of initial components: epoxy resin and anhydrite. In order to approve the results of calibration tests, this paper considers the theoretical and experimental solution of the elasticity test problem for a composite disk, one of the areas of which is uniformly heated. The obtained data are used for calibration tests in determining the actual loads and mechanical characteristics in the model sections.

1 Introduction

Photoelasticity method [1-8], which is a continual method, and the method of "defrosting" forced deformations, as its subsection, allow obtaining a stress-strain state in the area of stress concentration on the contact surface of the composite structures elements with a jump in forced deformations on models made of optically - responsive material. The

[*] Corresponding author: lfrishter@mail.ru

experimental method of photoelasticity in combination with numerical and theoretical methods allows detailed study of the areas with geometrical concentration of structures stresses.

The model of the photoelasticity method is made of mesh polymeric material by curing epoxy resin with anhydrite. The properties of the initial components: epoxy resin and hardener may differ slightly for different batches. Relevance of the research is to analysis properties of the material, from which to made the structural model, on single calibration model.

The method of defrosting forced deformations [4, 6, 7], using the procedure of preliminary freezing of model elements with subsequent defrosting of the entire model, is an effective method for simulation of stresses as a result of specified forced deformations.

When using the "defrosting" method of the composite model, it is important to determine the resulting forced deformations that occur in the model in order to transfer the experimental data to the full-scale construction.

The experimental solution of engineering problems by the photoelasticity method using the "defrosting" effect of forced deformations [1-8] includes calibration tests of the samples allowing to determine the following actual loads: forced deformations, pressure, as well as optical and mechanical characteristics of the model material: modulus of elasticity, material fringe value.

The purpose of calibration tests is to determine the loads acting on a cylinder when it is studied by the method of "defrosting" free temperature deformations. Elements obtained from the longitudinal section of a hollow cylinder are used to construct structures models by the method of photoelasticity and defrosting deformations [1-3, 6, 7].

2 Materials and Methods

2.1 Methods of calibration tests

This paper considers an experimental solution of the elasticity problem for a disk consisting of Ω_1, Ω_2 areas with modulus of elasticity E, Poisson's ratio ν, linear thermal expansion coefficient α. The Ω_2: $a \le r \le b$ area is not loaded, is in its natural state. Forced deformations $\varepsilon_r = \varepsilon_\theta = \varepsilon_0$ act in the Ω_1: $0 \le r \le a$ area.

Model with specified dimensions 2a = 2.25 cm; 2b = 4.33 cm is glued together using two elements. Element of the Ω_1 area – is the internal disk, it is cut from the cross-section of a long cylinder, which is under the action of a uniformly distributed load – external pressure. The cut blank of the internal disk is in uniform stress state - uniform compression:

$$\sigma_r = \sigma_\theta = -P_i, \quad \varepsilon_r = \varepsilon_\theta = -\frac{1-\nu}{E} P_i = \varepsilon_0, \quad u = -\frac{1-\nu}{E} P_i \, r .$$

The sum of normal stresses $\sigma_r + \sigma_\theta = -2P_i$ is constant the absence of loads on the ends of the cylinder $\sigma_z = 0$. The remaining components of the stress - strain state are equal to zero due to the axial symmetry of the cylinder and the constant load in the axial direction.

The cross section of the long cylinder is made at a distance from the ends of the cylinder, one of which moves freely, therefore, for this section, $w = Cz = 0$ can be assumed. The section thickness is t = 0.35 cm, the internal element of the disk can be considered under the conditions of a plane stress state.

Disk Ω_1 with specified forced deformations:

$$\varepsilon_0 = -\frac{1-\nu}{E} P_i , \tag{1}$$

is glued to an external disk Ω_2. A jump (finite discontinuity) of forced deformations $\Delta\varepsilon = \varepsilon_0$ is created in the contact area of elements $r = a$.

After annealing – "defrosting", the unknown stress-strain state is created in the composite disk due to the finite discontinuity (jump) of the specified forced deformations (1) or the effect of temperature deformations $\varepsilon_0 = \alpha T$, caused by uniform heating of the internal disk. The pattern of interference fringes (isochrome) obtained in the model and diagrams of fringes order m are shown in fig. 1.

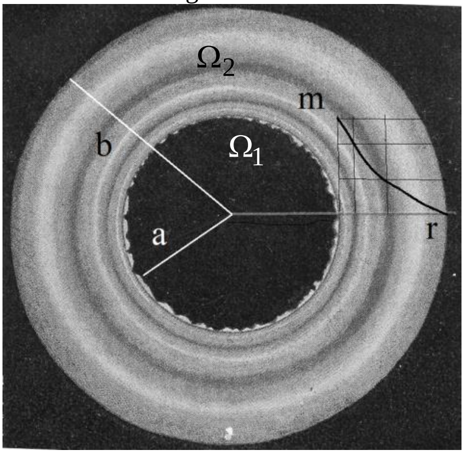

Fig. 1. Isochrome pattern in the composite disk, diagram of fringes order *m* in the external disk Ω_2.

2.2. Theoretical solution of the problem

The paper considers stress-strain state of the disk (fig. 1), in one of the areas of which $\Omega_1 :\ 0 \le r \le a$, forced deformations ε_0 of the form (1) are created.

Stress-strain state of the internal disc [9], caused by forced deformations and the resulting internal pressure in the areas contact, has the following form:

$$\sigma_r = -\frac{E\varepsilon_0}{2} + \frac{E}{2(1-\nu)} A, \quad \sigma_\theta = -\frac{E\varepsilon_0}{2} + \frac{E}{2(1-\nu)} A, \quad u = \frac{1+\nu}{2}\varepsilon_0\, r + \frac{A}{2} r . \tag{2}$$

Stress-strain state of the external disk $\Omega_2 :\ a \le r \le b$, caused by internal pressure occurring in the areas contact due to a jump of forced deformations, has the following form:

$$\sigma_r = \frac{E}{2(1-\nu)} C_1 - \frac{E}{(1+\nu)} C_2 , \ \sigma_\theta = \frac{E}{2(1-\nu)} C_1 + \frac{E}{(1+\nu)} C_2 , \ u = \frac{r}{2} C_1 r + \frac{1}{r} C_2 , \tag{3}$$

where $C_1,\ C_2,\ A$ are unknown constants.

The load is equal to zero at the boundary of the external cylinder:

$$\sigma_r\big|_{r=b} = 0 . \tag{4}$$

The continuity conditions of normal stresses and displacements are performed along the line of areas contact $r = a$:

$$\sigma_r\big|_{a+} = \sigma_r\big|_{a-} , \qquad u\big|_{a+} = u\big|_{a-} . \tag{5}$$

The continuity of the remaining stress and displacement components is performed automatically.

For $r = b$, condition (4) with regard to (3) defines the ratio:

$$C_2 = \frac{b^2(1+\nu)}{2(1-\nu)} C_1. \tag{6}$$

Taking into account the ratio (6), stress-strain state of the external disk will be as follows:

$$\sigma_r = \frac{E}{2(1-\nu)} C_1 - \frac{Eb^2}{2(1-\nu)\ r^2} C_1,$$
$$\sigma_\theta = \frac{E}{2(1-\nu)} C_1 + \frac{Eb^2}{2(1-\nu)} \frac{1}{r^2} C_1, \tag{7}$$
$$u = \frac{r}{2} C_1 r + \frac{b^2(1+\nu)}{2(1-\nu)} \frac{1}{r} C_1.$$

The continuity conditions (5) at the areas contact, taking into account (2), (7), will be rewritten:

$$\begin{cases} -\dfrac{E\varepsilon_0}{2} + \dfrac{E}{2(1-\nu)} A = \dfrac{E}{2(1-\nu)} C_1 - \dfrac{Eb^2}{2(1-\nu)\ r^2} C_1, \\ \dfrac{1+\nu}{2} \varepsilon_0\, r + \dfrac{A}{2} r = \dfrac{r}{2} C_1 r + \dfrac{b^2(1+\nu)}{2(1-\nu)} \dfrac{1}{r} C_1. \end{cases} \tag{8}$$

The unknown constants are determined from the solution of system (8), taking into account (6):

$$A = (1-\nu)\frac{a^2}{b^2}\varepsilon_0, \qquad C_1 = (1-\nu)\frac{a^2}{b^2}\varepsilon_0. \tag{9}$$

Taking into account the determined constants (9), stress-strain state of the external disk will be as follows:

for Ω_1: $0 \le r \le a$,

$$\sigma_r = -\frac{E}{2}\left(1-\frac{a^2}{b^2}\right)\varepsilon_0, \quad \sigma_\theta = -\frac{E}{2}\left(1-\frac{a^2}{b^2}\right)\varepsilon_0, \quad u = \left(\frac{1+\nu}{2} + \frac{(1-\nu)a^2}{2b^2}\right)\varepsilon_0\, r, \tag{10a}$$

for Ω_2: $a \le r \le b$,

$$\sigma_r = \frac{E}{2}\frac{a^2}{b^2}\left(1-\frac{b^2}{r^2}\right)\varepsilon_0, \ \sigma_\theta = \frac{E}{2}\frac{a^2}{b^2}\left(1+\frac{b^2}{r^2}\right)\varepsilon_0, \ u = \frac{a^2}{2b^2}\varepsilon_0\left((1-\nu)r + (1+\nu)\frac{b^2}{r}\right). \tag{10b}$$

2.3. Comparison of theory and experiment data

Maximum tangential stresses are written in the form:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma^{1.0}\, m}{2t}, \tag{11}$$

where m is the order of the interference fringe of the model according to fig. 1, $\sigma^{1.0} = 0.341$ kg / cm is the material fringe value, t = 0.35 cm is the thickness of the model.

For the internal disk according to (10a): $\tau_{\max} = \dfrac{\sigma_1 - \sigma_2}{2} = 0$.

For the external disk:

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma^{1.0}\, m}{2t} = -\frac{a^2}{r^2} E\, \varepsilon_0 \ . \qquad (12)$$

It follows from the relations (12) that the fringes orders are correlated:

$$\frac{m_i}{m_{i+1}} = \left(\frac{r_{i+1}}{r_i}\right)^2 \ . \qquad (13)$$

The obtained relation is checked using the data in fig. 1:

$$\frac{m_1}{m_2} = \frac{1}{2} = 0,5\ ; \quad \left(\frac{r_{i+1}}{r_i}\right)^2 = \left(\frac{13,2}{18,2}\right)^2 = 0,52\ .$$

Diagrams of the fringes orders constructed according to the experimental data (fig. 1) coincide with the data obtained by formula (13), which confirms the validity of the solutions given.
The relation (12) is applicable for determining the specified forced deformations ε_0 or the pressure acting on the cylinder from which the internal disk is cut under the conditions of a uniform stress state:

$$\varepsilon_0 = \frac{\sigma^{1.0}\, m}{2t\ E} \frac{r^2}{a^2}, \qquad (14)$$

where the modulus of elasticity is E = 200 kg/cm^2. According to fig. 1 using formulas (14), (1) for $\nu = 0.5$, the following is obtained:

$$\varepsilon_0 = 0.0048, \quad P_i = -\frac{E\, \varepsilon_0}{1-\nu} = -1.92\ \text{kg/cm}^2.$$

3 Results

Conducted calibration tests determine the loads acting on the cylinder during the manufacture of the model by defrosting free temperature deformations. Elements derived from the longitudinal section of a hollow cylinder with known frozen deformations are used in the methods of photoelasticity and “defrosting” of deformations to create structural models.

4 Discussion

The model of the photoelasticity method is made of mesh polymeric material by curing epoxy resin with anhydrite. The properties of the initial components: epoxy resin and hardener may differ slightly for different batches. Relevance of the research is to analysis properties of the material, from which to made the structural model, on single calibration model.

Obtained calibration tests of the composite disk determine the forced deformations created in the model, which allows transferring the experimental data obtained on the model to a full-scale construction. The data of calibration tests of the material of the model depend on the properties of the composite, made of epoxy resin and hardener of a specific batch and the various applied loads. The reliability of the model parameters obtained in the calibration tests is determined by the coincidence of the theoretical and experimental solutions of the test problem for the composite disk.

5 Conclusions

The method of composite disks calibration tests allows determining forced deformations and pressure acting in the model of the method of photoelasticity and defrosting of forced deformations. The obtained data of calibration tests are used for the experimental solution by the method of photoelasticity of elasticity problems with discontinuous forced deformations [10-17] and the simulation of composite structures with forced deformations. The accuracy of the obtained model parameters is determined by the accuracy of the experimental photoelasticity method. Conducted calibration tests allow determining the properties and loads of the model of the photoelasticity method, which is important when transferring the experimental data to the full-scale construction.

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