

# Determination of dynamic durabilities and dynamic strength of armature in the elements of the constructive system at its flash structural restructuring

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**Abstract.** The results of modeling and computational analysis of the static-dynamic deformation of the reinforced concrete frame-and-rod system are presented for special emergency actions caused by the sudden removal of one of the supporting elements. On the basis of energy, without the apparatus of the dynamics of structures, analytical dependences are constructed to determine the increments of the dynamic extensions in the stretched armature and the dynamic strength of the reinforcement in the sections of the frame elements under the indicated effects on the first half-wave of the structure's oscillations. Verification of the proposed analytical dependencies is performed by comparing the theoretical values of the calculated parameters with the experimental data. It is shown that the constructed analytical dependencies allow to determine quite strictly the investigated dynamic parameters of deformation of the loaded reinforced concrete framed structural systems of buildings and structures under their dynamic overloading by special emergency action associated with sudden structural reorganization of the structural system..

## 1 Introduction

Protection of buildings and structures against progressive collapse requires their calculation in a static-dynamic setting. The purpose of this calculation is to determine the preloads of the structural system elements as a consequence of the sudden removal of one of the supporting structures. In scientific researches of domestic and foreign authors [1-5], software complexes for calculating structures and in normative documents of different countries [6-9], various proposals are given on the estimation of the time of impact of the changed power flow in the structure from the sudden removal of one or another design from constructions. However, they are mostly staged in nature or in them, we will consider some particular solutions. In this connection, in this paper, analytical dependencies are constructed to determine the limiting time for dynamic reinforcement of working reinforcement in sections of reinforced concrete elements of a structural system caused by a sudden structural rearrangement in this system, and the results of modeling and calculation analysis of the deformation of the reinforced concrete frame-and-rod system, as well as the results of comparison with data from experimental studies.

## 2 Formulation of the problem

On the basis of the general Kelvin-Voigt deformation model and the condition of constancy of the total specific energy of deformation of the elements of the reinforced concrete structural system, to construct analytical dependences for determining the dynamic strength of reinforcement and dynamic extensions in it under sudden structural reorganization in a structural system of reinforced concrete after the sudden removal from it of one of the supporting structures. Perform modeling of static-dynamic deformation of a reinforced concrete frame loaded with a floor-defined operating load and emergency loading by the sudden removal of one of the vertical structures (Figure 1). In this case, it is required to establish how the duration of removal of the structural element affects the stress-strain state of the frame elements.

For the possibility of direct resistance of the calculated and experimental parameters under study, we will consider the model of a reinforced concrete frame, the results of which are given in [10, 11].

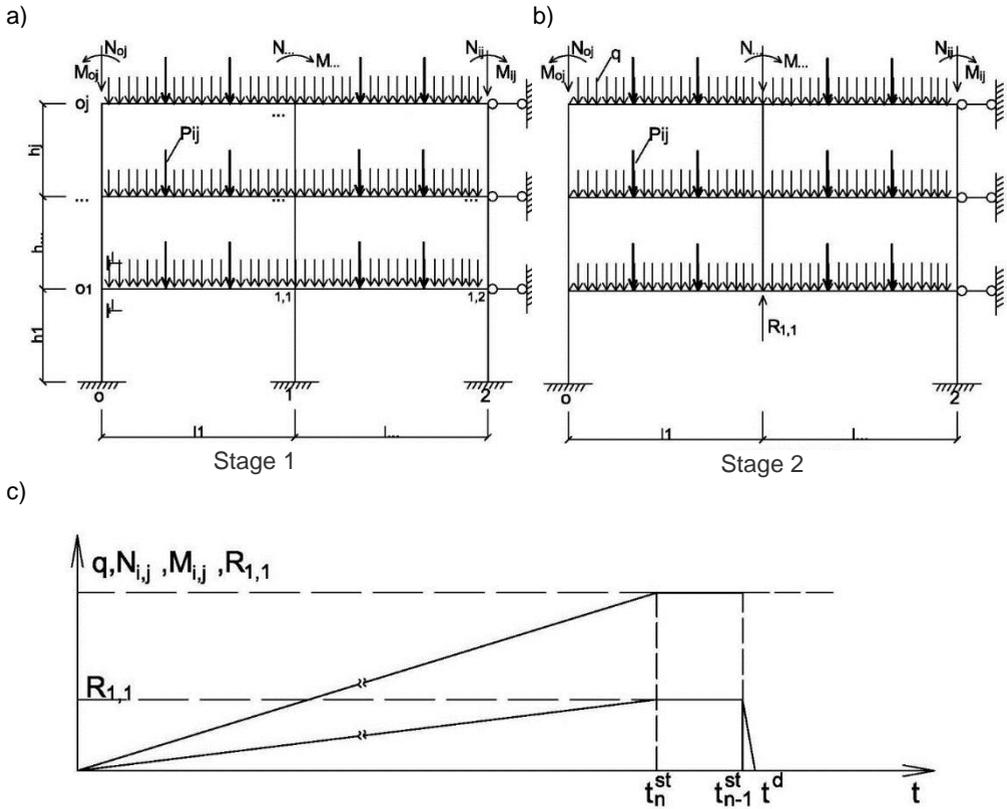
## 3 Method of solution

The loading of frame structures was carried out in two stages. At the first stage, the floor load  $P_{ji}$  and  $q_j$  was applied to the frame crossbars, where  $j$  is the floor number,  $i$  is the span number (see Figure 1a). She modeled the operational load in the frame of a multi-storey building (taking into account the accepted scale of the experimental design and the designated reinforcement).

At the second stage, a special emergency effect was simulated in the form of a sudden removal of the middle or extreme column. The mode of this loading was adopted as follows (Fig. 1 b, c).

At the beginning, in the initial  $n$ -time, a statically indeterminate frame with a distributed load of its own weight  $q$ ; load  $P_{ji}$  in stages in the mode of short-term loading  $t = brought$  to the design level of the operational load. In this case, the reaction  $R_{ji}$  in the removed middle or extreme column in the time interval  $0-$  also increased in proportion to the growth of the load  $P_{ji}$ . Further, the structure was briefly maintained for these loads  $P_{ji}$ ,  $q$  for the time from  $t_0$  (where is the time at the time the frame frame was removed and its static indeterminacy  $n$  decreased by one). Then, an instantaneous, within hundredths of a second from before, the removal from the calculation scheme of the reference reaction  $R_{ji}$  was carried out. This loading mode causes dynamic fluctuations in the structural system. In accordance with [12], at the time of removal of the support, the dynamic simulation of this process is performed by applying the reaction  $R_{ji}$  to the constructive system at the point of the remote column with the opposite sign. In real constructions, switching off the structure takes some time from  $t_0$  (impact, explosion, failure of the base) The magnitude of this time depends on the intensity of structural loading or the dynamic strength of the material. For example, when a column is struck by blow, this time is from experimental data [9-13], measured in hundredths of a second. In this case, the reaction  $R_{ij}$  with the opposite sign is transmitted to the remaining elements of the structural system, and additional dynamical forces and displacements arise as response reactions in the sections of the structural elements of the frame.

To determine these reactions, let us turn to the solution of the well-known problem of S.P. Tymoshenko about the fall on the beam of cargo from zero height, or to a similar problem G.A. Genieva about the vibrations of a two-component reinforced concrete rod with the sudden destruction of a brittle concrete matrix [14].



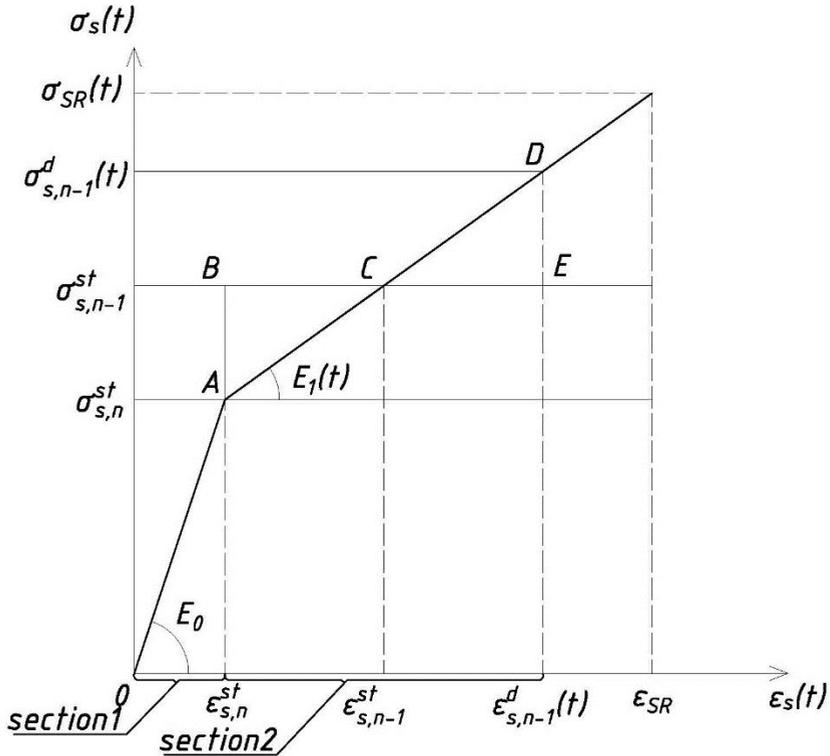
**Fig. 1.** Primary (a) and secondary (c) design diagram of the model of the fragment of the reinforced concrete frame and its loading axle (c) caused by the sudden switching off of the central support.

In the case of sudden removal (destruction) of a frame-and-rod structural system of one of the rods, for example, of the middle pillar, in the frame of the frame above it, a "reaction of response" occurs. Based on the conservation of the energy of the system in state 1 and state 2 (Figure 2). In accordance with [12, 15], we must replace the removal of the column by the reaction in this column on the first half-wave of oscillations of the system with the opposite sign. The diagram of the static-dynamic deformation of the working reinforcement in an arbitrary cross-section of the reinforced-concrete element of the structural system in state 1 (before removal of the column) is shown in Fig. 2.

Using the energy method, without resorting to the formulation and solution of differential equations of the dynamics of rod systems, we can determine the values of response reactions (dynamic stresses) in any section of the elements of the reinforced concrete constructive system using the diagram method [16-18]. To determine the dynamic stresses in the working reinforcement of the reinforced concrete element of the structural system, we consider a static-dynamic diagram of steel deformation (see Figure 2) at two stages of its deformation: Stage 1 - Static deformation mode for short-term loading of the structure to a specified load level (section  $OA$ ); Stage 2 - the mode of dynamic deformation after a sudden structural reorganization of the structural system (section  $AD$ ). The deformation mode in this section in accordance with the deformation theory [12] and the Kelvin-Voigt model in the general case of force resistance of reinforced concrete can be represented in parallel by the connected elements  $A$  and  $B$ . The first of them is

characterized by a diagrammatic work constructed from the results of standard tests, a purely viscous element, which is characterized by a viscous resistance module.

Under the static-dynamic loading regime under consideration in step 1, the viscous element helps to inhibit the development of deformations initiated in element A and in phase 2, the dependence between  $\sigma_s$  and is described by any of the known deformation models of reinforcing steel, for example [16,17].



**Fig. 2.** The diagram of the static-dynamic deformation of reinforcement in reinforced concrete elements n (section 1) and n-1 (section 2) of a time-indeterminate constructive system before and after its structural adjustment.

Taking into account that the effective work of the element A for concrete ends at very small values of the time  $t$  ( $\sim 0,1-0,01$  c) measured from the application of dynamic stresses (point A in the diagram), dynamic stresses in the armature with sufficient the degree of accuracy reflecting the deformation of the second section of the diagram can be written in the form:

$$\sigma_{s,n-1}^d(t) = \sigma_{s,n}^{st} + E_1(t) \cdot (\epsilon_{SR}^d - \epsilon_{s,n-1}^{st}), \quad (1)$$

when  $\sigma_s < \sigma_{s,n}^{st} < R_s$  the destruction of concrete does not occur with any duration of external impact. In the case  $\sigma_s > \sigma_{s,n}^{st} > R_s$  it is possible to establish the relationship between the dynamic strength limit of reinforcing steel and the maximum permissible time of dynamic impact.

According to the adopted two-element model:

$$\epsilon_s = \epsilon_A = \epsilon_B; \quad \sigma_s = \sigma_A + \sigma_B \quad (2)$$

Assuming as a constant in the section AD the viscous resistance module  $K$  for  $\sigma_s$  :

$$\sigma_B = K \frac{\partial \varepsilon}{\partial t}, \tag{3}$$

Then equation (3) for the first deformation section can be opened as follows:

$$\frac{\sigma_s}{K} = \frac{\partial \varepsilon}{\partial t} + \frac{E_0 \varepsilon_s}{K}, \tag{4}$$

The general solution of equation (4) under initial conditions for the first section  $OA$   $t = 0$ ; has the form  $\varepsilon_s = 0$

$$\varepsilon_s = \varepsilon(t) = \frac{\sigma_s}{E_0} \left( 1 - e^{-\frac{E_0}{K}t} \right), \tag{5}$$

and the meaning:

$$t_A = \left( \frac{E_0}{K} \right)^{-1} \ln \frac{\sigma_s}{\sigma - \sigma_{s,n}^{st}}. \tag{6}$$

For the second section of the deformation diagram (see Figure 2), based on the dependencies ((1), (2), (3))

$$\frac{\partial \varepsilon_s}{\partial t} + \frac{E_1(t)}{K} = \frac{\sigma_{s,n-1}(t) - \sigma_{sn}^{st} + E_1(t) \varepsilon_{s,n-1}^{st}}{K}. \tag{7}$$

The general solution of equation (7) under the initial conditions for the second section of  $AD$   $t = t_{\varepsilon}^{st}$ ,  $\varepsilon_s = \varepsilon_{s1}^{st}$  has the form:

$$\varepsilon_s = \varepsilon_s(t) = \frac{\sigma_s^{(t)} - \sigma_{s,n}^{st}}{E_1} \left[ 1 - \left( \frac{\sigma^{st}}{\sigma_s(t) - \sigma_{s,n}^{st}} \right) \frac{E_0}{E_1(t)} e^{-\frac{E_1(t)}{K}t} \right]. \tag{8}$$

If we accept the deformation criterion of strength for concrete, then its destruction will occur at the values of the relative design parameters:

$$\varphi = \frac{\sigma_{s,n-1}^d(t)}{\sigma_{s,n}^{st}}; \quad \theta = \frac{\varepsilon_s R}{\varepsilon_{s1}^{st}}; \quad \delta = \frac{E_1(t)}{E_0}; \quad \xi = \frac{E_0}{K}t. \tag{9}$$

Solving equation (9) with respect to  $\xi = \xi^d = \frac{E_0}{K}t^d$ , an analytical relationship is established that establishes the relationship between the limiting time  $t^d$ , the relative value of the dynamic impact intensity  $\varphi_s$  and the relative parameters of the operation of the armature  $\theta$  and  $\delta$  :

$$\xi^d = \frac{E_0}{K}t^d = \ln \frac{\varphi_s}{\varphi_s - 1} + \delta^{-1} \ln \left[ \frac{(\varphi_s - 1)}{(\varphi_s - 1) - \delta(\theta - 1)} \right]. \tag{10}$$

Relation (10) allows for a given value of stress  $\sigma_{s,n-1} = \sigma_{s,n-1}^d$ , determine the time limit  $t^d$ , or, from a given value  $t^d$ , determine the corresponding dynamic strength limit of the reinforcement  $\sigma_{sR}(t)$ .

The level of potential energy in the structural system after the structural adjustment in the reinforcement relative to the point of static equilibrium is determined by the expression:

$$\Phi(\varepsilon_s) = \int_0^{\varepsilon} \sigma_s(\varepsilon_s) d\varepsilon_s. \quad (11)$$

Then the value of the work of external forces during the dynamic deformation of the reinforcement in an arbitrary section of the reinforced concrete element  $\sigma_{s,n-1}$  is defined as the product by the corresponding displacement. The condition for the constancy of the total specific energy leads to the following analytical expression for the sought value of the dynamic stresses  $\sigma_{s,n-1}^d$ :

$$\Phi(\varepsilon_{s,n-1}^d) - \Phi(\varepsilon_{s,n}^{st}) = \sigma_{s,n-1}^{st} (\varepsilon_{s,n-1}^d - \varepsilon_n^{st}). \quad (12)$$

Dependence (12) expresses the equality of the areas of curvilinear  $\varepsilon_{s,n}^{st} AD \varepsilon_{s,n-1}^d$  and rectilinear  $\varepsilon_{s,n}^{st} BE \varepsilon_{s,n-1}^d$  of trapezoids (see Figure 2).

Assuming that the deformation of the reinforcing steel is linear in the dynamic deformation section of  $AD$ , the condition of equality of the areas of the triangles  $ABC$  and  $CDE$  leads to the relation

$$\varepsilon_{s,n-1}^d - \varepsilon_{s,n}^{st} = \varepsilon_{s,n-1}^{st} - \varepsilon_{s,n}^{st} \quad (13)$$

whence the required dynamic stress

$$\sigma_{s,n-1}^d = 2\sigma_{s,n-1}^{st} - \sigma_{s,n}^{st}. \quad (14)$$

At  $\sigma_s = \varepsilon_s E_1$  and  $\Phi(\varepsilon) = E_1 \varepsilon_s^2 / 2$  the dependence (12) written in the stresses takes the form:

$$\frac{\left[ \left( \sigma_{s,n-1}^d \right)^2 - \left( \sigma_{s,n}^{st} \right)^2 \right]}{2} = \sigma_{s,n-1}^{st} \left( \sigma_{s,n-1}^{st} - \sigma_{s,n}^{st} \right). \quad (15)$$

## 4 A numerical example and discussion of the results

For the numerical estimation of the partial time of dynamic preloading and increments of dynamic stresses in the reinforcement of reinforced concrete elements of the structural system with its sudden structural rearrangement, let us consider, as an example, the calculation of the reinforced concrete frame-and-rod system presented in Figure 1. The frame is made of concrete B25. Cross-section of crossbars is accepted rectangular with the sizes 100x50mm. The reinforcement is adopted by flat frames with symmetrical arrangement of working reinforcement diameter of 6 mm, class A400.

The dimensions of each of the frame spans are 1000 mm, the height of the floor is 700 mm. The design for the computational analysis was chosen on the basis that a physical

experiment was carried out for it [10] and it is possible to directly compare the experimental and calculated values of the investigated parameters.

Let for the reinforcement A400 on the section  $OA$  (see Figure 2)  $\varepsilon_s = \sigma_s / E_0$ ; in this case, on the basis of formula (11) for the potential energy, we can write

$$\Phi(\varepsilon_s) = \int_0^{\varepsilon_s} \sigma_s(\varepsilon_s) d\varepsilon_s = \frac{1}{2} E_0 \varepsilon_s^2 = \frac{1}{2} \sigma_s / E_0. \quad (16)$$

The condition for the constancy of the total energy (12) takes the form:

$$\frac{1}{2} \left[ \left( \sigma_{s,n-1}^d \right)^2 - \left( \sigma_{s,n}^{st} \right)^2 \right] = \sigma_{s,n-1}^{st} \left( \sigma_{s,n-1}^d - \sigma_{s,n}^{st} \right). \quad (17)$$

According to the results of the static calculation of the frame design in question (see Figure 1), the relative stresses  $\sigma_{s,n} / R_s^d$  in the working reinforcement in section I-I in the initial  $n$ -dently indeterminate frame amounted to  $0,26\sigma_{s,R}$ , in the system  $(n-1)$  to  $0,54\sigma_{s,R}$ , so  $\sigma_{s,n-1}^{st} = 2,08\sigma_{s,n}^{st}$ .

Then solving the quadratic equation (17) with respect to the required voltage  $\sigma_{s,n-1}^d$ , we obtain:

$$\sigma_{n-1}^d = 3,55\sigma_{s,n}^{st} \text{ or } \sigma_{n-1}^d = 3,55 \cdot 0,26\sigma_{s,R} = 0,923\sigma_{s,R}.$$

To determine the limiting time of the limiting time of the dynamic action, using formulas (9), we compute successively:

$$\varphi = \sigma_{s,n-1}^{d(t)} / \sigma_{s,n}^{st} = 3,54;$$

$$\theta = \frac{\varepsilon_{sR}}{\varepsilon_{s1}^{st}} \frac{0,025}{0,00052} = 48,08,$$

where the value  $\varepsilon_{sR} = 0,0025$  was taken according to the recommendations [19];

$$\sigma_s^{st} = \frac{\sigma_{s,n}^{st}}{E_0} = \frac{0,26\sigma_{sR}}{2 \cdot 10^5} = 0,00052.$$

Following the formula (9) the parameter  $\delta = \frac{E_1(t)}{E_0}$ , then by the formula (10) the value

is determined  $\xi^d$ . In the case when  $E_1(t) = 0$  and, accordingly,  $\delta = 0$  we get

$$\xi^d = \ln \frac{\varphi_s}{\varphi_s - 1} + \frac{(\theta - 1)}{(\varphi_s - 1)} = \ln \frac{\varphi_s}{\varphi_s - 1} + \frac{\theta - 1}{\varphi_s - 1} = \ln \frac{3,55}{3,55 - 1} + \frac{48,08 - 1}{3,55 - 1} = 18,79$$

In the case  $\delta = 0,6$  we get

$$\xi^d = \ln \frac{3,55}{3,55 - 1} + 0,6^{-1} \ln \left[ \frac{(3,55 - 1)}{(3,55 - 1) - 0,6(48,08 - 1)} \right] = 3,11.$$

Knowing the parameter  $\xi^d$  it is possible to determine the limiting time of dynamic preloading with the known experimental value of the viscous resistance module  $K$ .

## 5 Conclusions

The proposed analytical dependencies allow for the given parameters of the static-dynamic deformation diagrams in a system of  $n$  times statically indeterminate and  $n-1$  times statically indeterminate systems to determine the dynamic reinforcement of the reinforcement and the limiting time of this preload in any section of the structural system and reinforced concrete.

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