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Convective-diffusion model of particle deposition in modeling of building structures

Vadim Akhmetov[1,*] and *Valentina* Akhmetova[1]

[1] Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia

Abstract. The problem of the motion of a swirling flow in a vortex chamber with particle deposition effects at the lateral surface is numerically investigated. Flowfields with formation of recirculation zones are analyzed on the basis of the Navier-Stokes equations. Two cases of the transport process for solid particles of different sizes with deposition on the lateral surface of the channel are studied (passive impurities are assumed). In the first case, the particles are injected into the peripheral part of the channel together with the swirling flow; in the second case, this process is performed through an annular slot without swirling. The results are compared with available experimental data and asymptotic solutions.

1 Introduction

Mathematical modeling of two-phase flows containing particles of different sizes is of practical importance for numerous industrial applications, such as the intensification of heat and mass exchange [1-3], the ecologically pure burning of fuel [4], the stabilization of combustion processes [5-6], and the spraying of aerosols [7]. The stability of the swirling axisymmetric flows is considered in [8-12]. In particular, swirling flows are used to remove finely divided impurities from gases and to separate particles of powders by size [13-14].

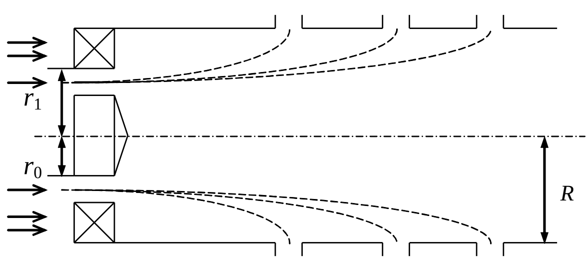

Fig. 1. Schematic of the problem.

The corresponding experimental device can schematically be represented as a long cylindrical channel of radius *R;* a gas swirling flow enters its peripheral part (Figure 1). A

* Corresponding author: vadim.akhmetov@gmail.com

polydisperse powder is injected into the flow either separately through a relatively narrow annular slot of width $\delta = r_1 - r_0$ ($\delta/R \ll 1$) or together with this flow. Under the action of the centrifugal force, the particles of the powder tend to the wall of the channel and are removed from the channel through a number of annular slots.

In this paper, a numerical method [15] is used to study the flow field structure for the previously mentioned device. The simplified convection-diffusion model is presented to describe particle transport and deposition.

2 Hydrodynamic flow model

The flow field calculation results for air phase have been obtained as the solutions of the Navier-Stokes equations under the assumption that the flow is axisymmetric and incompressible. The most important properties of the flows under consideration can be explained by the appearance of near-axial recirculation zones [16-17]. In the cylindrical coordinate system r, φ, z the Navier-Stokes equation can be represented in terms of the stream function ψ, the vorticity Ω and azimuthal velocity V_φ in form

$$\frac{1}{r}\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi}{\partial r}\right) = -\Omega \tag{1}$$

$$\frac{\partial \Omega}{\partial t} + \frac{\partial}{\partial z}(V_z \Omega) + \frac{\partial}{\partial r}(V_r \Omega) = \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 \Omega}{\partial z^2} + \frac{\partial^2 \Omega}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{\Omega}{r}\right)\right] + G^2 \frac{1}{r}\frac{\partial (V_\varphi)^2}{\partial z} \tag{2}$$

$$\frac{\partial V_\varphi}{\partial t} + \frac{\partial}{\partial z}(V_z V_\varphi) + \frac{1}{r}\frac{\partial}{\partial r}(r V_r V_\varphi) + \frac{V_r V_\varphi}{r} = \frac{1}{\mathrm{Re}}\left[\frac{\partial^2 V_\varphi}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V_\varphi}{\partial r}\right) - \frac{V_\varphi}{r^2}\right] \tag{3}$$

$$V_r = -\frac{1}{r}\frac{\partial \psi}{\partial z}, \quad V_z = \frac{1}{r}\frac{\partial \psi}{\partial r}, \quad \Omega = \frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r} \tag{4}$$

System (1) – (4) is written in conservative form and contains the two dimensionless parameters: the Reynolds number $\mathrm{Re} = UR/\nu$ (where ν *is* the kinematic viscosity) and the swirl number $G = W_0/U$. Here the radius R is taken as the characteristic length, the axial and radial velocities are related to a given axial velocity U at the channel inlet, and the azimuthal velocity is related to its maximal value W_0 at the channel inlet.

The flow is considered in the cylindrical domain D $(0 \le z \le z_k,\ 0 \le r \le 1)$. The boundary conditions include the specification of the rigidly rotating flow with the uniform profile of the velocity V_z as well as the formulation of no-slip conditions on the rigid surfaces of the channel and the formulation of the symmetry conditions on the axis $r = 0$. In addition, soft boundary conditions should be given at the outlet section $z = z_k$. Thus, the set of boundary conditions can be written down as

$$\psi = 0, \quad V_\varphi = 0, \quad \frac{\partial \psi}{\partial z} = 0, \quad 0 \le r \le r_0, \quad z = 0 \tag{5}$$

$$\psi = f_1(r), \quad V_\varphi = 0, \quad \frac{\partial \psi}{\partial z} = 0, \quad r_0 \le r \le r_1, \quad z = 0 \tag{6}$$

$$\psi = f_1(r), \quad V_\varphi = f_2(r), \quad \frac{\partial \psi}{\partial z} = 0, \quad r_1 \le z \le 1, \quad z = 0 \tag{7}$$

$$\psi = f_1(1), \quad V_\varphi = 0, \quad \frac{\partial \psi}{\partial r} = 0, \quad z_0 \le z \le z_k, \quad r = 1 \tag{8}$$

$$\psi = 0, \quad V_\varphi = 0, \quad \Omega = 0, \quad 0 \le z \le z_k, \quad r = 0 \tag{9}$$

$$\frac{\partial \psi}{\partial z} = \frac{\partial \Omega}{\partial z} = \frac{\partial V_\varphi}{\partial z} = 0, \quad 0 \le r \le 1, \quad z = z_k \tag{10}$$

where $f_1(r) = 0.5(r^2 - r_0^2)$, $f_2(r) = (r - r_1)/(1 - r_1)$.

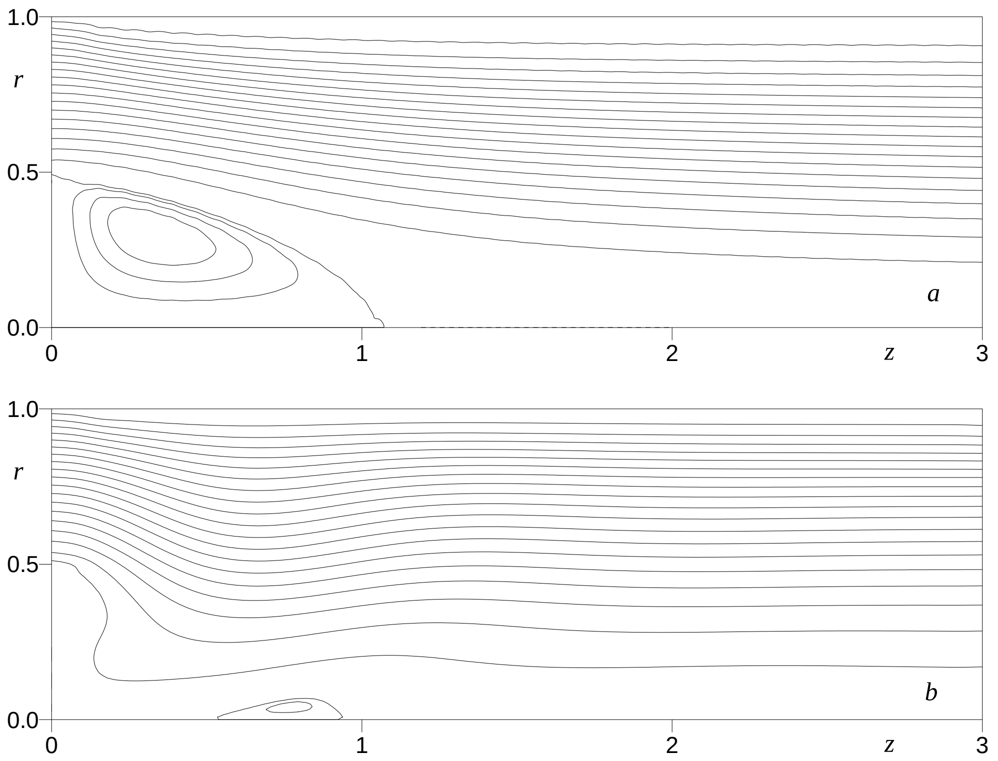

Fig. 2. Streamlines for $r_0 = r_1 = 0.5$: *a* – Re = 100, $G = 0$; *b* – Re = 500, $G = 3$.

The finite-difference relaxation method was used to solve boundary value problem (1) – (4) with boundary conditions (5) – (10); this method was successfully applied in [3] for the analysis of swirling flows of various types. We used a uniformly spaced grid 41x129 with the grid spacing $z_k = 10$. The time step δt was taken from the range between 0.05 and

0.2. Values of Re and G were taken from the ranges between 100 and 1000 and between 0 and 8, respectively. The typical streamline patterns $\psi = \mathrm{const}$ are represented in Figure 2.

3 Mass transfer of solid particles

Among all the forces acting on a particle in vortex chambers the Stokes viscous drag force is the most significant [13]. In cylindrical coordinates the equations of motion of the particles under the action of this force have the dimensionless form:

$$\frac{dV_{zs}}{dt} = \frac{1}{2\mathrm{St}}(V_z - V_{zs}) \tag{11}$$

$$\frac{dV_{rs}}{dt} = G^2 \frac{V_{\varphi s}^2}{r} + \frac{1}{2\mathrm{St}}(V_r - V_{rs}) \tag{12}$$

$$\frac{dV_{\varphi s}}{dt} = -\frac{V_{rs}V_{\varphi s}}{r} + \frac{1}{2\mathrm{St}}(V_\varphi - V_{\varphi s}) \tag{13}$$

Here the parameter $\mathrm{St} = (\rho_s r_s^2 U_0)/(9\nu\rho R)$ is the Stokes number and the subscript s indicates the variables associated with particles. For St << 1 from (11) and (13) it follows that the axial and azimuthal particle velocities V_{zs} and $V_{\varphi s}$ coincide with the corresponding velocities of the basic flow. The radial particle velocity V_{rs} can be determined from (12). Under the conditions St<<1 and $G^2\,\mathrm{St} = O(1)$ from this equation we obtain

$$V_{rs} - V_r = 2\mathrm{St}G^2 \frac{V_{\varphi s}^2}{r} \tag{14}$$

As the flow in question includes the recirculation zone and significantly inhomogeneous velocity and concentration fields, we also investigated two approaches using averaging with respect to r for calculating V_{rs}. In the first approach the deposition rate was determined as a function of $V_{rs}(z)$ by averaging (14) with respect to r in the form

$$V_s(z) = 2StG^2\left\langle V_\varphi^2 / r\right\rangle + \left\langle V_r\right\rangle \tag{15}$$

In the second approach the effective deposition rate V_{rs}^* was introduced by means of the following relation

$$\frac{1}{r}\int_0^r V_{rs}\, crdr = V_{rs}^* \int_0^r c\,dr \tag{16}$$

Here, on the left side we have the flux of $V_{rs}\,cr$ averaged over the interval $[0, r]$ and on the right an effective flux calculated on the basis of the velocity $V_{rs}^* = V_{rs}^*(z,t)$ and the total concentration of the particles on the interval $[0, r]$. This approach was used previously in [15].

We will consider the process of particle transfer for the flows calculated. Following the method based on the convection-diffusion model for gas mixtures with fine low-inertia particles, we can neglect the inverse effect of the particles on the fluid flow [7]. This passive admixture approximation will be employed in the present paper. The velocity field can be found on the basis of (11) – (13). The particle-mass conservation equation can be transformed into the diffusion equation for a passive scalar

$$\frac{\partial c}{\partial t}+\frac{\partial}{\partial z}(V_z c)+\frac{1}{r}\frac{\partial}{\partial r}(rV_{rs} c)=\frac{1}{\mathrm{Re\,Sc}}\left[\frac{\partial^2 c}{\partial z^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right)\right] \tag{17}$$

Here c is the concentration of particles, $\mathrm{Sc}=\nu/\mathrm{D}$ is the Schmidt number, and D is the diffusion coefficient. For example, in [18-19] the convection-diffusion equation (17) was used to simulate mixture flows with loss of particle mass due to deposition on the walls.

In order to determine V_{rs} we will consider three models in which in (17) the deposition rate is determined either as $V_{rs}(z)$ from (15), or as $V^*_{rs}(z,t)$ from (16), or as $V_{rs}(z,r,t)$ from the solution of (12) with the conditions $V_{rs}(z,0,t)=0$.

The boundary conditions for equation (17) can be written down as

$$z=0: \quad \frac{\partial c}{\partial z}=0, \quad 0\le r\le r_1; \quad c=1, \quad r_1\le r\le 1 \tag{18}$$

$$z=z_k: \quad \frac{\partial c}{\partial z}=0, \quad 0\le r\le 1 \tag{19}$$

$$r=0,\ r=1: \quad \frac{\partial c}{\partial r}=0, \quad 0\le z\le z_k \tag{20}$$

Here we do not take into account the effects of repulsion of particles during the process of deposition on the lateral surface; instead we assume that the flow of particles $j=D\partial c/\partial r+V_{rs}c$ is determined only by the convective component without the influence of diffusion $(\partial c/\partial r=0)$.

The second case: the particles of a powder enter the channel without swirling through an annular slot ($r_0\le r\le r_1$) in such a way that their axial velocity is equal to the velocity of the main flow. When determining the velocity field of particles, from (11) we obtain $V_{zs}=V_z$ if $\mathrm{St}\ll 1$. The radial and azimuthal velocities V_{rs} and $V_{\varphi s}$ of particles are determined from equations (12) and (13), since $\mathrm{St}\,dV_{\varphi s}/dt=O(1)$. The distribution obtained is substituted into (17). The boundary conditions for $z=0$ can be written down as

$$z=0: \quad \frac{\partial c}{\partial z}=0, \quad 0\le r\le r_0, \quad r_1\le r\le 1; \quad c=1, \quad r_0\le r\le r_1 \tag{21}$$

Boundary value problems (17) – (20) and (16), (19) – (21) were solved by the finite-difference relaxation method used commonly for transport equations (2) and (3). The fields of concentrations were analyzed for $\mathrm{St}=10^{-5}–10^{-1}$, $\mathrm{Sc}=1$.

The basic calculations were carried out using approach (15).

4 Calculation results and discussion

A quartz powder with particles of diameters $d_s = 10 - 100$ mu was used in the experiments considered in [13]. The following parameters were chosen: $L = 1$ m, $\mathrm{Re} \sim 10^5$ (where Re is the Reynolds number taken relative to the mean velocity and the channel radius $R = 0.05$ m), $G = 5.33$. The main peculiarity discovered in these experiments consists in the following. When the powder enters the channel together with the swirling flow, the particles deposit on the channel wall rather quickly: first, the particles of larger diameters ($d_s \sim 100$ mu) and, then, the particles of smaller diameters ($d_s \sim 10$ mu). The reverse process can be observed when the powder enters the channel through the annular slot together with the nonswirling flow. The use of the former procedure for introducing the powder is not meaningful for the fractional separation of a mixture. The values of the distance z between the point where the particles of radius $r_s = d_s / 2R$ deposit and the inlet section are shown in Figure 3 (circles and triangles correspond to the first and second cases, respectively).

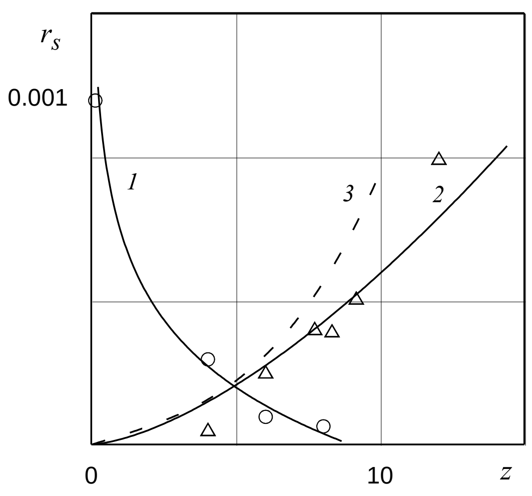

Fig. 3. Comparison of the particle distribution along the channel length with the experiments [13] for $r_0 = r_1 = 0.5$ (curve 1); $r_0 = 0.4$, $r_1 = 0.6$ (curve 2) and asymptotic calculations [20] (curve 3).

The flowfield calculated for $\mathrm{Re} = 500$, $G = 5.33$, $z_k = 15$ was compared with experimental data given in [13]. Then, the field of concentration of particles and the distribution of concentrations $Q_w(z) = (V_{rs}c)\big|_{r=1}$ on the wall were determined (note that this distribution characterizes the process of deposition of particles). The value of z that corresponds to the maximal value of $Q_w(z)$ was adopted as a point of deposition of particles on the wall.

Our numerical results are presented in Figure 3 by curve *1* (when the particles enter the peripheral part of the channel with swirling) and curve *2* (without swirling through the annular slot). The dotted curve represents the dependence obtained by asymptotic methods for the case of rigidly rotating flow when the second way of introducing the particles was used [20].

5 Conclusions

Our comparisons demonstrate the validity of our results and confirm the usefulness of our approach based on the equation of convective diffusion for modeling the transfer processes and the deposition of particles in vortex chambers.

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