Stress-strain state of a composite near rectangular inclusion

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Abstract. In this research we developed a technique for calculating the stress-strain state of a model construction from a thermoplastic composite material with an embedded piezoactuator. Numerical simulations of the model construction stress-strain state with different arrangement of piezoactuators: upper and middle, were performed. Numerical simulations were carried out in a three-dimensional setting taking into account the complete technological scheme of laying and anisotropy of the properties of reinforcing layers. The results of numerical experiments revealed the areas of maximum stress. Recommendations for the MFC's embedding into composite materials were formulated.

1 Introduction

Recently, in aviation technology, there has been a trend towards the use of SMART structures with controlled geometry. This makes it possible to significantly improve the operational characteristics of aircraft, by reducing vibroacoustic loading of highly loaded elements such as the wing of an airplane or helicopter blades. The development of SMART structures from thermoplastic composite materials (TCM) based on built-in piezoelements will, on the one hand, reduce the weight of the structure, on the other hand, will allow to exclude mechanical drives and swash plates, which will significantly reduce the cost of maintenance of structures with controlled geometry and fuel consumption of aircrafts.

With great efficiency SMART structures can be used to create aviation wings with variable geometry, which will optimally adapt to the aerodynamic parameters of the air flow. It is known to use SMART technologies to damp vibrations with helicopter blades [1]. At the present time, the possibility of using SMART technologies for creating fan blades and a straightening device for an aircraft engine is considered.

Interest in the problem of creating aviation structures with controlled geometry on the basis of SMART technologies, in one respect, is due to the fact that the elemental base has recently expanded considerably. A large number of new piezoelements and piezoactuators on their basis appeared, as well as a number of compact systems for composite materials monitoring. Another reason is composites have become widely used in the design of aviation technology. For example, the American aircraft Boeing 787 Dreamliner and the European Airbus 350 XWB for more than 50% consist of composite materials that are used in the manufacture of highly loaded elements of the airframe and the engine [2].

It is the composite materials that give a wide range of possibilities for creating a whole new class of SMART structures that have self-diagnostic functions, self-recovery (in case of reversibility), geometry control. Technologically, they allow to embed various sensors and control elements (Figure 1a) into the material structure at the stage of fabrication of structures, thus creating various types of SMART structures (Figure 1b) capable of responding to external factors.

Thus, composite materials provide a unique opportunity to create materials not only with a set of properties, but also with properties that can be changed during operation. In this they differ drastically from traditional materials - metals and alloys.

Fig. 1. Piezoactuator (a) general view (b) in the structure of the controlled helicopter blade.

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At present, some works describing the theoretical basis for the creation of SMART structures from TCM with controlled geometry are known, in addition, there are cases when laboratory specimens and operating models of such structures were created. However, the scientific problem that limits the creation of SMART structures is the lack of scientifically based approaches and proven methods for their design, modeling, calculation, the lack of experimental technologies for their manufacturing and the lack of test methods. The problem of introducing piezoactuators into a structure remains unresolved. The introduction of a foreign body inside a layered composite structure can lead to a decrease in the strength of the structure, up to its destruction [3, 4]. To solve this problem, it is necessary to conduct a complex of computational studies aimed at developing new approaches and mathematical models of composite materials with piezoelectric structural elements (piezocomposites).

Within the framework of the present research, the problem of numerical calculation of the stress-strain state of model samples made of TCM is considered, with different versions of the piezoactuator arrangement: upper and in the center of the layers of the fragment of the composite material.

2 Numerical model

We surveyed the model samples consisting of six anisotropic layers of woven TCM with weaving of the textile framework 5H Satin. To prepare the samples, a HTA40-based prepreg was used with a volume content of 58% and a thermoplastic PEEK binder. The thickness of one layer of prepreg is 0.3 mm, height 1.8 mm, length and width 30 mm. The bending type actuator was set in the form of a parallelepiped with the dimensions of 10x10x0.3 mm. The volume content of the actuator in the TCM was 1.85%. In the framework of computational experiments, the following reinforcement schemes were considered [Actuator $0^0/90^0/45^0/-45^0/90^0/0^0$] and [$0^0/90^0/45^0$ /Actuator $-45^0/90^0/0^0$].

The mathematical formulation of the problem corresponded to the theory of elasticity of an anisotropic body. In the variational formulation this formulation consists in finding the minimum of the Lagrangian functional with additional conditions in the form of geometric Cauchy relations [5]. The variation of the functional in the absence of mass forces has the following form:

$$\delta J_u = \int_V \varepsilon_{ij} C_{ijkl} \delta \varepsilon_{kl} dV - \int_{S_f} F_i \cdot \delta u_i dS, \quad (1)$$

where ε_{ij} and $\delta \varepsilon_{ij}$ is the tensor and the variation of the strain tensor, C_{ijkl} is the tensor of the elastic modulus, δu_i is the variation of the displacement vector, and F_i is the vector of external forces.

Additional conditions for the functional (1) are geometric equations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (2)$$

For each orthotropic layer of the thermoplastic subdomain $V^{(l)}$, the components of the elastic modulus tensor $C_{ijkl}^{(l)}$ depend not only on the material, but also on its orientation relative to the global coordinate system of the structure.

We introduce the local coordinate system of the orthotropic carbon fiber layer $OX_1X_2X_3$. In it, the axis OX_1 coincides with the direction of the base, OX_2 - weft, OX_3 - perpendicular to the plane of the layer. In the established local coordinate system, the components of the tensor $\tilde{C}^{(l)}$ will be determined through the technical elastic constants of the layers according to the formulas [6]:

$$\begin{aligned} C_{1111}^{(l)} &= \frac{1}{E_{22}A} \left(\frac{1}{E_{33}} - \frac{\nu_{23}^2}{E_{22}} \right), \\ C_{2222}^{(l)} &= \frac{1}{E_{33}A} \left(\frac{1}{E_{11}} - \frac{\nu_{31}^2}{E_{33}} \right), \\ C_{3333}^{(l)} &= \frac{1}{E_{11}A} \left(\frac{1}{E_{22}} - \frac{\nu_{12}^2}{E_{11}} \right), \\ C_{1122}^{(l)} &= \frac{1}{E_{33}A} \left(\frac{\nu_{31}\nu_{23}}{E_{22}} + \frac{\nu_{12}}{E_{11}} \right), \\ C_{1133}^{(l)} &= \frac{1}{E_{33}A} \left(\frac{\nu_{12}\nu_{31}}{E_{11}} + \frac{\nu_{23}}{E_{33}} \right), \\ C_{2233}^{(l)} &= \frac{1}{E_{11}A} \left(\frac{\nu_{23}\nu_{31}}{E_{33}} + \frac{\nu_{12}}{E_{22}} \right), \end{aligned} \quad (3)$$

$$\begin{aligned} C_{2323}^{(l)} &= G_{23}^{(l)}, \quad C_{1313}^{(l)} = G_{13}^{(l)}, \\ C_{1212}^{(l)} &= G_{12}^{(l)}, \end{aligned}$$

$$A = \frac{1}{E_{11}E_{22}E_{33}} (1 - 2\nu_{12}\nu_{23}\nu_{31} - \frac{E_{11}}{E_{33}}\nu_{31}^2 - \frac{E_{22}}{E_{11}}\nu_{12}^2 - \frac{E_{33}}{E_{22}}\nu_{23}^2)$$

To convert the tensor $\tilde{C}^{(l)}$ components from the local coordinate system to the global one, we use the formula:

$$C_{i'j'k'l'}^{(l)} = C_{ijkl}^{(l)} \alpha_{i'i} \alpha_{j'j} \alpha_{k'k} \alpha_{l'l}, \quad (4)$$

where $\alpha_{i'i}$ is the matrix of the cosines of the angles between the direction of the axes of the local and global coordinate systems for each layer. In this case, the cosine matrices $\alpha_{ij}^{(k)}$ are used to convert components.

The elastic properties of transversally isotropic yarns based on carbon fibers and a thermoplastic binder used

in numerical modeling are taken from the previous work [5].

The sample considered is orthotropic and is characterized by nine independent elastic constants: Young's modulus E_X , E_Y , E_Z , Poisson's coefficients ν_{XY} , ν_{YZ} , ν_{XZ} , and shear modulus G_{XY} , G_{YZ} , G_{XZ} . Where x is the direction of the base, y is the direction of the weft. An ideal contact is used between the interacting threads of the model. For the representative fragment under consideration, six types of experiments were realized: stretching along the X , Y , Z axes to determine the effective Young's modules and Poisson's ratios and a net shift in the planes XY , XZ , YZ in order to determine the shear modules. Effective modules were obtained by volume averaging [4] using the following dependences:

$$E_x^* = \frac{\langle \sigma_X \rangle}{\varepsilon_X}, E_y^* = \frac{\langle \sigma_Y \rangle}{\varepsilon_Y}, E_z^* = \frac{\langle \sigma_Z \rangle}{\varepsilon_Z},$$

$$G_{XY}^* = \frac{\langle \tau_{XY} \rangle}{\gamma_{XY}}, G_{YZ}^* = \frac{\langle \tau_{YZ} \rangle}{\gamma_{YZ}}, G_{XZ}^* = \frac{\langle \tau_{XZ} \rangle}{\gamma_{XZ}}, \quad (5)$$

$$\nu_{XY} = \left| \frac{\varepsilon_Y}{\varepsilon_X} \right|, \nu_{YZ} = \left| \frac{\varepsilon_Z}{\varepsilon_Y} \right|, \nu_{XZ} = \left| \frac{\varepsilon_Z}{\varepsilon_X} \right|,$$

where the deformations ε_X , ε_Y , ε_Z , and γ_{ij} were predetermined in numerical experiments.

Table 1. Elastic properties of piezoactive element layers.

Material	E_x , hPa	E_y , hPa	E_z , hPa	ν_{xy}	ν_{yz}	ν_{xz}	G_{xy} , hPa	G_{yz} , hPa	G_{xz} , hPa
Prepreg	60.9	61.1	8.616	0.14	0.34	0.34	4.123	4.195	3.2
MFC	30	15.5	15.5	0.35	0.4	0.4	5.7	10.7	10.7

3 Results of numerical simulation

Based on the results of the numerical solution of relations (1-6), the fields of distribution of the normal (σ_{xx} , σ_{yy} , σ_{zz}) and the tangential (τ_{xy} , τ_{xz} , τ_{yz}) stresses were obtained, for the macrostrain level of 0.1% (Figure 2-13). To analyze the effect of an embedded piezoactive element with an upper location on the stress-strain state of the structure under consideration, stress diagrams were constructed, passing through the center of the MFC located in the first layer of the sample, in the laying plane of the layers (shown in Fig. 2-7 a).

The analysis of the results obtained for the upper location of the activator revealed that in the vicinity of the piezoactive element, the tension decreases along the X and Y axes for cases of tension. For stresses σ_{xx} the tension decreases by 35%, for σ_{yy} - by 65%. When shifting in the XY plane, the increase in the stresses τ_{xy} by 42% is observed. For normal tear-off stresses σ_{zz} , the increase of 32.7% on the layer-piezoactive element boundary is observed at the elongation along the Z axis. With shear in the planes XZ and YZ in the area of the piezoactive element, the stresses τ_{xz} and τ_{yz} increase by 45 and 39%, respectively.

The average stresses and deformations were determined from the following relations:

$$\langle \sigma_{ij} \rangle = \frac{\sum_{k=1}^n \sigma_{ij}^k V^k}{V}, \langle \varepsilon_{ij} \rangle = \frac{\sum_{k=1}^n \varepsilon_{ij}^k V^k}{V}; i, j = x, y, z, \quad (6)$$

where n is the total number of finite elements, σ_{ij}^k is the stress in the finite element, ε_{ij}^k is the deformation in the finite element, V^k is the volume of the finite element, V is the total volume of the model.

When constructing the finite element model, 20-node SOLID186 elements were used. Based on the results of the convergence study for this task, we determined the maximum and minimum size of a finite element. The maximum size is 1 mm, the minimum dimension is 0.5 mm. The calculated area was divided into 1 million elements. Calculation of the stress-strain state of the sample, for six boundary-value problems, was performed in the software complex Ansys Workbench Mechanical, using the software module developed by the authors. For each loading option, macrodeformations of 0.1% were set. The elastic effective properties of materials used in the numerical calculation are presented in Table 1.

The effective elastic properties of the monolayer of the layered composite were determined on the cell of the periodicity of the 5H Satin tissue in the studies performed earlier. The technical elastic constants of the piezo element are taken from the previous works [7].

Fig. 2. The stress fields of the normal stresses σ_{xx} (MPa) (b) and the stress distribution diagram σ_{xx} (a) under tension along the X axis, the sample with the upper position of the activator.

a)

b)

Fig. 3. The stress fields of the normal stresses σ_{yy} (MPa) (b) and the stress distribution diagram σ_{yy} (a) when stretching along the Y axis, a sample with an upper position of the actuator.

a)

b)

Fig. 5. The stress fields of the shear stresses τ_{xy} (MPa) (b) and the stress distribution diagram τ_{xy} (a) under shear in the XY plane, the sample with the upper position of the actuator.

a)

b)

Fig. 4. The stress fields of the normal stresses σ_{zz} (MPa) (b) and the stress distribution diagram σ_{zz} (a) when stretching along the Z axis, the sample with the upper position of the actuator.

a)

b)

Fig. 6. The stress fields of the shear stresses τ_{xz} (MPa) (b) and the stress distribution diagram τ_{xz} (a) under shear in the XZ plane, the sample with the upper position of the actuator.

a)

b)

Fig. 7. The stress fields of the shear stresses τ_{yz} (MPa) (b) and the stress distribution diagram τ_{yz} (a) under shear in the YZ plane, the sample with the upper position of the actuator.

a)

b)

Fig. 9. The stress fields of the normal stresses σ_{yy} (MPa) (b) and the stress distribution diagram σ_{yy} (a) when stretching along the Y axis, the sample with the middle position of the actuator.

a)

b)

Fig. 8. The stress fields of the normal stresses σ_{xx} (MPa) (b) and the stress distribution diagram σ_{xx} (a) when stretching along the X axis, the sample with the middle position of the actuator.

a)

b)

Fig. 10. The stress fields of the normal stresses σ_{zz} (MPa) (b) and stress distribution diagram σ_{zz} (a) when stretching along the Z axis, the sample with the middle position of the actuator.

Fig. 11. The stress fields of the shear stresses τ_{xy} (MPa) (b) and the stress distribution diagram τ_{xy} (a) under the shear in the XY plane, the sample with the middle position of the actuator.

Fig. 12. The stress fields of the shear stresses τ_{xz} (MPa) (b) and the stress distribution diagram τ_{xz} (a) under the shear in the XZ plane, the sample with the middle position of the actuator.

To analyze the influence introduced into the center of the layers of the piezoactuator on the stress-strain state, stress diagrams were constructed, (shown in Fig. 8-13 a). The distribution fields of the normal ($\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$) and the

tangential ($\tau_{xy}, \tau_{xz}, \tau_{yz}$) stresses of the sample with the average position of the actuator are shown in Fig. 2-7 b.

Fig. 13. The stress fields of the shear stresses τ_{yz} (MPa) (b) and the stress distribution diagram τ_{yz} (a) under the shear in the YZ plane, the sample with the middle position of the actuator.

The analysis of the results obtained for the middle location of the activator revealed that in the vicinity of the piezoactive element, for cases of stretching along the X axis, the increase in stress is observed, and for the stretching along the Y axis - the decrease. For stresses σ_{xx} the increase of 18% is observed, for stresses σ_{yy} - the decrease of 38%. When shifting in the XY plane, the increase in the stresses τ_{xy} by 42% is observed. For normal tear-off stresses, tensile stress along the Z axis shows the increase of 32.7% at the layer-piezoactive element boundary. With the shear in the planes XZ and YZ in the area of the piezoactive element, the stresses τ_{xz} and τ_{yz} increase by 54 and 37%, respectively.

The comparative analysis of the results of the sample obtained with the upper and middle positions of the actuator revealed that, with the middle position of the actuator, the increase in the maximum stresses τ_{xz} by 10% is observed. At the same time, for the sample with the upper location of the activator, the increase in the maximum stresses by 3.3% is observed.

Conclusion

Thus, within this research, a methodology for calculating the stress-strain state of a model sample from TCM with an embedded piezoactuator was developed. According to the developed methodology, the numerical calculation of the SSB of the sample with different arrangement of the piezoactuator (upper and middle) is carried out in a

three-dimensional setting taking into account the complete technological layout and anisotropy of the properties of the reinforcing layers and piezoactuator. According to the results of computational experiments, it is found that the region of maximum stresses is observed at the junction of layers of a thermoplastic and a piezoactuator. Analysis of the stresses in the layers of the model sample and TCM revealed that the most dangerous, defining the safety margin of the structure, are the interlayer stresses σ_{zz} and the interlayer tangential stresses τ_{xz} and τ_{yz} . Based on the results of the studies, the piezoactuator is recommended to be placed on the upper and lower surfaces of the sample.

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