

# Multi-objective hydropower purchase optimization method for inter-provincial power grids under time-sharing electricity price

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**Abstract:** The existing electric grid purchase ratio research focuses on the methods of various types of combinations of purchasing and selling business of electricity, and the methods of reasonable risk allocation between different businesses, First, it lacks the consideration of the sensitivity of customer requirements to electricity price changes. Second, it lacks the consideration of optimization of the revenue and risk of hydropower purchase in the provincial electric grid.

Therefore, this paper establishes the multi-objective time-of-use power price model based on customer requirements for time-of-use power price response. It introduces a VaR-based risk assessment method. Also, it proposes a multi-objective optimization model that maximizes the expected revenue on electricity purchase and minimizes the risk of purchasing electricity. The electricity purchase ratio scheme and the electricity purchase risk scheme are jointly optimized to obtain the electric grid inter-provincial electricity purchase risk decision and the optimal electricity purchase ratio. The results show that the grid company will obtain greater economic benefits while avoiding risks as much as possible after using the power purchase optimization method of this paper.

## 1 Establishment of time-price model

### 1.1 Customer response curve

Drawing on the operation mode of the power market in East China, the power sales side and the power purchase side will be divided into peak and low valley sections for operation. The peak time is from 8:00 to 22:00 every day, and the trough time is from 22:00 to 8:00 every day. The peak and valley time-selling electricity price, that is, the peak and the low valley are implemented with different sales price.

According to the principle of consumer psychology, users will have different responses to different electricity prices. Thus changing their power consumption mode, resulting in the effect of peak shaving and valley filling. Therefore, the user's reaction to the electricity price can form a user's reaction curve. The user response curve is studied. After the peak -valley price difference is adjusted multiple times, the electricity price difference and the power difference before and after the adjustment can be counted; then the regression fitting is performed to obtain the response curve of the user in a certain area:

$$y = f(t, x) \quad (1)$$

Y is the ratio of the electricity consumption after the peak-valley time-sharing electricity price to the electricity consumption and the electricity consumption before the implementation, x is the ratio of the electricity price to the

original electricity price after using the peak-to-valley time-of-use price.

In this paper, we use the electricity price comprehensive reaction curve of the literature [2] to fit the formula (2), as shown below:

$$\begin{cases} y = 0.4319x^3 - 0.4794x^2 - 0.4191x + 1.4629 (x \leq 1) \\ y = 0.329x^3 - 1.8315x^2 - 2.5134x + 0.0111 (x > 1) \end{cases} \quad (2)$$

According to the peak and valley time division, the corresponding model is established for optimization.

### 1.2 Establishment of objective function

#### 1.2.1 The first objective function

Regarding the objective function of time-sharing electricity price [1], there are generally multiple targets with minimum peak load, maximum valley load, and minimum peak-to-valley difference. All of them can achieve the purpose of balancing system load:

(1) Minimize system peak load:

$$\min_{x_f, x_g} \max_t L = \frac{Q_t f(t, x_f, x_g)}{\Delta t} \quad (3)$$

Maximize system valley load:

$$\max_{x_f, x_g} \min_t L = \frac{Q_t f(t, x_f, x_g)}{\Delta t} \quad (4)$$

(3) Minimize system peak-to-valley difference:

$$\min_{x_f, x_g} \left[ \max_t \frac{Q_t f(t, x_f, x_g)}{\Delta t} - \min_t \frac{Q_t f(t, x_f, x_g)}{\Delta t} \right] \quad (5)$$

$Q_t (t=1,2,3,\dots,T)$  are respectively the electricity consumption in each period before the implementation of the peak-to-valley time-of-use price, and  $T$  is the total number of time periods,  $\Delta t$  is the time of each time period.

This paper chooses to target the minimum peak-to-valley difference.

### 1.2.2 The second objective function

The satisfaction rate of electricity expenses is taken as the user satisfaction index. The formula is expressed as:

$$1 - \frac{\min_{x_f, x_g} (M_1 - M)}{M} \quad (6)$$

The  $M$  is the user's electricity purchase fee before using the time-of-use electricity price,  $M_1$  is the user's electricity purchase fee after using the time-of-use tariff

### 1.3 Constraints

① The total number of users is unchanged:

$$Q_f + Q_g = Q'_f + Q'_g \quad (7)$$

$Q_f$ 、 $Q_g$ 、 $Q'_f$ 、 $Q'_g$  are respectively the total electricity consumption for each period before and after the implementation of the peak-to-valley time-of-use price..

② peak and valley time-of-use electricity price constraints:

Considering the cost of electricity production, the level of social economy, and the affordability of power users, there is a certain limit on the price of electricity:

$$\begin{cases} l_1 p < x_f < l_2 p \\ l_3 p < x_g < l_4 p \end{cases} \quad (8)$$

$l_1$ 、 $l_2$ 、 $l_3$ 、 $l_4$  are respectively the upper and lower limits of the range of user price changes during peak and valley periods

$$2 \leq \frac{x_f}{x_g} \leq 5 \quad (9)$$

$x_f$ 、 $x_g$  are respectively the peak hour electricity price and the valley time electricity price respectively, and  $p$  is the original electricity price of the region.

### 1.4 Simplified model:

$$\begin{cases} \min_{x_f, x_g} \left[ \max_t \frac{Q_t f(t, x_f, x_g)}{\Delta t} - \min_t \frac{Q_t f(t, x_f, x_g)}{\Delta t} \right] \\ \min_{x_f, x_g} (M_1 - M) \\ 1 - \frac{M_1 - M}{M} \end{cases} \quad (10)$$

$$s.t. \begin{cases} 1.2p \leq x_f \leq 1.8p \\ 0.3p \leq x_g \leq 0.8p \\ Q'_f + Q'_p + Q'_g = Q_f + Q_p + Q_g \\ 2 \leq \frac{x_f}{x_g} \leq 5 \end{cases} \quad (11)$$

The NSGA2 algorithm solves the problem with the highest user satisfaction and the minimum peak-to-valley difference, and obtains the optimal peak period electricity price and valley time price.

## 2 Introduction of the NSGA-II algorithm

The NSGA-II algorithm was proposed by Srinivas and Deb on the basis of NSGA in 2000. It is superior to the NSGA algorithm: it uses a fast non-dominated sorting algorithm, and the computational complexity is much lower than that of NSGA; congestion and congestion are adopted. The comparison operator replaces the shared radius shareQ that needs to be specified, and serves as the winning criterion in the peer comparison after quick sorting, so that individuals in the quasi-Pareto domain can be extended to the entire Pareto domain and evenly distributed, maintaining the Diversity of population .It introduces an elite strategy, expands the sampling space, prevents the loss of the best individual, and improves the computational speed and robustness of the algorithm.

NSGA-II is based on the first generation of non-dominated sorting genetic algorithm, and its improvement is mainly for the three aspects mentioned above:

① A fast non-dominated sorting algorithm is proposed, which reduces the computational complexity on the one hand, and merges the parental population with the progeny population on the other hand, so that the next generation of populations are selected from double space, thus retaining The best of all individuals;

② Introduce an elite strategy to ensure that some excellent individual individuals will not be discarded during the evolution process, thus improving the accuracy of the optimization results;

③ The use of congestion and congestion comparison operators not only overcomes the deficiencies in NSGA that require artificially assigned parameters, but also serves as a comparison criterion between individuals in the population, so that individuals in the quasi-Pareto domain can be uniformly extended to the entire Pareto. The domain ensures the diversity of the population.

## 3 Multi-objective optimization model of benefit-risk [3]of electricity purchasing ratio

The goal of grid company risk management is to eliminate the volatility of its profits, and managers must be able to assess potential losses in grid operations. The power purchase risk of the grid company refers to the possible adverse changes in the value of the assets due to changes in the price of the electricity market and changes in the volatility of the price of electricity. The risk is generally

measured in the form of price volatility, which will cause changes in the grid company's rate of return.

This paper starts with the application of portfolio theory in the risk analysis of purchasing power, assuming that there is no price fluctuation in the contract market of peak and valley segments, and regards the cost of purchasing electricity in the peak and valley segments of the futures market as a portfolio and using VaR value to analysis its fluctuations in electricity purchase costs.

### 3.1 Calculation of portfolio VaR<sup>[4]</sup>

VaR refers to the worst expected loss of a certain portfolio under certain confidence levels in the future, under the normal fluctuation of the market, which can be expressed as  $P(\Delta V \leq V) = 1 - c$ , In this formula:  $\Delta V$  is the change in the value of the portfolio when the market changes unfavorably;  $c$  is the confidence level;  $V$  is the value at risk under the confidence level  $c$ . The maximum loss measured by  $\Delta V$  is equal to the VaR value of the portfolio at a given confidence level  $c$ .

We take the purchase cost of the spot market peak period as the first investment, and the purchase cost of the spot market during the valley period as the second investment<sup>[5]</sup>, assuming that  $\beta_1$ 、 $\beta_2$  are respectively the ratio of the first investment and the second investment purchase to the total purchase price of the futures market. The average electricity price of the first investment is  $p_{fx}$ , the fluctuation standard deviation is  $\sigma_1$ ; the average electricity price of the second investment is  $p_{gx}$ ; the fluctuation standard deviation is  $\sigma_2$ ; the total electricity purchase is  $Q_x$ ;

The total investment cost is:

$$G_x = (\beta_1 p_{fx} + \beta_2 p_{gx}) Q_x \quad (12)$$

The variance of investment fluctuations is:

$$\sigma^2 = \beta_1^2 \sigma_1^2 + \beta_2^2 \sigma_2^2 + 2\beta_1 \beta_2 \sigma_1 \sigma_2 p \quad (13)$$

$p$  is the correlation coefficient of the electricity price in the peak and valley period.

The value of risk is:

$$Var = \alpha S_1 \sqrt{T} \sqrt{\beta_1^2 \sigma_1^2 + \beta_2^2 \sigma_2^2 + 2\beta_1 \beta_2 \sigma_1 \sigma_2 p} \quad (14)$$

### 3.2 Establishment of the income maximization model

The purchase side is divided into the same peak-to-valley period, corresponding to different price levels; from the provincial, provincial monthly contract market, spot market power purchase, analysis the daily purchase of electricity, using mathematical functions expressed as follows:

$$\max R = \max \{E(S - G)\} \quad (15)$$

$S$  is the income of electricity sales,  $G$  is the cost of electricity purchase,  $R$  is the expected return on electricity purchase;  $E$  is the mean operator;

① Electricity sales revenue: In the case of implementing peak-to-valley time-of-use tariffs on the electricity-selling side, the sales revenue is related to the

peak-to-valley time-of-use tariff plan and the corresponding load response. The sales revenue  $S$  is:

$$S = x_f Q_f + x_g Q_g; \quad (16)$$

$Q_f$  is the load demand for the peak period;  $Q_g$  is the load demand for the valley period;  $x_f$  is the electricity price during the peak period of the electricity sales side;  $x_g$  is the electricity price during the valley period of the electricity sales side

② Cost of electricity purchase: Calculate the cost of electricity purchase, the cost of electricity purchase  $G$  is:

$$G = (1 - \lambda_1) Q_f p_{fh} + \lambda_1 Q_f p_{fx} + (1 - \lambda_2) Q_g p_{gh} + \lambda_2 Q_g p_{gx}; \quad (17)$$

$\lambda_1$ 、 $\lambda_2$  are respectively the ratio of the purchased electricity in the spot market to the total purchased electricity in the peak and valley periods respectively;  $p_{fh}$ 、 $p_{fx}$  are respectively the peak-time contract market and the spot market hydropower unit purchase cost;  $p_{gh}$ 、 $p_{gx}$  are respectively the electricity purchase expenses for hydropower in the valley period contract market and the spot market.

### 3.3 Simplified model:

Establishing the multi-objective optimization model of benefit-risk of electricity purchasing ratio, solving the expected value of electricity purchase, the risk of purchasing electricity, purchasing ratio of peaks and valleys in the provinces and provinces.

Objective function:

$$\begin{cases} \max R = \max \{E(S - G)\} \\ \min VaR = \alpha \sqrt{T} \sqrt{\beta_1^2 \sigma_1^2 + \beta_2^2 \sigma_2^2 + 2\beta_1 \beta_2 \sigma_1 \sigma_2 p} \end{cases} \quad (18)$$

Constraints: Electricity balance, purchase power constraints.

① Electricity balance

$$\sum_{i=1}^4 (Q_i / Q) = 1 \quad (19)$$

$Q_1, Q_2$  are respectively the electricity of Contract market in peak and valley time,  $Q_1, Q_2$  are respectively the electricity of spot market in peak and valley time,  $Q$  is the total load demand:

② purchase power constraints

$$X_i, \min \leq X_i / Q \leq X_i, \max \quad (i = 1, 2, \dots, 4) \quad (20)$$

$X_i, \max$ 、 $X_i, \min$  is the largest and smallest ratio of  $X_i$  accounts for the total load demand;

### 3.4 Solve

Solve the corresponding power purchase ratio scheme and its benefits and risks through NSGA-II algorithm; obtain the power purchase ratio scheme under different expected benefits for decision makers with different risk preferences.

## 4 Examples analysis

**4.1** This example uses a set of summer typical load data in a certain area for simulation calculation. The specific data is shown in Table 1.

**Table 1.**A set of summer typical load data in an instance calculation

Time	Load/MW	Time	Load/MW
00: 00—01:00	394	12: 00—13:00	527
01: 00—02:00	411	13: 00—14:00	534
02: 00—03:00	402	14: 00—15:00	532
03: 00—04:00	379	15: 00—16:00	530
04: 00—05:00	419	16: 00—17:00	562
05: 00—06:00	437	17: 00—18:00	568
06: 00—07:00	460	18: 00—19:00	601
07: 00—08:00	497	19: 00—20:00	599
08: 00—09:00	554	20: 00—21:00	564
09: 00—10:00	566	21: 00—22:00	526
10: 00—11:00	581	22: 00—23:00	449
11: 00—12:00	518	23: 00—24:00	426

From the above information, we can recognize:

The maximum load is 601 MW, the minimum load is 379 MW, and the peak-to-valley difference is 222 MW.

**4.2** After studying the user response curve and adjusting the peak-to-valley price difference multiple times, the electricity price difference and the power difference before and after the adjustment can be counted; then the regression fitting is performed to obtain the response curve of the user in the area

$$\begin{cases} y = 0.4319x^3 - 0.4794x^2 - 0.4191x + 1.4629(x \leq 1) \\ y = 0.329x^3 - 1.8315x^2 + 2.5134x - 0.0111(x > 1) \end{cases}$$

Before the implementation of the time-of-use tariff, the user purchase price is 70.147\$/MWh. Based on the electricity market time division in East China, the purchase and sale of electricity are divided into peak time and valley time: The peak hours are from 8:00 to 22:00 (excluding 22:00) and the trough is from 22:00 to 8:00 (excluding 8:00). According to the economic development level, user affordability and power cost analysis of the place, the local price control department stipulates the user peak and valley time price:

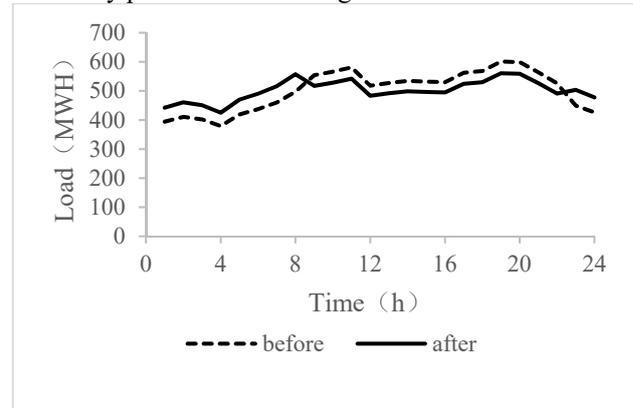
$$\begin{cases} 1.2p \leq x_f \leq 1.8p \\ 0.3p \leq x_g \leq 0.8p \end{cases}$$

The goal is to minimize the peak-to-valley difference and the highest user satisfaction in the system:

$$\begin{cases} \min_{x_f, x_g} [\max_t \frac{Q_t f(t, x_f, x_g)}{\Delta t} - \min_t \frac{Q_t f(t, x_f, x_g)}{\Delta t}] \\ \max_{x_f, x_g} [1 - \frac{M_1 - M_0}{M_0}] \end{cases}$$

The optimal peak-time electricity price is 87.76\$/MWh and the valley-time price is 43.53\$/MWh by NSGA-II algorithm. After using the time-sharing electricity price, the system peak-to-valley difference is 135.7979MWh,

the satisfaction is 100%, and the peak-to-valley time-sharing is used. The load comparison before and after the electricity price is shown in Figure 2.



**Figure 2.** Load comparison chart before and after using peak and valley time-of-use electricity price

**4.3** According to the trial operation data of East China Power Market, the average electricity price distribution obeys the normal distribution. The peak market price is 45\$/MWh, the real-time electricity price is 36.925 \$/MWh, the standard deviation is 3.693 \$/MWh; the market price of the valley contract is 33.35\$/MWh, real-time electricity price average 30.441 \$ / MWh, standard deviation 2.52 \$ / MWh.

Using the NSGA-II algorithm to calculate the expected revenue maximization model and the risk minimization model, the corresponding purchase ratio scheme and its benefits and risks are obtained; The power purchase ratio scheme is chosen by decision makers with different risk preferences. The data is shown in Table 2.

**Table 2.**The power purchase ratio scheme derived from the calculation of the example and its benefits and risks

Expected revenue(\$/MWh)	Conditional value-at-risk(\$/MWh)	Peak time spot market purchase ratio	Low-spot period spot market purchase ratio
358504.5935	0.0000	0.0000	0.0000
359284.1653	690.9455	0.0095	0.0162
361253.0561	2130.0935	0.0437	0.0139
362634.9691	3140.5231	0.0689	0.0071
365654.3750	5948.8991	0.0982	0.1008
367323.4530	6712.4182	0.1467	0.0169
369070.0225	8029.2284	0.1766	0.0169
371006.6208	9475.4805	0.2107	0.0129
373383.6626	11276.6783	0.2508	0.0152
375141.7690	12713.0635	0.2738	0.0445
376410.7758	13668.9064	0.2955	0.0445
377660.5300	14865.5099	0.3035	0.1008
379616.6069	15931.3106	0.3610	0.0001
382335.9360	18311.8240	0.3866	0.0873
383686.1922	19388.1260	0.4065	0.1008
385522.2590	20424.9240	0.4590	0.0121

386993.0422	21534.7136	0.4842	0.0121
389172.4251	23141.9502	0.5243	0.0000
391343.8165	24787.2703	0.5609	0.0023
391913.0398	25216.8055	0.5707	0.0023
395518.6818	28047.1179	0.6242	0.0363
397856.4302	29712.7636	0.6714	0.0059
398839.8480	30465.1444	0.6874	0.0093
399782.5789	31411.7750	0.6873	0.0776
400439.1223	32092.7704	0.6874	0.1240
402444.0490	33256.4297	0.7437	0.0317
403819.2763	34337.0239	0.7642	0.0445
405086.3844	35292.9212	0.7858	0.0445
405935.4366	36976.0288	0.7437	0.2821
408811.9063	38209.7074	0.8423	0.0748
408938.3306	38309.9796	0.8441	0.0762
410781.1312	39640.3066	0.8796	0.0595
413137.3313	41276.8504	0.9301	0.0167
415205.6915	42802.3222	0.9682	0.0052
417306.0496	44458.0003	0.9987	0.0280
418011.1425	45141.6218	1.0000	0.0731
418382.7519	45942.5705	0.9751	0.2043
420477.7690	47749.5535	0.9987	0.2555
422942.6438	51719.9812	0.9502	0.6358
426162.6466	54240.9796	1.0000	0.6577
426268.5825	54416.4312	0.9982	0.6728
427569.0740	55978.4817	0.9987	0.7642
428627.0025	57948.5460	0.9761	0.9346
428989.2576	58278.2207	0.9802	0.9436
430934.8416	60109.0296	1.0000	1.0000

## 5 Conclusion

In this paper, based on the sensitivity of users to electricity price changes, considering the goal of minimizing system peak-to-valley load and maximizing user satisfaction, a multi-objective time-of-use electricity price model based on user response is established, which improves the feasibility of time-sharing electricity price scheme. It laid the foundation for establishing the largest expected model of power purchase. In the establishment of the largest model of expected revenue from electricity purchases, the inter-provincial contract and the electricity purchase ratio of the spot market were considered, and the conditional risk value based on the portfolio theory was introduced as the risk index, and the expected return benefit of the purchase of electricity and the risk of purchasing electricity were proposed. The minimized multi-objective optimization model realizes the joint optimization of the electricity purchase ratio scheme and the power purchase risk scheme, and coordinates the economic benefits of the user and the grid.

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