

Analysis of non symmetric FG sandwich plates under Thermo-mechanical loading using a novel shear deformation theory with stretching effect

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Abstract. The analysis of non-symmetric functionally graded sandwich plates under thermo-mechanical loading is developed using a novel hyperbolic shear deformation theory and considering thickness stretching effects. This theory accounts for adequate distribution of the transverse shear strains in the thickness of the plate and satisfies the traction free boundary conditions on the top and bottom surface of the plates, thus a shear correction factor is not required. The governing equations of equilibrium of non-symmetric functionally graded sandwich plates can be obtained using virtual work principle and the closed form solutions are obtained by using Navier technique. The accuracy of the present results is established by comparing those with well known trigonometric shear deformation theories. The results are presented for deflections and stresses of non-symmetric simply supported square plates.

1 Introduction

Composite laminated materials are finding wide application in many engineering fields. Therefore, a number of approximate analytical and numerical methods have been developed in this area. Such composites are made of two or more materials to obtain a good structural performance, which the constituent does not show individually. Recently, advanced composite materials known as functionally graded material have attracted much attention in many engineering applications due to their advantages of being able to resist high temperature gradient while maintaining structural integrity [1]. The functionally graded materials (FGMs) are microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. They are usually made from a mixture of ceramics and metals to attain the significant requirement of material properties. FGMs are widely used in many structural applications such as mechanics, civil engineering, optical, electronic, chemical, mechanical, biomedical, energy sources,

nuclear, automotive fields and ship building industries to eliminate stress concentration and relax residual stresses and enhance bond strength.

Several studies have been performed to analyze the behavior of functionally graded plates and shells. Reddy [2] analyzed the static behavior of functionally graded rectangular plates based on his third-order shear deformation theory of plates.

Cheng and Batra [3] related the deflections of a simply supported functionally graded polygonal plate given by the first-order shear deformation theory and a third-order shear deformation theory to those of an equivalent homogeneous Kirchhoff plate. Cheng and Batra [4] also presented results for the buckling and steady-state vibrations of a simply supported functionally graded polygonal plate based on Reddy's plate theory. Loy et al. [5] studied the vibration of functionally graded cylindrical shells using the Love shell theory. Employing the sinusoidal shear formation theory (SSDT),

Zenkour [6] presented the Navier analytical solution for FG plates.

Li et al. [7] developed a three dimensional solution for free vibration of multi-layer FGM system-symmetric and unsymmetric FGM sandwich plates using the Ritz method. However, various higher-order shear deformation theories are developed using five unknown functions. The well-known higher-order plate theories with five unknown functions are as follows: (i) parabolic shear deformation plate theory (PSDPT) [8], (ii) sinusoidal shear deformation plate theory (SSDPT) [9], and (iii) exponential shear deformation plate theory (ESDPT) [10].

In this study, a novel higher order shear deformation theory for linear analysis of moderately thick functionally graded sandwich plate is examined. In this model the displacement varies as a hyperbolic function across the plate thickness and satisfies zero shear stress condition at the top and bottom surfaces of the plate, thus a shear correction factor is not required. The accuracy of the present results is established by comparing those with well known trigonometric shear deformation theory.

2 Problem formulation

Let us consider the case of a sandwich plate composed of three microscopically heterogeneous layers as shown in Fig. 1. Rectangular Cartesian coordinates are used to describe infinitesimal deformations of a three-layer sandwich elastic plate occupying the region $[0, a] \cdot [0, b] \cdot [-h/2, +h/2]$ in the unstressed reference configuration. The mid-plane of the composite sandwich plate is defined by $z = 0$ and its external bounding planes being defined by $z = \pm h/2$, the equations will be derived in tensorial notations and specialized afterwards for the problem under consideration. The top and bottom layers of non-symmetric sandwich plate are made of an isotropic homogeneous material ceramic and metal, respectively, while the core layer is made of a mixture of (ceramic and metal) materials.

The effective material properties for each layer, like Young's modulus and Poisson's ratio, can be expressed as:

$$P^{(n)}(z) = P_m + (P_c - P_m)V^{(n)} \quad (1)$$

where P_m and P_c denote the property of the bottom and top faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$ ($n = 1, 2, 3$). Note that P_m and P_c are, respectively, the corresponding properties of the metal and ceramic of the FGM sandwich plate.

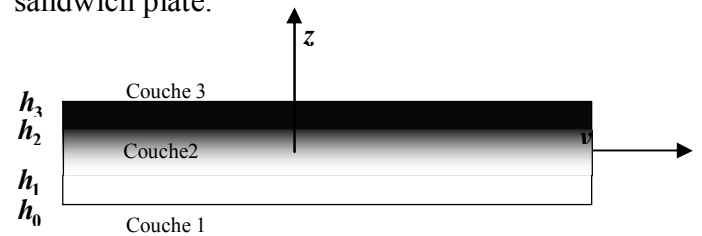


Figure 1. Geometry of the FGM sandwich plate.

3 Governing equations for FG sandwich plate

The displacement field for FG sandwich plates is described in the following equations:

$$u(x, y, z) = u_0(x, y) - zw_{,x} + f(z)\theta_x \quad (2a)$$

$$v(x, y, z) = v_0(x, y) - zw_{,y} + f(z)\theta_y \quad (2b)$$

$$w(x, y, z) = w_0(x, y) + \hat{f}(z)\theta_z \quad (2c)$$

where, u, v, w are displacements in the x, y, z directions, u_0, v_0 and w_0 are midplane displacements, θ_x, θ_y and θ_z rotations of the yz, xz , and xy planes due to bending, respectively. $f(z)$ represents the shape function determining the distribution of the transverse shear strains and stresses along the thickness. It is given by

$$f(z) = \frac{\cosh(\pi/2)}{[\cosh(\pi/2)-1]}z - \frac{(h/\pi)\sinh\left(\frac{\pi}{h}z\right)}{[\cosh(\pi/2)-1]} \quad (2d)$$

with $\theta_z \neq 0$.

The linear strain expressions derived from the displacement model are as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + f(z) \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} \quad (3a)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = f'(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \quad (3b)$$

Using the principle of virtual work, the following expressions can be obtained:

$$\int_{-h/2}^{h/2} \int_{\Omega} [\sigma_x \delta \epsilon_x + \sigma_y \delta \epsilon_y + \sigma_z \delta \epsilon_z + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz}] d\Omega dz - \int_{\Omega} q \delta \quad (5)$$

The governing equations of equilibrium can be derived easily from Equation (5) which is expressed as.

$$[C]\{\Delta\} = \{F\} \quad (6)$$

where $\{\Delta\} = \{U, V, W, X, Y, Z\}$ and $[C]$ is the stiffness matrix and $\{F\}$ is the force vector.

4 Numerical procedure

The thermomechanical bending analysis is conducted for combinations of metal and ceramic. For simplicity, Poisson's ratio of the two materials is assigned the same value.

Table1 contains the dimensionless center deflection \bar{w} for a FG sandwich plate subjected to thermo-mechanical loads. The deflections are considered for $k=3$ and different FG plate types. It can be seen from the table1 that the results of the present theory are very close to those of the other shear deformation theories. It can be observed that the HSDP overestimates the deflections comparatively to RHSDT and this, is due to the thickness stretching effect. For a sandwich plate, the deflections decrease when the dimensionless ratio (a/b) increases.

Table 2 shows the influence of power law index k , and different plate types on the transverse shear stresses of non-symmetric sandwich plates. As is observed, the increase of the power law index k leads to a decrease of stresses of the non-symmetric sandwich plates, and the thickness stretching effect leads also to a reduction of stresses.

Table1. Effect of aspect ratio a/b on the dimensionless deflection of the FGM sandwich plates ($k=3$)

Schem	Theory	a/b=1	a/b=2	a/b=3	a/b=4
(0-1-0)	RHDPT	0.991418	0.386195	0.189408	0.109104
	HDPT	1.04333	0.410224	0.203668	0.119285
	SSDPT	1.01673	0.399467	0.198203	0.115999
	TSDPT	1.03131	0.405363	0.201199	0.117801
	FSDPT	1.14369	0.450781	0.224252	0.131638
(1-3-1)	RHDPT	0.962845	0.376115	0.184793	0.106656
	HDPT	0.997264	0.392717	0.195100	0.114330
	SSDPT	0.971882	0.382438	0.194076	0.111191
	TSDPT	0.985791	0.388073	0.192741	0.112913
	FSDPT	1.09287	0.431354	0.214671	0.126040
(2-1-2)	RHDPT	0.959755	0.375429	0.184632	0.106685
	HDPT	0.990245	0.390314	0.194009	0.113755
	SSDPT	0.965082	0.380135	0.188853	0.110670
	TSDPT	0.978873	0.385715	0.191680	0.112363
	FSDPT	1.08505	0.428589	0.213340	0.125275
(1-0-1)	RHDPT	0.959500	0.375360	0.184615	0.106689
	HDPT	0.989882	0.390189	0.193956	0.113731
	SSDPT	0.964718	0.380017	0.188807	0.110654
	TSDPT	0.978509	0.385593	0.191630	0.112341
	FSDPT	1.08468	0.428454	0.213275	0.125235

Table2. Dimensionless transverse shear stresses τ_{xz} of the different sandwich square plates

k	Theory	τ_{xz}		
		(0-1-0)	(1-3-1)	(2-1-2)
0	RHDPT	0.142430	0.205538	0.170443
	HDPT	0.175814	0.227649	0.205372
	SSDPT	0.1716038	0.235615	0.198813
	TSDPT	0.1744812	0.231653	0.203034
	FSDPT	0.1736235	0.191050	0.212861
1	RHDPT	0.136266	0.140042	0.140973
	HDPT	0.169464	0.171405	0.171788
	SSDPT	0.163779	0.165731	0.166155
	TSDPT	0.167460	0.169410	0.169807
	FSDPT	0.174786	0.175641	0.176173
2	RHDPT	0.114226	0.122363	0.124937
	HDPT	0.149472	0.153330	0.152449
	SSDPT	0.140626	0.146338	0.147845
	TSDPT	0.146059	0.150751	0.150873
	FSDPT	0.165933	0.160245	0.154643
3	RHDPT	0.104469	0.114044	0.116725
	HDPT	0.140350	0.144499	0.142463
	SSDPT	0.130468	0.137199	0.138410
	TSDPT	0.136471	0.141781	0.141103
	FSDPT	0.160582	0.151749	0.143556
4	RHDPT	0.100046	0.109840	0.112740
	HDPT	0.136337	0.140173	0.137604
	SSDPT	0.126029	0.132632	0.133851
	TSDPT	0.132264	0.137348	0.136362
	FSDPT	0.158488	0.147725	0.138061
5	RHDPT	0.0981636	0.107666	0.110923
	HDPT	0.134806	0.138122	0.135375
	SSDPT	0.124290	0.130319	0.131795
	TSDPT	0.130635	0.135185	0.134204
	FSDPT	0.158235	0.146089	0.135419

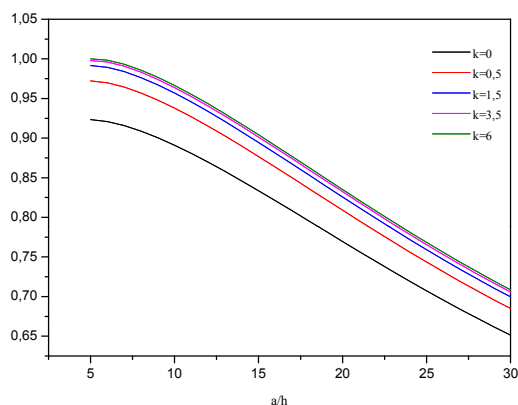


Fig.2 Dimensionless center deflection \bar{w} as a function of side-to-thickness ratio a/h for sandwich plate type (1-3-1)

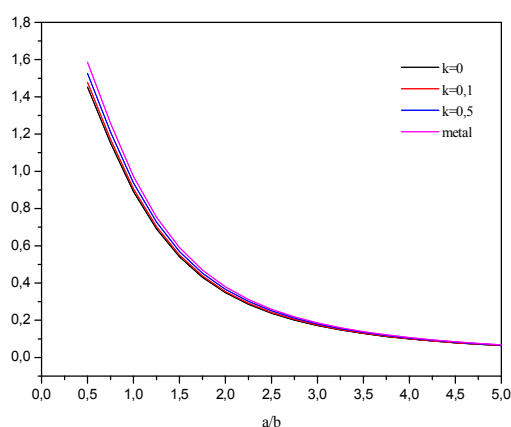


Fig.3 : Effect of the aspect ratio a/b on dimensionless center deflection \bar{w} for sandwich plate type (1-3-1)

Figure 2 shows the variation of the center deflection \bar{w} with side-to-thickness ratio a/h of sandwich plate type (1-3-1) with different volume fraction exponent k . The deflections of the sandwich plates decrease as a/h increases. Figure 3 shows the variation of the center deflection \bar{w} with the aspect ratio a/b of sandwich plate type (1-3-1). The deflection of the ceramic plate is found to have the smallest magnitude and that of the metallic plate the largest one. The increase of the aspect ratio a/b leads to a decrease of deflections of the homogeneous and FG sandwich plates.

Conclusion

A novel hyperbolic shear deformation theory for thermo-mechanical bending of non-symmetric sandwich plates is presented. The theory accounts for the stretching and shear deformation effects without requiring a shear correction factor. The

gradation of properties through the thickness is assumed to be of the power law type and comparisons have been made with homogeneous isotropic plates. The governing equations and boundary conditions are derived by employing the principle of virtual work. These equations are then solved via Navier-type solution. Numerical results for stresses and deflection are obtained and investigated for different plate configurations. The effect of power-law index, span to thickness ratio and aspect ratio are studied. It has been confirmed that the inclusion of thickness stretching effect makes the plate stiffer, and hence, leads to a reduction of deflections and stresses.

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