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Crack identification based on the nonlinear response of plates with variably oriented surface crack.

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Abstract. In order to secure structural and operational safety of structures, it is important to implement a structural health monitoring (SHM) strategy to issue early warnings on damage or deterioration prior to costly repair or even catastrophic collapse. Developing a SHM strategy for structures enables evaluating structural integrity, durability and reliability of the monitored structure. Hence, the main objective of this work is to develop a damage detection procedure based on a plate's dynamic response and the Hilbert transform. Rectangular plates are considered and assumed to contain a surface crack which is centrally located, with a depth of h_0, a length of 2C and inclined with an angle β. Von Karman plate theory is adopted herein, and the crack is modeled through the line spring model given by fracture mechanics. The plate is assumed to behave nonlinearly due to large deformation. The differential quadrature method is used to investigate the linear and nonlinear dynamic behaviors of cracked plates. The influence of crack's parameters on modal properties is discussed. The eigenfrequencies of cracked plates with respect to crack half length C and orientation β are performed. For crack characterization, Hilbert transform is applied to the obtained linear and nonlinear time responses. It is shown throughout this paper that identified backbones describe changes in crack orientation.

1 Introduction

Vibration based structural health monitoring is the process of identifying the health of structures based on changes in dynamic behavior caused by damage [1]. Many techniques have been extensively discussed in literature such as: optimization techniques, regression models, signal processing, etc. Thanks to its use for vibration signal's identification [2], the Hilbert Transform is adopted in this work to detect crack presence. The dynamic behavior of cracked structures was described by many models, for 1D solids, i.e. beams containing one or multiple open cracks [3, 4], or breathing ones. Vibration of damaged 2D solids such as plates and shells was investigated in case of horizontal crack [5,6]. The oriented crack case was considered in [7,8] where the line spring model was adopted and the flexibilities were calculated based on fracture mechanics developments [9,10].
To the best of the authors' knowledge crack identification of plates is not well addressed as beams, hence Hilbert transform is investigated in this paper, where performed nonlinear forced responses are Hilbert transformed. The free vibration of the cracked plate is first investigated numerically by using the differential quadrature method. It is shown that the developed numerical procedure is accurate since it matches well with the semi-analytical results and the identified backbones describe changes in crack orientation.

2 Vibration of plates with variably oriented surface crack

A rectangular cracked plate of dimensions L_x, L_y and H and material properties of E, ρ, υ, is considered that is subjected to bending and tensile stresses. The plate is assumed to contain a crack at the centre of its top surface, inclined with an angle β with respect to x axis, having as depth h, and length 2C (see figure 1).

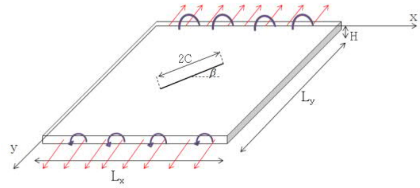

Fig. 3. Rectangular plate with inclined surface crack

For transverse vibration analysis, the considered problem is modeled through the following partial differential equation

$$
\begin{aligned}
&D\nabla^4 w=-\rho H\frac{\partial^2 w}{\partial t^2}+N_x\frac{\partial^2 w}{\partial x^2}+F\\
&+\frac{CD(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\left(\frac{\partial^4 w}{\partial y^4}+\upsilon\frac{\partial^4 w}{\partial x^2\partial y^2}\right)\\
&\quad+\frac{2CD\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\left(\frac{\partial^4 w}{\partial x\partial y^3}+\upsilon\frac{\partial^4 w}{\partial x^3\partial y}\right)\\
&-\frac{C(1+\cos(2\beta))}{(6\alpha_{bt}+\alpha_{tt})(1-\upsilon^2)h+2C}N_0\frac{\partial^2 w}{\partial y^2}-\frac{2C\sin(2\beta)}{(6C_{bt}+C_{tt})(1+\upsilon)h+2C}N_0\frac{\partial^2 w}{\partial y\partial x}
\end{aligned}
\tag{1}
$$

where $w=w(x,y,t)$ is the transverse displacement response, ∇^4 is the bi-harmonic operator and $D=\frac{Eh^3}{12(1-\upsilon^2)}$ is the plate's flexural rigidity. N_x is the in plane force per unit length and N_0 is the in plane force caused by the presence of the crack. The compliance coefficients α_{bb} , α_{tt} and $\alpha_{bt}=\alpha_{tb}$ are used to match the stretching and bending resistance for symmetric loading and they can be found in [9]. The other compliance coefficients C_{bb}, C_{tt} and $C_{bt}=C_{tb}$ can be found in [10], they are chosen to take into account the anti-symmetric mode loading.

The boundary conditions that must be satisfied by an edge parallel to **x** edge for example are as follows:

$$\text{Clamped edge: } w=0\ ;\ \frac{\partial w}{\partial y}=0 \tag{2-a}$$

$$\text{Pinned edge: } w=0\ ;\ M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)=0 \tag{2-b}$$

$$\text{Free edge: } M_y=0\ ;\ \left(\frac{\partial^3 w}{\partial y^3}+(2-\nu)\frac{\partial^3 w}{\partial y\partial x^2}\right)=0 \tag{2-c}$$

3 Free vibrations of cracked plates based on the differential quadrature method (DQM)

Thanks to the efficiency of the differential quadrature method for solving partial and differential equations, it is used here for plate's problem. A short overview is first given and the DQM formulation for the inclined cracked plate problem is presented.

3.1 An overview of the DQM

The DQM requires to discretize domain of the problem into N points. The derivatives at any point are approximated by a weighted linear summation of all the functional values along the discretized domain, as follows [3]

$$
\left.\frac{d^m f(x)}{dx^m}\right|_{x=\bar{z}_k}=\frac{d^m}{dx^m}\begin{bmatrix} f(\bar{z}_1)\\ f(\bar{z}_2)\\ f(\bar{z}_3)\\ .\\ .\\ .\\ f(\bar{z}_N)\end{bmatrix}=c_{kj}^{(m)}\begin{bmatrix} f(\bar{z}_1)\\ f(\bar{z}_2)\\ f(\bar{z}_3)\\ .\\ .\\ .\\ f(\bar{z}_N)\end{bmatrix}
\tag{3}
$$

where 'm' is the order of the highest derivative appearing in the problem, $f(\bar{z}_k)$ are the values of the function at the sampling points $\bar{z}_k$ relating the m[th] derivative to the functional values at $\bar{z}_k$. These coefficients can be determined by making use of Lagrange interpolation formula as follows:

$$f(x)=\sum_{i=1}^{N}\frac{L(x)}{(x-\bar{z}_i)L_1(\bar{z}_i)} \tag{4}$$

where: $L(x)=\prod_{j=1}^{N}(x-\bar{z}_j),\quad L_1(\bar{z}_i)=\prod_{j=1}^{N}(\bar{z}_i-\bar{z}_j)$

The weighting coefficients for the first order derivative to the functional values at $\bar{z}_k$ can be obtained as:

$$
c_{kj}^{(1)}=\begin{cases}\dfrac{L_1(\bar{z}_i)}{(\bar{z}_k-\bar{z}_j)L_1(\bar{z}_j)} & k\neq j\\[2ex] -\displaystyle\sum_{j=1,j\neq k}^{N}c_{kj}^{(1)} & k=j\end{cases}
\tag{5}
$$

The second, third and higher derivatives can be calculated as:

$$c_{kj}^{(m)}=\sum_{l=1,j\neq k}^{N}c_{kl}^{(1)}c_{lj}^{(m-1)} \tag{6}$$

For accurate results, we adopt the Chebychev-Gauss-Lobatto mesh distribution given for the interval [a, b] by:

$$\bar{z}_k=\frac{(b-a)}{2}\left[1-\cos\left(\frac{k-1}{N-1}\pi\right)\right]\quad k=1,2,\ldots,N \tag{7}$$

It should be noted that this method has been used and deeply tested in case of multi-cracked beams [3, 4].

3.2 Free vibration analysis using DQM

The free vibration problem is first solved by using 2D differential quadrature method. By assuming a harmonic motion, the time-space partial differential equation (1) is reduced to the following partial differential equation :

$$
\begin{aligned}
&D\nabla^4 w=-\rho H\omega^2 w+\frac{CD(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\left(\frac{\partial^4 w}{\partial y^4}+\upsilon\frac{\partial^4 w}{\partial x^2\partial y^2}\right)\\
&\quad+\frac{2CD\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\left(\frac{\partial^4 w}{\partial x\partial y^3}+\upsilon\frac{\partial^4 w}{\partial x^3\partial y}\right)
\end{aligned}
\tag{8}
$$

In order to use the DQM, the plane of the plate is discretized by an (n_e x m_e) Gauss-Lobatto-Chebychev grid. At a given point **(x_i ,y_j)**, of the plate, each partial derivative in Eq.(10) is written, in the DQM analog, as follows:

$$\left.\frac{\partial^p w(x,y)}{\partial x^p}\right|_{x=x_i}=\sum_{k_x=1}^{ne}c_{ik_x}^{(p)}w(x_{k_x},y_j) \tag{9-a}$$

$$\left.\frac{\partial^q w(x,y)}{\partial y^q}\right|_{y=y_j} = \sum_{k=1}^{me} b_{jk_y}^{(q)} w(x_i, y_{k_y}) \qquad (9\text{-b})$$

$$\left.\frac{\partial^{p+q} w(x,y)}{\partial x^p \partial y^q}\right|_{y=y_j} = \sum_{k_x=1}^{ne} c_{ik_x}^{(m)} \sum_{k_y=1}^{me} b_{jk}^{(m)} w(x_{k_x}, y_{k_y}) \qquad (9\text{-c})$$

After some mathematical developments and using a matrix form, the following eigenvalue problem is obtained

$$[K]\{w\} = \Omega^2 \{w\} \qquad (10)$$

After some mathematical developments and boundary conditions incorporation, one gets:

$$\begin{bmatrix} A & A_b^t \\ A_b & 0 \end{bmatrix} \begin{bmatrix} \{w\} \\ 0 \end{bmatrix} = \Omega^2 \begin{bmatrix} \{w\} \\ 0 \end{bmatrix} \qquad (11)$$

The matrix A_b is the matrix generating the considered boundary conditions. This resulting eigenvalue problem is to be solved; and $\{w\}$ is a (n_e x m_e) row vector given by :

$$\{w\}^T = \{w(x_1,y_1) \; ... \; w(x_1,y_{me}) \; ..... \; w(x_{ne},y_1) \; ...w(x_{ne},y_{me})\} \qquad (12)$$

The numerical solution of this eigenvalue problem allows getting the eigenfrequencies and eigenmodes of plates with various crack characteristics. The impact of both crack length and orientation on natural frequencies and eigenmodes are investigated.

4 Nonlinear free response of cracked plates with inclined crack

In order to solve the nonlinear vibration problem, Galerkin method is first used:

$$w(x, y, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} u_i(x) v_j(y) q_{ij}(t) \qquad (13)$$

where $u_i(x)$ and $v_j(y)$ are the eigenmodes parts in x and y directions of the cracked rectangular plate, respectively. By inserting Eq.(13) into Eq(1), the following equation is obtained:

$$\begin{aligned} D\nabla^4 w = \rho H \omega^2 w(x,y) &+ \frac{CD(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C} \frac{\partial^4 w}{\partial y^4} \\ &+ \frac{CD\upsilon(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C} \frac{\partial^4 w}{\partial x^2 \partial y^2} \\ &+ \frac{2CD\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C} \frac{\partial^4 w}{\partial x \partial y^3} \\ &+ \frac{2CD\upsilon\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C} \frac{\partial^4 w}{\partial x^3 \partial y} \end{aligned} \qquad (14)$$

The resulting in-plane forces are formulated according to Berger formulation as follows [7]:

$$\begin{cases} N_x = \dfrac{6D}{h^2 L_x L_y} q_{ij}^2(t) \int_0^{L_x}\int_0^{L_y} \left[\left(\dfrac{\partial u_i(x)}{\partial x}\right)^2 v_j^2(y) + \upsilon \left(\dfrac{\partial v_j(y)}{\partial y}\right)^2 u_i^2(x) \right] dxdy \\ N_0 = \dfrac{6D}{h^2 L_x L_y} q_{ij}^2(t) \int_0^{L_x}\int_0^{L_y} \left[\left(\dfrac{\partial v_j(y)}{\partial y}\right)^2 u_i^2(x) + \upsilon \left(\dfrac{\partial u_i(x)}{\partial x}\right)^2 v_j^2(y) \right] dxdy \end{cases} \qquad (15)$$

Based on the 1-mode analysis and by multiplying both sides of Eq(14) by $u_i(x)$ and $v_i(y)$ and integrating over the plate's area, the following nonlinear differential equation is obtained:

$$M_{ii}\ddot{q}_{ii}(t) + \mu \dot{q}_{ii}(t) + K_{ii} q_{ii}(t) + G_{ii} q_{ii}^3(t) = 0 \qquad (16)$$

Note that a weak classical linear viscous damping μ is introduced. M_{ii} is the modal mass and K_{ii}, G_{ii} are modal rigidity of the cracked plate respectively. They are given by:

$$M_{ii} = \frac{\rho H}{D} \int_0^{L_x}\int_0^{L_y} u_i^2(x) v_i^2(y) dxdy \qquad (17\text{-a})$$

$$K_{ii} = \int_0^{L_x}\int_0^{L_y} \begin{pmatrix} u_i^{iv}(x)v_i(y) + 2u_i''(x)v_i''(y) + u_i(x)v_i^{iv}(y) \\ -\dfrac{CD(1+\cos(2\beta))}{((\alpha_{bt}/6)+\alpha_{bb})(3+\upsilon)(1-\upsilon)h+2C}\left(u_i(x)v_i^{iv}(y) + \upsilon u_i^{iv}(x)v_i(y)\right) \\ -\dfrac{2CD\sin(2\beta)}{((C_{bt}/6)+C_{bb})(1+\upsilon)h+2C}\left(u_i'(x)v_i'''(y) + \upsilon u_i'''(x)v_i'(y)\right) dxdy \end{pmatrix} u_i(x)v_i(y)dxdy \qquad (17\text{-b})$$

$$G_{ii} = \int_0^{L_x}\int_0^{L_y} \begin{pmatrix} -P_{1ii}u_i''(x)v_i(y) + \dfrac{C(1+\cos(2\beta))}{(6\alpha_{bt}+\alpha_{tt})(1-\upsilon^2)h+2C} P_{2ii}u_i(x)v_i''(y) \\ +\dfrac{2C\sin(2\beta)}{(6C_{bt}+C_{tt})(1+\upsilon)h+2C} P_{2ii}u_i'(x)v_i'(y) \end{pmatrix} u_i(x)v_i(y)dxdy \qquad (17\text{-c})$$

Numerical solution of the obtained nonlinear differential equation is performed for different crack parameters and various initial conditions. The resulting nonlinear time response is analyzed using the Hilbert transform.

5 Crack detection using Hilbert transform

The Hilbert Transform (HT) of a signal s(t), is an integral transformation, from time domain to time domain, defined by [2] :

$$H(s(t)) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{s(\tau)}{t-\tau} d\tau \qquad (18)$$

The real signal s(t) and its HT h(t), form an analytical complex signal $\tilde{S}(t)$ of the form :

$$\tilde{S}(t) = s(t) + i\,h(t) = A(t)e^{i\theta(t)} \qquad (19)$$

The instantaneous amplitude $A(t)$ and phase $\theta(t)$ change with time. They are given by:

$$A(t) = \sqrt{(s(t))^2 + (h(t))^2} \quad ; \quad \theta(t) = \mathrm{Arctan}(\frac{h(t)}{s(t)}) \qquad (20)$$

Instantaneous frequency (IF) is defined as the derivative of the phase:

$$\omega(t) = \frac{d\theta(t)}{dt} \qquad (21)$$

It measures the rate and direction of a phase in the complex plane. It can be estimated by different algorithms [2], the one adopted in this work is based on the following formulae:

$$\omega(t) = \frac{s(t)\dot{h}(t) - \dot{s}(t)h(t)}{2\pi(s^2 + h^2)} \qquad (22)$$

The 'Freevib' method is developed by Feldman [2] to analyze systems' vibrations. Based on the Hilbert Transform, it offers a direct way to plot the system skeleton curve, which includes modal frequency and non-

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linearity in spring characteristic. The 'Freevib' method proposes equations for identifying instantaneous characteristics of the dynamic system (Eq.20 and Eq.22)

5 Numerical results and discussion

In this section a rectangular simply supported (SSSS) cracked plate is considered. The isotropic material properties are taken as E=7.03 10^9 N/m^2, ρ=2660 kg/m^3, υ=0.3. In order to prove accuracy of the developed numerical method, linear eigenfrequencies are first calculated by using DQM and compared with literature. Note that the plate is discretized into (12x12) points. According to Table 1, it is shown that the method is accurate, and the results match very well with the semi-analytical ones given in [7].

Table 1: Calculated ω_1 for different β and C

	Lx=0.3 Ly=0.3			
	C=0.003 (m)		C=0.0075 (m)	
	Present (DQM)	[7]	Present (DQM)	[7]
β=0	251.81	251.81	244.649	244.65
β=20	252.607	252.61	246.291	246.32
β=40	254.618	254.64	250.440	250.51
β=60	256.905	256.92	255.135	255.18
β=80	258.396	258.40	258.184	258.19
β=90	258.601	258.60	258.601	258.60

By calculating eigenfrequencies of the cracked plate, it is judicious to investigate influence of crack parameters on the modal frequency. The first normalized eigenfrequency is depicted in figure 2 with respect to crack length and orientation. It is shown that a plate with a crack, either of length C=0 or β=90°, behaves as an undamaged one. The horizontal crack (β=0°) is the most severe one. In order to illustrate the applicability of the HT to crack identification, the former plate is considered. The plate's dimensions are L_x=L_y=1; H=0.01. The crack is assumed to have a length of 2C=0.4, and a depth h_0 = 0.6H. The obtained forced responses are analyzed by using the Hilbert transform, and then the instantaneous frequency and amplitude are calculated. By relating the instantaneous frequencies to instantaneous amplitudes, the backbone curves corresponding to the nonlinear free vibration behavior are obtained.
It is shown that the identified backbones by the Hilbert Transform describe the cracked plate nonlinear free vibration behavior. The presence of the crack is indicated since the plate's identified linear eigenfrequency is decreasing. According to figure 3, the horizontal crack (β=0°) is the most severe and the plate nonlinear behavior is clearly affected by the crack orientation. According to obtained curves, it is shown that a plate with vertical crack (β=90°) behaves as an undamaged one.

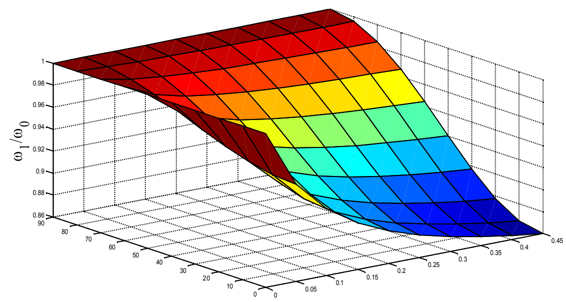

Fig. 2. First eigen frequency with respect to crack half length C and orientation β

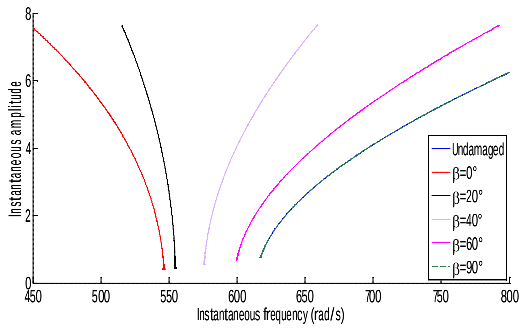

Fig. 3. Identified backbones for different crack angles

6 Conclusion

Through this work, the numerical study of cracked plate's vibrations is performed based on the differential quadrature method. The nonlinear free response of the cracked plate is investigated as well. Based on the obtained time responses and the Hilbert transform, identified backbones can describe the cracked plate nonlinear behavior due to the in plane forces. The obtained backbones indicate the presence of damage since the identified modal frequency is decreasing. Moreover, the plate's nonlinear behavior is affected by the presence of the crack. In a further work, dynamic behavior of plates with different excitation types can be investigated, as well as the quantification of crack parameters based on the nonlinear response of the plate.

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