A review on the application of the theory of critical distance towards concrete

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Abstract. Theory of Critical Distance (TCD) is one of Fracture Mechanics numerical model that has gone through tremendous laboratory works and validation. Hence, it has been proven to be precise in broad perspectives in the field. Recently, TCD research related to fracture, especially fatigue on concrete are growing but the depth of study is still shallow and deficient compared to metal and steel. Thus, this made the fracture assessment in concrete obscures and governs by uncertainties. Previous efforts have managed to optimize TCD but the results only valid if the water-cement ratio of a concrete specimen in its optimum level. When the water-cement ratio is adjusted to a higher or lower from its optimum level, the output errors showed inconsistency as reported by Luca Susmel (2016). Therefore, this research aims to optimize the Theory of Critical Distance (TCD) by incorporating water-cement ratio and the interaction of microstructure matrix. The optimization involves few stages and finite element. If Theory of Critical Distance (TCD) can be improved by considering concrete's additional element in its mathematical expression, it will definitely contribute to betterment in assessing concrete infrastructure around the globe.

1 Introduction

 "The Versailles Train Crash of 1842" is always said that this tragedy is the departure point in understanding the mechanism of cyclic fatigue. At that time, people did not realise that build-up of small stress cycles could lead to a crack and sudden failure [1]. Following the tragedy, the study in fatigue is progressing until current. But up to current progress, fatigue is known dominantly on metal and steel [2]. There are steady guidelines and formulations established for fatigue and fracture in steel and metal components. The sound studies in fatigue especially on metal and steel make the engineers today have high confidence to use and incurred fatigue element in their design – it is easy to understand and implement.

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In reality, not only steel or metals but concrete structures also continually experienced repeated loading or also called fatigue loading. It is known that the study on concrete has been broadly branched until current and it continues [3][4]. However, there are some fragment of concrete study is noticed to be deficit, which is fatigue and fracture mechanics.

Although at the very beginning of the last century to the latest study in fatigue and fracture taking place tremendously, there are still no recognized agreements in methods to perform the fatigue assessment of concrete – and no organized works done to cultivate specific method or standard to suit the condition of detrimental effect of notches on plain concrete subjected to cyclic loading [5]. Fatigue is being very subjective on its application on concrete. While engineers are confident on concrete study and its applications and contributions to the world's constructions, some of them overlooked the design and might neglect the consequences from the repeated cyclic loading.

In this challenging situation, the study endeavours of using the so-called Theory of Critical Distances (TCD) to perform the repeated and static cycle fatigue assessment of notched plain concrete. Recently, researches relating fatigue on concrete are growing but the depth of study is still shallow and lacking compared to metal and steel [6]. The trivialities in fatigue in concrete will be projected to improvise the Theory of Critical Distance (TCD) either from its conceptual or numerical perspectives with attuning to the variation of water-cement ratio.

The Theory of Critical Distance (TCD) has been proven to be accurate in various perspectives of Fracture Mechanics [7]. Professor Luca Susmel (2006) and Professor David Taylor (2007) claimed that TCD is suitable for practical interest like industrial engineers and it is well-proven. As to implant confidence in the mechanics, every study relating TCD will be controlled over minimal allowance of error.

However, based on one of the recent findings in Professor Luca Susmel in year 2016 where he applied the formalisation on two batches of concrete specimen – both batches of concrete are made of different water-cement ratio, it was found out the error on the same notch modes but different water-cement ratio, are severely high. Therefore, the Theory of Critical Distance (TCD) must be investigated further – to identify its consistency and application on different water-cement ratio in concrete composition.

Through this study, it will definitely increase the understanding in the Theory of Critical Distance (TCD) and enhance its application in future. This will embark the journey of static and fatigue study in concrete to become more emphasize. The study is suggested to be more careful and thorough when making the concrete specimen. It is because fracture testing especially concrete might be bare to many mistakes and uncertainties.

The study aims to propose a characterise crack in concrete governs by water-cement. The characterisation is based on the Theory of Critical Distance (TCD) framework. The technique based on the use of local stresses suitable for estimating fatigue damage in notched concrete components subjected to fatigue and static loading.

Generally, the study of fatigue in concrete is important to ensure the design of a structure includes defect-tolerant design philosophy. Secondly, it is essential for engineers to understand the phases in fatigue cracks. Through the study, either engineers or researchers are able to use Theory of Critical Distances (TCD) and apply it in concrete study with confidence and zero-reluctance. Therewithal, the reason and parameter affecting

TCD's precision when bump into cases of static and fatigue with different water-cement ratio in concrete will be revealed – and TCD will be more sensitive to its application on concrete. Besides, unexpected and sudden failure can be avoided by understanding purely the material's fatigue endurance limit. Despite addressing stress in every design calculation, material's toughness and endurance limit should not be put aside. Engineers have to realise that static and fracture mechanics are related and should not distinguish them apart as what happened in previous decades – structures will be better in quality in any way.

2 Literature Review

The components in concrete structures often never are as perfect as the engineered component. Surface irregularities, metallurgical of metal structures, and sizes are some of the features that engineers will need to inspect before installation. The worry arises when these imperfections create stress concentrations through notches and cracks. The stress concentrations are sensitive – if a material having a notch or crack, applied with any external force even if the force is small, the force at the notch or crack tip will be greater amplified [8] – and this is the study related to fatigue, which is often left behind compared to steel and metal when comes to fatigue analysis.

In structural engineering realm, engineers always wanted the most practical and high accuracy formulation for them to incurred into their design calculations. Elasto-plastic fracture mechanics (EPFM) is one of the well-known extension of Fracture Mechanics which consider further plasticity energy at the notch tips [9]. Nevertheless, the challenging part of this technique is it requires high level of experience and its numerical modelling is massive [10].

In the study, concrete is the material of matter – concrete is a classified as a brittle material although it does allow severe plasticity [11]. To be more detail from modern fracture mechanics perspective, concrete is considered as a quasi-brittle material as it does have some dissimilarity to ideal brittle material [12]. Hereby, the motivation of the study will be shifted to concentrate on linear-elastic fracture mechanics by obeying its law.

Research about fatigue and fracture on its application towards concrete progressively recognized in the recent modern practice because the design safety factors are reducing from the past [13]. Based on engineering understanding, the lower the safety factor means lower the ultimate stress, consequently smaller size of concrete structural component and less concrete used. The decrement in size of concrete structural components leads to more problems involving fatigue and fracture – it has higher chance for local stresses to occur, making the structures more prone to fatigue [14].

Concerning fatigue, sometimes it might increase slightly the usage of concrete – unfortunately it is not a good news for the healthy environment. Currently, the world is rushing towards sustainability as to save the planet. Thus, researchers on concrete especially who involves closely in making the procedures of the Eurocode (EC), or American Concrete Institute (ACI) are trying their best to reduce the concrete consumption in their calculations and practice without jeopardizing its strength [15]. Henceforward, fatigue study in concrete structure no longer be neglected [16].

2.1 Theory of Critical Distance

The idea in analysing fatigue and fracture being addressed (by Taylor (2010) interpreted from Neuber (1936) and Peterson (1959) work) is measuring the maximum stress at the notch root - the high stress will then steadily decrease over distances away from the notch tip – the concept is called the Theory of Critical Distances [17].

The Theory of Critical Distances (TCD) is a group of formalization used to predict the effect/fatigue strength at notches or other stress concentration features region [7]. TCD has been used for years and recently it has been brought up by linking it with LEFM – TCD is based on critical distance from notch root and represents the extension of linear elastic fracture mechanics (LEFM) principles [18].

The Theory of Critical Distances (TCD) successfully analyses and explain notch sensitivity study in a vast scope. TCD is effectively workable in different notch modes; from the sharp notch cases like cracks [19][20] to the blunt notch like surface irregularities [21][22][23][24]. TCD can corresponds very fine under entire range of cycles; low-cycles fatigue loading [25], medium-cycle fatigue loading [7][26][27], and high-cycle fatigue loading [28][29][5][18]). In term of morphological viewpoint, TCD is a formalization which is suitable for brittle and quasi-brittle ([30][31][32]).

There are also other approaches catering fracture in concrete [33][34][35][36]). But in the latest trending of the research in this scope is getting rid by two models; Size Effect by Zdeněk Pavel Bažant (Bažant) and Boundary Effect by Xiaozhi Hu/ Kai Duan (Hu/Duan).

Bažant's emphasized when the failure did not happened at crack initiation then the concrete specimen are subjected to size dependency [37]. Based on Bažant, if this case occurs, the model should properly handle the macro-crack growth and able to explain from energy outlook [38]. Conversely, Hu claimed that Bažant's Size Effect model was not that well-organized as he did not particularly stressed on the effect of grain size in studying fracture behaviour in concrete [39]. Meanwhile Hu's Boundary Effect was being very specific and detail capturing fracture in concrete to the extent of incurring aggregate size and fictitious crack mode in their analysis [40].

It is undeniable Hu's Boundary Effect Model is astonishing but the model did not reach the study of fatigue or cyclic loading prior to time onto concrete. Additionally, Hu-Duan's Boundary Effect was criticized by few professionals such as Christian G. Hoover and Bažant himself declaring the Boundary Effect was only precise in a tiny scope of the field and unreliable enough if to be applied throughout the ground [41]. This give a lead in Theory of Critical Distances (TCD) in catering cases as mentioned. Enhancing TCD in its formulation considering more practical factor will certainly makes good to the field.

Throughout the study, Theory of Critical Distances (TCD) will be the fundamental – and based on few examples of research done as stated above, one should realise TCD has good evidences and great benefits in studying fracture mechanics field compared to several conventional method which these characterisations are not as vibrant as TCD. The Theory of Critical Distances is one of the best well-known formalizations and it satisfies cases in this study harmoniously with the condition of high-cycle fatigue loading on brittle and quasi-brittle material like concrete [18]. However, the study arises the concern when the error in TCD applications variate due to one of the change of one of the concrete's basic mix design –water-cement ratio. Thus, the aim of the study will help the research to cater

issues on sensitivity of the Theory of Critical Distances (TCD) on its application to the concrete under fatigue condition with different water-cement ratio.

2.2 Research Progress in Fatigue and Fracture in Concrete

Analysis of structures under fatigue loading with fluctuating stresses and strains amplitude with respect to time on concrete structure has been discovered since 1900 – however, it was studied by distinctive phenomena [42].

Personally, the study would give a special acknowledgement in developing fracture mechanics field from various aspects - Charles Edward Inglis (1875-1952), Alan Arnold Griffith (1893 - 1963), George Rankin Irwin (1907 - 1998), and Harold Malcolm Westergaard (1888 - 1950). Then, followed by the researchers who directly get involves in the research of fracture in concrete and especially to whom utilizing the Theory of Critical Distance (TCD).

Sir Charles Edward Inglis is one of the most remarkable researchers in fracture mechanics field and deliberated as he was considered the earliest to break through the field. In year 1913, he published his infamous discussion at that time titled "Stress in a plate due to the presence of cracks and sharp corners". Later in year 1921 by an English aeronautical engineer, Alan Arnold Griffith of the Royal Aircraft Establishment in the United Kingdom with the lead of the mathematical work of Professor Sir Charles Edward Inglis. He became fascinated in effect of scratches and surface finish on metals subjected to fatigue loading [12]. Then, Dr. George Rankin Irwin, an American expert in fracture mechanics and materials [43] extended Griffith's approach by refining the definition of energy release within the plastic zone [44][12]). Nextly, Professor Harold Malcolm Westergaard went deep into studying bearing effect and cracks, and eventually explored the theory of elasticity and plasticity [45].

Table 1. Research Development in Fatigue and Fracture in Concrete (in general)

Citation Reference Number	Bibliography	Input
[46]	The Aberdeen Group, "Some Notes on Concrete Fatigue," #C620293 (1962).	**Issue** i. Much work is needed on the influence of moisture, curing, rest periods, freezing and thawing, temperature cycles, admixture, air entrainment and many other factors. ii. Metal behaviour under fatigue has been studied for a very long time compared to concrete. a. Consequently, many fatigue study in concrete were based on hypotheses and assumptions. **Output** i. Experiments on the fundamental properties of fatigue and the study on why fatigue occurring concrete is needed. ii. Demand to understand behaviour of concrete not only under static loading but also fatigue
[47]	H. W. Reinhardt, D. A. Hordijk, and H. A. Cornelissen, "Tensile Tests and Failure Analysis of Concrete," *J. Struct. Eng.*, vol. 112, no. 11, pp. 2462–2477 (1986).	**Issue** i. As the exploration in the concrete from its durability aspect is being concerned, the tensile strength of concrete also comes across to be more potent. ii. Application of pure fracture mechanics is found to be insufficient except for certain cases. **Output** i. Normal and lightweight concrete have been tested in static, repeated, and cyclic loading a. To provide more accurate tensile behaviour of concrete and practical computational analysis.

	ii. Numerical methods involving fracture mechanics application in concrete require a comprehensive description of material behaviour. iii. Non-linear fracture mechanics and numerical methods need a complete stress deformation relation. iv. Future of Fracture Mechanics shall enhance a. The interaction of crack Mode I and Mode II b. The effect of loading rate under impact and under creep conditions. *Issue* i. Paris Law (the crack length increment per cycle is a power function of the stress intensity factor amplitude) is valid only for one specimen size
[48] Z. P. Bazant and K. Xu, "Size Effect in Fatigue Fracture of Concrete," *ACI Mater. J.*, vol. 88, no. 4, pp. 390–399 (1991).	*Output* i. The study emerged Paris Law with the Size-Effect law for fracture under monotonic loading ii. Paris law for fatigue fracture is applicable to big-scale concrete specimen only. iii. The combination between Paris Law and Size-Effect Law must be tested as follows, in near future: a. with variation of stress intensity factor b. when lower limit of the cyclic load not zero c. variation of number of cycles per minute d. different specimen geometries and notch lengths are measured.
[49] A. Milenkovic and M. Pluis, "Fatigue of Normal-weight Concrete and Light-weight Concrete," (2000).	*Issue* i. The study objective is to enhance European concrete and construction industry to its capabilities in terms of cost-effective and environmentally friendly construction, combining the building of lightweight structures. ii. It is a desk study on fatigue of lightweight aggregate concrete (LWAC)

	Output
	 i. The study came up with a proposal for testing LWAC.
	    a. There are certain shape and dimensions that should be followed to avoid the effect of the testing equipment on the stress distribution and achieving a pure tensile or compression at the mid-zone.
	    b. Fatigue tests on normal density concrete (NDC) has shown significant influence of humidity on fatigue life. Thus, it should be checked on LWAC.
	    c. NDC have shown no significant difference between constant amplitude and variable amplitude tests. Therefore, LWAC is proposed to be exerted by constant amplitude stress.
	    d. Additional research is needed in concrete creep tests at mean stress level, shrinkage tests, and development of the modulus of elasticity under fatigue loading.

[50] P. B. Cachim, J. A. Figueiras, and P. A. A. Pereira, "Fatigue Behavior of Fiber-Reinforced Concrete in Compression," *Cem. Concr. Compos.*, vol. 24, no. 2, pp. 211–217 (2002).	***Issue*** i. An experimental program has been carried out to evaluate the performance of plain concrete and fiber-reinforced concrete under compressive fatigue loading. ***Output*** i. Two types of hooked-end steel fibers have been tested and compared. ii. Wöhler Diagram were determined, cyclic creep curves were plotted and the development of the secant stiffness was also assessed for the tested specimens. iii. Strong linkage was found between the secondary creep rate and the number of cycles to failure. iv. Monotonic stress–strain curves might be used as a deformation failure criterion for concrete under fatigue loading. v. The strategic to improve fatigue life of concrete is to add fibers seems to be related with the distribution of the fibers in concrete.

	Reference	Description
[51]	J. Zhang and Q. Liu, "Determination of Concrete Fracture Parameters from a Three-Point Bending Test," *Tsinghua Sci. Technol.*, vol. 8, no. 6, pp. 726–733 (2003).	**Issue** i. This paper presents a method to determine the cracking strength and tensile strength material parameters and stress-crack width relationship from a three-point bending test. **Output** i. Load-crack mouth opening displacement curves is determined experimentally. ii. Crack strength was then deduced through a numerical method. a. The process can be used to evaluate the effect of coarse aggregate and cementitious matrix strength on the stress-crack width relationship, tensile strength, and fracture energy of concrete. iii. This method relies on the fictitious crack model with cracking strength as the crack propagation criterion. iv. The more the coarse aggregate, the higher the tensile strength and the fracture energy; but become less brittle.
[52]	C. Gaedicke, J. Roesler, and S. Shah, "Fatigue Crack Growth prediction in Concrete Slabs.," *Int. J. Fatigue*, vol. 31, no. 8–9, pp. 1309–1317 (2009).	**Issue** i. A new method proposed in the study to predict crack growth and fatigue life of concrete slabs is presented based on similar models for 2-D specimens. **Output** i. Four large-scale concrete slabs on ground subjected to three different stress ranges have been tested to fatigue failure. ii. Geometric correction factors are derived based on 3-D finite element modelling. iii. Geometric correction factors used to calculate the effective crack length and stress intensity factor from the experimental fatigue compliance iv. The proposed method is able to predict the fatigue life and the fatigue resistance of several independent slabs cast with the same geometry and material.

		v. The analysis of the tested slabs shows that load pulses with higher minimum loads generated more fatigue damage. vi. The principles of this slab fatigue crack growth procedure can be stretched to estimate life in un-cracked or partially-cracked concrete slabs on ground.
[53]	P. Stroeven, "Low-cycle Compression Fatigue of Reinforced Concrete Structures.," *Procedia Eng.*, vol. 2, no. 1, pp. 309–314 (2010).	***Issue*** i. The study reports on experiments performed in the low-cycle compression fatigue domain, considering two relatively high upper load levels and several lower ones. ***Output*** i. Amplitude has a minimal effect on life span of concrete at lower frequencies in the low-cycle fatigue domain. ii. At higher frequencies, small amplitude loading leads to an increase in life span as compared to permanent loading under the upper load level.
[54]	F. Aslani and R. Jowkarmeimandi, "Stress–strain model for concrete under cyclic loading," *Mag. Concr. Res.*, vol. 64, no. 8, pp. 673–685 (2012).	***Issue*** i. A hysteretic stress–strain model is developed for unconfined concrete with the intention of providing efficient modelling for the structural behaviour of concrete in seismic regions. a. The proposed model is based on the findings of previous experimental and analytical studies. ***Output*** i. The model for concrete subjected to monotonic and cyclic loading comprises four components in compression and tension:- a. A control curve; b. an unloading curve; c. a reloading curve; and d. a transition curve ii. The model was certified by comparing the results with a series of tests. a. Generally, the proposed model shows satisfactory agreement with the experimental results.

Ref	Content
	iii. The proposed constitutive model was suitable simulation of the response of concrete subjected to cyclic loadings in both compression and tension.
[31] R. Louks, H. Askes, and L. Susmel, "Static Assessment of Brittle/Ductile Notched Materials: An Engineering Approach based on the Theory of Critical Distances," *Frat. ed Integrita Strutt.*, vol. 30, pp. 23–30 (2014).	**Issue** i. The study reformulates the Theory of Critical Distances based on two simple material parameters – Ultimate Tensile Strength and the Fracture Toughness. **Output** i. An experimental data base was compiled from the technical literature. ii. The samples include sharp V-notches to blunt U-notches, and a range of materials that exhibit brittle, quasi-brittle and ductile mechanical behaviour. iii. Failure predictions were about 30% conservative; less than 10% were non-conservative; and less than 2% were out of the 20% allowable error boundary. iv. In assessing components under Mode I loading using Von Mises stress should incurred minimum 1.5 as a safety factor v. On the other hand, assessing components under Mode I loading using Maximum Principal stress should incurred minimum 1.2 as a safety factor
[5] L. Susmel, "High-cycle Fatigue of Notched Plain Concrete," in *Procedia Structural Integrity XV Portuguese Conference on Fracture, PCF 2016*, vol. 2, pp. 3447–3458. (2016)	**Issue** i. The research investigates the accuracy of Theory of Critical Distances (TCD) in modelling high-cycle fatigue regime and its effect to the behaviour of notched plain concrete. **Output** i. Based on TCD, the high-cycle fatigue assessment is performed using a scale length parameter, which is treated as a material property. ii. The accuracy of TCD was checked through a number of experimental results generated by testing, under four-point bending with notched specimens of plain concrete. iii. The study proven TCD is successful in estimating the high-cycle

		fatigue strength of notched concrete beams, resulting in predictions falling within an error interval of about ±15%.
[55]	Y. Wang, X. Hu, L. Liang, and W. Zhu, "Determination of Tensile Strength and Fracture Toughness of Concrete using Botched 3-p-b Specimens," *Eng. Fract. Mech.*, vol. 160, pp. 67–77 (2016).	**Issue** i. Maximum loads of three-point bending test under notched specimens with constant height are often used to determine the tensile strength and fracture toughness of concrete. ii. Generally, heterogeneous composite structures of concrete are unable to get strength and fracture toughness parameters through three-point bending test; because most of the formalisations related to three-point bending test are commonly assumed as homogeneous which might be inaccurate. **Output** i. A simple fracture model is established, linking fictitious crack to the maximum aggregate size so that the concrete structure and fictitious crack formation can be considered together. ii. Maximum loads of three-point bending test specimens with different initial notches containing different influence of the tensile strength and fracture toughness are analysed. 　a. A simple methodology is developed in analysing quasi-brittle fracture of small notched 3-p-b concrete to get tensile strength f_t and fracture toughness K_{IC}. 3-p-b test and going through few formulas that two parameters can be obtained accurately.
[56]	S. Ayyad and M. Alawneh, "Effect of Concrete Parameters on Local Fracture Energy of Concrete," *Int. J. Appl. Eng. Res.*, vol. 12, no. 5, pp. 793–796 (2017).	**Issue** i. The study focus on determining the influence of concrete parameters on local fracture energy and failure mode of concrete beams. ii. Relationship between failure mode, fracture energy and size effect of existing fracture models related were reviewed and few findings obtained.

Output

i. Three-point bending test on notched concrete beams have been performed

    a. value of fracture energy G_f increases by increasing water-cement ratio – increasing in concrete strength

    b. increasing beam depth and/or aggregate size show a considerable increasing in fracture energy G_f value

    c. It is confident to state that the cohesive crack model is the best performing fracture model for concrete and is widely used in the fracture analysis of concrete.

- *Cohesive Crack Model* is a model that developed the fracture process zone by the analysing the plot of stress versus crack opening relationship at the crack tip

3 Summary

The concern on fracture mechanics has begun in the early of the 20th century where Inglis (1913) studied on crack in plate. Then, the concept and formulations were sharpened as time goes by; fracture study has become even more important when World War II. But, almost every fracture study only involves steel and metal. In the middle of 20th century, researcher started to realise the importance of considering fatigue and fracture study in concrete. Starting that point, more researchers begin to determine and analyse fracture properties of concrete. The tests on concrete involve every range of scope; lightweight, normal-weight, and heavy-weight concrete. In fact, some of the experiments were testing concrete under fatigue loading. As the concrete's fracture parameters were mostly well-defined at that time, the derivation grew even deeper by analysing the fracture process zone, fatigue crack growth, critical number of cycles to failure, so on and so forth. As fracture mechanics in concrete getting matured, some of the researchers took the opportunity to simplify the testing involving concrete's fracture because fracture test on concrete was not easy. The simplifications were meant to benefit the industry. The simplifications were not only meant for experimental scope but it developed by simplifying the concept and formulations too.

Fracture mechanics has many more brances in the field such as Wöhler Curve method, Elastic-Plastic Fracture Mechanics (EPFM), Linear-Elastic Fracture Mechanics (LEFM), the Theory of Critical Distances (TCD), Critical Plane approach, Manson-Coffin Curve method (MCCM), Gradient Elasticity, Boundary Effect Method (BEM), Size Effect Method (SEM) etc. However, the Theory of Critical Distances (TCD) is lighter and more practical for industry [57][58].

The Theory of Critical Distances is one of the best well-known formalizations and it satisfies cases in this study harmoniously with the condition of high-cycle fatigue loading on brittle and quasi-brittle material like concrete [18]. However, the study arises the concern when the error in TCD applications variate with respect to one of the concrete fatigue strength's aspects – which is water-cement ratio. Of all researches as listed above, there were a little gap which might enhance fracture mechanics application in a better way. Thus, the aim of the study will help the research to cater issues on sensitivity of the Theory of Critical Distances (TCD) on its application to the concrete with different water-cement ratio.

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