Energy harvesting in a delayed Rayleigh harvester device

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Abstract. In the present work, we report on energy harvesting in a delayed Rayleigh oscillator coupled to a piezoelectric harvester device. Analytical investigation using the multiple scales method is performed to obtain approximation of the periodic and QP amplitude response. The influence of different parameters of the harvesting system on the output power coupling to periodic and QP vibrations is examined. Results show that for appropriate values of the delay parameters, QP vibrations can also be exploited to harvest energy with good performance.

1 Introduction

The efficiency of the energy harvesting (EH) has received great attention in the last few years. The main goal that EH wants to achieve is to reduce the requirements of an external source by considering simplified models of harvester device. One recent idea that has received growing attention is energy harvesting systems operating in a self-induced oscillations regime. In recent paper [1], QP vibration-based EH has been studied in a delayed van der Pol oscillator coupled to an electromagnetic EH device and later by considering a forced and delayed Duffing harvester device [2]. It was concluded that energy can be extracted from QP vibrations over broadband of excitation frequency away from the resonance.

A common method to design a wide band of EH is the inclusion of delay feedback in the position, in the velocity or in both. For instance, it was shown that for appropriate values of time delay and feedback gains the output power can be obtained from QP vibrations with a good performance [2,3]. Note that, the influence of time delayed feedback absorber has been used to enhance EH [4].

The purpose of this work is to study the EH performance in a delayed Rayleigh harvester device subjected to a harmonic excitation. The energy harvester system consists in a delayed Rayleigh oscillator coupled to an electrical circuit through a piezoelectric coupling mechanism. Special attention is paid to the effect of delay and coupling parameters on the EH performance taking advantage of large-amplitude QP vibrations produced in the system.

The paper is organized as follows: in section 2 we investigate periodic and the average power near the primary resonance. Section 3 is devoted to QP vibration-based EH and section 4 concludes the work.

2 Equation of motion and periodic energy harvesting

We consider a harvester device consisting of a nonlinear single spring-masse-damper system subjected to a harmonic

excitation near the primary resonance. The schematic model of the system is shown in Fig.1 and the governing equation for the harvester can be written in the dimensionless form as

Fig. 1. Description of the energy harvesting system.

$$\ddot{\eta}(t) + \omega_0^2 \eta(t) - [\alpha_1 \dot{\eta}(t) - \alpha_2 \dot{\eta}(t)^2] \dot{\eta}(t) - \theta v(t) = \lambda \dot{\eta}(t - \tau) + F \cos(\omega t)$$

$$\dot{v}(t) + \beta v(t) + \alpha \dot{\eta}(t) = 0 \quad (1)$$

where $\eta(t)$ is the relative displacement of the rigid mass m , $v(t)$ is the voltage across the load resistance, α_1 and α_2 are, respectively, the linear and nonlinear mechanical damping ratio, θ is the piezoelectric coupling term in the mechanical attachment, α is the piezoelectric coupling term in the electrical circuit, β is the reciprocal of the time constant of electrical circuit, F and ω are, respectively, the amplitude and the frequency of the excitation, while λ and τ are, respectively, the feedback gain and time delay.

To investigate the response of the harvester system (1) near the primary resonance we consider the resonance condition $\omega = \omega_0 + \sigma$ where σ is detuning parameter. The method of multiple scales [5] is implemented by introducing a bookkeeping parameter ε and scaling parameters as $\alpha_1 = \varepsilon \tilde{\alpha}_1$, $\alpha_2 = \varepsilon \tilde{\alpha}_2$, $\theta = \varepsilon \tilde{\theta}$, $\lambda = \varepsilon \tilde{\lambda}$, $F = \varepsilon \tilde{F}$, $\beta = \varepsilon \tilde{\beta}$, $\alpha = \varepsilon \tilde{\alpha}$, $\sigma = \varepsilon \tilde{\sigma}$. Thus, system (1) reads

$$\ddot{\eta}(t) + \omega_0^2 \eta(t) = \varepsilon \left[-\alpha_1 \dot{\eta}(t) - \alpha_2 \dot{\eta}(t)^3 + \theta v(t) + \right.$$

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$$\begin{aligned} & \lambda \dot{\eta}(t - \tau) + F \cos(\omega t) \\ & \dot{v}(t) + \beta v(t) + \alpha \dot{\eta}(t) = 0 \end{aligned} \quad (2)$$

To ultimately solve Eq. (2), steady-state solutions are expanded as

$$\begin{aligned} \eta(t) &= \eta_0(T_0, T_1, T_2) + \varepsilon \eta_1(T_0, T_1, T_2) + \\ & \quad \varepsilon^2 \eta_2(T_0, T_1, T_2) + O(\varepsilon^3) \\ v(t) &= v_0(T_0, T_1, T_2) + \varepsilon v_1(T_0, T_1, T_2) + \\ & \quad \varepsilon^2 v_2(T_0, T_1, T_2) + O(\varepsilon^3) \end{aligned} \quad (3)$$

where $T_0 = \tau$, $T_1 = \varepsilon \tau$ and $T_2 = \varepsilon^2 \tau$. In terms of the variables $T_i (i = 0, 1, 2)$, the time derivatives become $\frac{d}{dt} = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + O(\varepsilon^3)$ and $\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_1^2 + 2\varepsilon^2 D_0 D_2 + O(\varepsilon^3)$, where $D_i = \frac{\partial}{\partial T_i}$. Substituting (3) into (2), equating terms of different order of ε , resolving and eliminating the secular terms, we obtain the following slow flow of the amplitude and the phase:

$$\begin{cases} \frac{dr}{dt} = S_1 r + S_3 r^3 - h \sin \phi \\ r \frac{d\phi}{dt} = S_2 r - h \cos \phi \end{cases} \quad (4)$$

where $S_1 = -\frac{\alpha_1}{2} - \frac{D_c}{2\omega} + \frac{\lambda_s}{2\omega}$, $S_2 = \frac{\sigma}{2\omega} - \frac{\lambda_c}{2\omega} - \frac{D_s}{2\omega}$, $S_3 = -\frac{3}{8}\alpha_2\omega^2$, $h = \frac{F}{2\omega}$, $\lambda_c = \frac{\lambda\omega\cos\varphi}{\sqrt{\beta^2 + \omega^2}}$, $\lambda_s = \frac{\lambda\omega\sin\varphi}{\sqrt{\beta^2 + \omega^2}}$, $D_c = -\lambda\omega\cos\omega\tau$ and $D_s = \lambda\omega\sin\omega\tau$. Equilibria of this slow flow, corresponding to periodic solutions of Eq. (2), are determined by setting $\frac{dr}{dt} = \frac{d\phi}{dt} = 0$. This leads to the amplitude-frequency response equation:

$$S_3^2 r^6 + 2S_1 S_3 r^4 + (S_1^2 + S_2^2) r^2 - h^2 = 0 \quad (5)$$

An expression for the average power is obtained by integrating the dimensionless form of the instantaneous power $P(t) = \beta v(t)^2$ over the period of the excitation T . This is given by

$$P_{av}^P = \frac{1}{T} \int_0^T \beta v^2 dt \quad (6)$$

where $T = \frac{2\pi}{\lambda}$. Then, the average power expressed by $P_{av}^P = \frac{\beta v^2}{2}$ reads

$$P_{gen}^{av} = \frac{1}{2} \frac{\beta \alpha^2 \omega^2}{\beta^2 + \omega^2} r^2 \quad (7)$$

where the amplitude r is obtained from Eq. (5).

3 Quasi-periodic energy harvesting

Next, we use the second perturbation procedures to approximate periodic solutions of the slow flow (4) corresponding to QP responses of Eq. (2) and we analyze the influence of parameters on the QP output power. This can be done by transforming the modulation equations from the polar form (4) to the following Cartesian system using the variable change $u = a \cos \phi$ and $v = -a \sin \phi$

$$\begin{cases} \frac{du}{dt} = S_2 v + \mu \{S_1 u + S_3(u^2 + v^2)u\} \\ \frac{dv}{dt} = -S_2 u + h + \mu \{S_1 v + S_3(u^2 + v^2)v\} \end{cases} \quad (8)$$

where μ is a new bookkeeping parameter introduced in damping and nonlinearity. Following [1, 6], a periodic solution of the slow flow (8) can be sought in the form

$$\begin{aligned} u(t) &= u_0(T_1, T_2) + \mu u_1(T_1, T_2) + O(\mu^2) \\ v(t) &= v_0(T_1, T_2) + \mu v_1(T_1, T_2) + O(\mu^2) \end{aligned} \quad (9)$$

where $T_1 = t$ and $T_2 = \mu t$. Introducing $D_i = \frac{\partial}{\partial T_i}$ yields $\frac{d}{dt} = D_1 + \mu D_2 + O(\mu^2)$, substituting (9) into (8), collecting terms, resolving and eliminating secular terms, we obtain the following autonomous *slow-slow* flow system on R and ψ

$$\begin{aligned} \frac{dR}{dt} &= S_3 R^3 + \left(S_1 + \frac{S_3 h^2}{S_2^2}\right) R \\ R \frac{d\psi}{dt} &= 0 \end{aligned} \quad (10)$$

where R and ψ are, respectively, the amplitude and the phase of the slow flow limit cycle. Equilibria of this *slow-slow* flow corresponding to periodic solutions of the slow flow (8) determine the QP solution of the original equation (2). In addition to the trivial equilibrium, $R = 0$, the non-trivial equilibrium is obtained by setting $\frac{dR}{dt} = 0$ and given by

$$R = -\sqrt{\frac{S_1}{S_3} + \frac{h^2}{S_2^2}} \quad (11)$$

The approximate periodic solution of the slow flow (8) is given by

$$\begin{aligned} u(t) &= R \cos(\kappa t) + \frac{h}{S_2} \\ v(t) &= -R \sin(\kappa t) \end{aligned} \quad (12)$$

The approximate amplitude $a(t)$ of the QP oscillations is given by

$$a(t) = \sqrt{R^2 + h_1^2 + 2h_1 R \cos(\kappa t)} \quad (13)$$

while the modulation envelope of these QP oscillations is delimited by a_{min} and a_{max} given by

$$a_{min} = \min \left\{ \sqrt{R^2 + h_1^2 \pm 2h_1 R} \right\} \quad (14)$$

$$a_{max} = \max \left\{ \sqrt{R^2 + h_1^2 \pm 2h_1 R} \right\} \quad (15)$$

The average QP power is obtained from

$$P_{av}^{QP} = \frac{1}{2} \frac{\beta \alpha^2 v^2}{\beta^2 + v^2} a^2 \quad (16)$$

where a is derived from Eqs. (14), (15).

The influence of different system parameters on the maximum output powers is examined. In what follows, we fix the parameters $F = 0.1$, $\omega_0 = 1$, $\theta = 0.3$, $\beta = 0.1$, $\alpha = 0.2$, $\alpha_1 = 0.01$ and $\alpha_2 = 0.01$.

Figures 2, 3 and 4 show, respectively, the variation the output average power amplitudes versus the external frequency ω for fixed values of parameters and for $\lambda = 0$

(Fig.2), $\lambda = 0.07$ (Fig.3) and $\lambda = -0.07$ (Fig.4). The periodic response is given by (5) and the QP modulation envelope is obtained from Eqs. (18) and (19). Also, the average power for the periodic response is given by Eq. (7), while that for the QP response is deduced from (20). The solid lines represent stable solution and the dashed lines correspond to unstable ones. Figure 3 and 4 indicates

Fig. 2. Vibration and power amplitudes versus ω for $\lambda = 0$; analytical approximation: solid lines for stable and dashed line for unstable solution.

Fig. 3. Vibration and power amplitudes versus ω for $\lambda = 0.07$ and $\tau = 7.7$; analytical approximation: solid lines for stable and dashed line for unstable solution.

that for given values of the delay amplitude λ , the periodic vibration-based EH can be extracted in a small range near the resonance, thereby limiting considerably the bandwidth of the harvester. Instead, QP vibration-based EH can be extracted in relatively broadband of excitation frequencies far from the resonance. Also it can be observed that for positive values of λ , energy extracted from QP domain can be achieved for excitation frequencies prior to the resonance (left to the resonance peak; Fig 3b), while for negative values of λ QP vibration-based EH can be achieved beyond the resonance (right to the resonance peak; Fig.4).

Figure 5 represents analytical approximations of periodic and QP vibrations and inset in the figure is shown time

Fig. 4. Vibration and power amplitudes versus ω for $\lambda = -0.07$ and $\tau = 7.7$; analytical approximation: solid lines for stable and dashed line for unstable solution.

Fig. 5. Vibration amplitudes versus ω for $\tau = 7$ and (a): $\lambda = 0$ and (b): $\lambda = 0.05$; analytical approximation: solid lines for stable and dashed line for unstable solution; time histories corresponding to different regions.

history response using numerical simulations. Figure 6a shows the stability chart of the QP solution in the parameter plane (λ , τ) indicating the green regions where stable QP solutions exist and the gray region corresponding to unstable QP oscillations. In Fig. 6b are shown time histories response related to crosses (a), (b) and (c) picked from Fig. 6a.

4 Conclusions

We have studied EH performance in a delayed Rayleigh oscillator coupled to a piezoelectric harvesting device. The

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Fig. 6. Stability chart in the plane (λ, τ) and time histories corresponding to different regions; SQP (stable QP solutions); UQP (unstable QP solutions).

method of multiple scales is performed to obtain the slow-slow flows of the EH system. The periodic and QP solutions as well as the corresponding power amplitudes are obtained and the influence of different system parameters on the EH performance is reported. Results shown that for a small value of delay amplitude, the periodic vibration-based EH can be extracted in a narrow range around the resonance, while QP vibration-based EH can be extracted over wide ranges of excitation frequency away from the resonance at the left for positive values of the delay amplitude and at the right for negative ones.

References

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