Numerical Application of a Stick-Slip Control and Experimental Analysis using a Test Rig

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Abstract. Part of the process of exploration and development of an oil field consists of the drilling operations for oil and gas wells. Particularly for deep water and ultra deep water wells, the operation requires the control of a very flexible structure which is subjected to complex boundary conditions such as the nonlinear interactions between drill bit and rock formation and between the drill string and borehole wall. Concerning this complexity, the stick-slip phenomenon is a major component related to the torsional vibration and it can excite both axial and lateral vibrations. With these intentions, this paper has the main goal of confronting the torsional vibration problem over a test rig numerical model using a real-time conventional controller. The system contains two discs in which dry friction torques are applied. Therefore, the dynamical behaviour were analysed with and without controlling strategies.

1 General Introduction

1.1 Oilwell drilling and test-rig model overview

The actual organizational model of oil and gas industry is no longer sustainable with oil prices below \$50 a barrel. Christopher *et al.* (2016) [1] describes what they call the “potentially game-changing disruptions that may lead oil and gas companies to rethink their operating models fundamentally”. Then, to address the industry challenges, this paper focuses its efforts on modelling conventional drilling operations using a reduced scale set-up model.

Briefly, the drilling process involves rock failure by a rotating **drill bit**. To rotate this drill bit from the top-end position (surface), the drilling rig’s power source is used to turn a **rotary table**. A mechanism, consisting of **master bushing** and **kelly** is used to transmit the rotation from the rotary table (or **top drive**) to the **drill string**. The drill string is the subsurface component through which torque is transmitted from the surface to the bottom of the drilling system (downhole). It consists of connected lengths of **drill pipes**, the **bottom hole assembly (BHA)**, and the **drill bit**. The BHA is the portion of the drill string between the drill pipe and the drill bit. It is made up, primarily, of **drill collars (DC)** and **heavy weight drill pipe (HWDP)**.

Previous studies have identified three types of vibrations that may occur during drilling operations: **Torsional**, **Lateral** and **Axial**, as cited in [2–4].

In order to reproduce the phenomena inherent to the torsional vibrations present in the rotating drilling system

a reduced-scale test rig set-up model (shown in Fig. 1) was developed at the PUC-Rio Dynamics and Vibration Laboratory. The set-up model consists of an electrical and a mechanical subsystems. Electrical subsystem represents the drilling rig’s power source and the rotary table (or top drive), while the mechanical system represents the drill pipes, BHA and drill bit.

The test rig consists of three degrees of freedom: disc 1 (R1), disc 2 (R2) and the DC-motor inertia. Also, it contains an electric system which provides torque to the mechanical system.

Figure 1. Electromechanical system set-up model.

The electric subsystem is modelled as a voltage source connected in series with a resistor and an inductor, providing torque T_m . The angular velocity Ω_m imposed by T_m is eight times greater than the angular velocity Ω_3 transmitted to the mechanical subsystem due to the transmission factor $\eta = 8 : 1$.

The system presents two resistive torque sources. At disc 1 (R1), the friction means the bit/rock friction interaction. The intermediate friction source (at disc 2) may

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represent a friction raise from the interaction between drill pipes and borehole wall.

1.2 Motivation and objectives

Since vibration problems were detected and identified in drilling operations, several approaches have been suggested, both in industry and literature, to model and control these vibrations. Some of these approaches were conducted to the surface system (electric subsystem). Most of them dealt with the torsional behaviour and the suppression of the stick-phase of the stick-slip oscillations [5]. Moreover, the nonlinearity and uncertainty of the system parameter can be modelled in the control design processes. These facts are the motivation of this article.

Therefore, the scope of this paper is to minimize the torsional vibration problem of the test-rig set-up model using a control strategy. Specific objectives address:

- Open-loop analysis of the electrical and mechanical sub-systems.
- Application of a control strategy using different velocity inputs in a closed-loop system.
- Control of the torsional vibration considering the non-linearity due to friction interaction with the first and second inertia.
- Improvement on a developed experimental set-up for future verification and validation of the models.

1.3 Methodology

The methodology required to evaluate the performance of the control system is based on five steps [6]:

1. The modelling of the mechanical and mathematical representation of the test-rig set-up model.
2. An open-loop analysis is performed on a simplified model using a linear DC motor and a three degrees-of-freedom (DOF) system. In this system, a friction model capable of generating torsional vibrations is proposed acting on the first and second inertia, i.e., Stribeck friction.
3. The closed-loop analysis starts assuming a set of parameters.
4. Two situations are analysed concerning the control strategies:
 - (a) Only few parameters such as Motor Torque, J3 RPM, and Normal 1 are known.
 - (b) All the parameters (such as Normal forces and all angular velocities) are known.

Initially, the Proportional and Integral (PI) is the control strategy used in this comparison.

2 Preliminary concepts

2.1 Open-loop system

Generally, there are two open-loop system models: controlled or not controlled. In other words, for an open-loop analysis in a drilling system or in the proposed test rig set-up, the process may be equipped with a controller plus an actuator (Fig. 2a) or just with an actuator driven by an input signal Fig. 2b. For the purpose of this paper, the open-loop system will be considered without the controller.

Figure 2. Block diagrams of open-loop systems.

The S_{input} is the input signal of the controller in the block diagram of Fig. 2a. The V_{input} and u_c are the voltage input signal of the actuator, the u is the torque input signal of the drilling system. The y is the output angular velocity of the system.

2.2 Closed-loop system

For this paper the closed-loop analysis of the test rig set-up will be performed with disturbances and without disturbances. Furthermore, the closed-loop control system, also known as feedback control system, is represented by the block diagram shown in Fig. 3.

Figure 3. General block diagrams of a closed-loop system.

The Ω_{ref} is the reference signal of the system to be compared to the measured angular velocity y_m in the block diagram of Fig. 3 and generates the error signal $E(t)$. The U_m is the voltage input signal of the actuator, the T_m is the torque input signal of test rig set-up. The y is the output angular velocities of test rig set-up.

2.3 Automatic control systems

The control strategy (PI controller) used in the current paper is a well known industrial closed-loop controller, chosen according to their control actions, as explained in more details in [6].

2.3.1 Proportional and integral controller (PI)

The principle used in the SoftTorque control system (PI controller) is based on the proportional and integral control theory. So, based on the general block diagram of Fig. 3, the mathematical control law has the form:

$$\mathbf{u}_c(t) = k_p e(t) + k_i \int_0^t e(t) dt \quad (1)$$

where u_c is the control signal, y_m is the controlled variable, r is the manipulated variable and $e(t)$ is the control error ($e(t) = [r(t) - y_m(t)]$). k_p is the proportional gain. k_i is the integral gain and can be represented as $k_i = \frac{k_p}{T_i}$. T_i is the integral time constant.

3 Mathematical modelling of the test rig set-up model

A simple schematic of the reduced scale test rig set-up model is illustrated in Fig. 1.

In this case, the torsional set-up model is divided in two dynamic subsystems: the *electrical system* and the *mechanical system*. The first part (electrical), is represented by a DC motor, delivering torque on the third inertia. The motor dynamics has a considerable impact on the full dynamical behaviour. On the other hand, the mechanical part consists of three inertia. Both subsystems are physically connected to perform the torsional dynamics and will be investigated separately.

Figure 4. A test rig set-up modelled as a torsional pendulum.

The corresponding equations of motion of the test rig set-up model are represented by the model of torsional dynamics in Eq. 2.

$$\begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} \begin{bmatrix} \ddot{\Omega}_1 \\ \ddot{\Omega}_2 \\ \ddot{\Omega}_3 \end{bmatrix} + \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} -T_1 \\ -T_2 \\ T_3 \end{bmatrix} \quad (2)$$

where φ_1 , φ_2 and φ_3 are the angular displacements of the three discs, Ω_1 , Ω_2 and Ω_3 are the angular velocities measured at the three discs. J_1 , J_2 and J_3 are the equivalent mass moment of inertia of the three discs. C_1 , C_2 and C_3 are the equivalent viscous damping coefficients, k_1 and k_2

is the equivalent torsional stiffness of the wire, and T_1 and T_2 are the nonlinear functions representing the resistive torque at discs 1 and 2. T_3 is the supplied torque to the mechanical system.

The behaviour of the DC electronic motor has its dynamical principles represented by two linear models: (1) the mechanical equation of free-body diagram of the rotor, and (2) the electric circuit equation of the armature.

The equation of motion of the motor dynamics is presented as:

$$L \dot{I} + R I + V_{emf} = U_m \quad (3)$$

$$V_{emf} = k_e \eta \Omega_3 \quad (4)$$

$$T_m = k_t I - C_m \eta \Omega_3 - T_{ff} - J_m \eta \Omega_3 \quad (5)$$

where $T_3 = \eta T_m$ due to the gearbox factor. I is defined as armature current, R and L are electrical resistance and electrical inductance, respectively. U_m is the motor input voltage. V_{emf} represents the back-electromotive force (back-emf) related to the angular velocity Ω_3 . k_e and k_t parameters are respectively the electromotive force constant and the motor torque constant. T_{ff} represents a internal friction torque of the DC motor.

3.1 Friction torque modelling

For this reason, the Eq. 6 describes how the resistive torque at the drill bit is modelled based on the friction law presented in [7, 8].

$$T_i = N_i r_i \left[\mu_k + (\mu_s - \mu_k) \cdot e^{-vb|\Omega_i|} \right] \cdot \text{sign}(\Omega_i) \quad (6)$$

where $i = 1, 2$, N_i is the normal force, also constant in the model, and r_i is the distance between the centre of the disc and the normal contact point (see Fig. 1). μ_s and μ_k are static and kinetic friction coefficient. The friction model curve is illustrated in Fig. 5.

Figure 5. Stribeck friction model.

4 Stick-slip severity (SSS)

The **Stick-Slip Severity (SSS)** is a severity criterion that evaluates the vibration amplitudes in relation to the nominal velocity. Therewith, SSS is performed for the used PI control strategy. In this paper, the SSS may be presented as a 2D and 3D grey scale colour-map that plots the

relation between $N1$ (normal at R1) and nominal angular velocity (RPM at DC motor), exposing limits where the previous defined torsional vibration (stick-slip) becomes severe according to the criterion.

$$SSS = \left(\frac{\Omega_1^{max} - \Omega_1^{min}}{2 \Omega_{ref}} \right) \cdot 100\% \quad (7)$$

where Ω_1^{max} and Ω_1^{min} are respectively the maximum and minimum angular velocity of the disc 1. Ω_{ref} is the reference angular velocity.

5 Analysis of the results

An optimization method is used to evaluate the effectiveness of the open-loop analysis in control design, as presented in the flowchart schematic in Fig. 6. This procedure considers the subsystems separately. Tab. 1 contains the parameters of the test rig set-up.

Figure 6. The iterative open-loop flowchart used in the analysis.

Multiple step responses $[\Omega_3, \Omega_2, \Omega_1]$ of the drill string system are shown in Figs. 7a and 7b. Remembering that in the open-loop analysis there is no need to satisfy any control criteria or to make consistent conclusions about steady state errors, because there is no pre-defined setpoint velocity (Ω_{ref}).

After substituting the parameters of Tab. 1 into the Eq. 2 and 5, a summary of the open-loop angular velocity responses of the test rig set-up and the DC motor systems, given in respect to time when the torque varies, are displayed in Tab. 2. Moreover, for each constant torque from $0.01 \text{ N} \cdot \text{m}$ to $0.30 \text{ N} \cdot \text{m}$ and the associated results, it is defined a Number (**Num.**) varying from 1 to 15, respectively, in the first column of the Tab. 2.

5.1 Open-loop results

A *Simulink* modelling is designed to simulate the dynamic of the mechanical part of test rig set-up, in other

Table 1. Simulation parameters values.

Parameter	Description	Value	Unit
J_1	Disc 1 (R1)	0.0289	kg m^2
J_2	Disc 2 (R2)	0.0197	kg m^2
J_m	DC motor inertia	$0.37 \cdot 10^{-3}$	kg m^2
C_1	R1 damping	0.0134	Nms/rad
C_2	R2 damping	0.0067	Nms/rad
k_1	R1-R2 stiffness	0.6441	Nm/rad
k_2	R2-R3 stiffness	0.1762	Nm/rad
R	Electrical resistance	0.33	Ohm
L	Electrical inductance	$1.1 \cdot 10^{-3}$	H
k_e	Electromotive force constant	$6.02 \cdot 10^{-2}$	V/rad/s
k_t	Motor torque constant	0.12	Nm/Amp
C_m	Motor speed constant	$1.91 \cdot 10^{-4}$	Nms/rad
T_f	Internal friction torque	0.1	Nm
η	Transmission factor	8	—
r_1	N1 contact point	0.047	m
r_2	N2 contact point	0.047	m
μ_s	Static coefficient	0.461	—
μ_k	kinetic coefficient	0.370	—
v_b	velocity weakening parameter	1.5	—
k_i	integral gain	1.4891	—
k_p	proportional gain	0.4408	—

Figure 7. Angular velocity and torque responses.

words, without a DC motor actuator. The simulations are performed to evaluate the system behaviour when a range of input torques are disturbed by a set of $N1$ and $N2$. The white zone indicates the zone where an input torque is effective to keep the angular velocity at a constant value. The black zone indicates the zone where the input torque

Table 2. Test rig set-up and DC Motor open-loop results.

Num.	Discs 1 and 2		DC Motor
	T_3^{input}	$[\Omega_1 \ \& \ \Omega_2]^{settled}$	
1	0.0100N·m	2.9499 rpm	0.1865 V
2	0.0307N·m	9.0605 rpm	0.5018 V
3	0.0514N·m	15.1710 rpm	0.8171 V
4	0.0721N·m	21.2816 rpm	1.1324 V
5	0.0929N·m	27.3921 rpm	1.4477 V
6	0.1136N·m	33.5026 rpm	1.7630 V
7	0.1343N·m	39.6132 rpm	2.0783 V
8	0.1550N·m	45.7237 rpm	2.3936 V
9	0.1757N·m	51.8343 rpm	2.7089 V
10	0.1964N·m	57.9448 rpm	3.0242 V
11	0.2171N·m	64.0554 rpm	3.3395 V
12	0.2379N·m	70.1659 rpm	3.6548 V
13	0.2586N·m	76.2765 rpm	3.9701 V
14	0.2793N·m	82.3870 rpm	4.2854 V
15	0.3000N·m	88.4976 rpm	4.6006 V

is not sufficient to suppress the resistive torque generated by the normal forces (N_1 and N_2).

The Hopf supercritical bifurcation diagrams of the test rig system are assumed in a non-controlled system for a set of N_1 and N_2 0 N and 10 N, as illustrated in Figs. 8a and 8b and Figs. 9a and 9b, respectively.

Figure 8. Bifurcation diagrams for $N_2 = 0$ N.

Figure 9. Bifurcation diagrams for $N_2 = 10$ N.

5.2 Closed-loop results: PI controller

Now, a *Simulink* modelling is designed to simulate the dynamic test rig set-up with a DC motor actuator. The torsional vibration map of the test rig system when subjected

to PI controller is illustrated in Figs. 10a and 10b. It is noticed that the system reaches amplitudes of vibration that are 4x (200%) the nominal velocity. The white zone indicates the zone where a PI controller is effective. For best understanding see [9].

Figure 10. SSS and Bifurcation diagrams for PI ($N_2 = 0$ N).

In this case, it is possible to say (see Fig. 10c) that, for a constant angular velocity ($\Omega_{ref} = 15.17$ rpm), it presents transition region from equilibrium point to periodic solution. It is a Hopf supercritical bifurcation diagram and may show the amplitude of vibration increasing when different N_1 are applied to the system PI controller, as illustrated in Fig. 10c. Also, using Ω_{ref} as nonlinear control parameter, Fig. 10d, for $N_1 = 60$ N, the periodic solution becomes equilibrium point around $\Omega_{ref} = 30$ rpm.

In order to analyse the influence of intermediate friction on the dynamic responses and the effectiveness zone of PI controller, $N_2 \neq 0$, Figs. 11a and 11b present the SSS maps. The vibration zone is clearly increased compared to Figs. 10a and 10b. Also, the bifurcation points have changed observing Figs. 10c and 10d comparing to 11c and 11d.

Figure 11. SSS and Bifurcation diagrams for PI ($N_2 = 10$ N).

Figs. 11b and 11c still show two nonlinear jumps before 20 rpm when $N1$ is varied. Following the same reasoning, Fig. 12 illustrates large variations of SSS. However, for all three $N2$ values, the greater influence occurs for low values of Ω_{ref} where it may be observed a decreasing of the effective zone of the PI controller.

Figure 12. SSS and Bifurcation diagrams for PI ($N2 = 20$ N).

Finally, a comparison between numerical and experimental values is performed. The system is settled to 30 and 50 rpm and the both transient are observed in Figs. 13a and 13b, respectively.

Figure 13. Numerical and Experimental Ω_3 comparison.

For this test, both data converge to the settled angular velocity and the step response characteristics of both systems show the following performance indicators (see Tabs. 3 and 4).

6 Conclusions

The drilling system provides a myriad of problems to investigate. Controlling torsional vibration of this kind of system is challenging.

It is possible to observe that the PI controller is effective in a certain zone. By varying the intermediate friction torque T_2 (via $N2$), the controllable zone changes mainly for low nominal velocities Ω_{ref} . PI controller has proven to be very ineffective when the system is in torsional vibration. This is because PI controller is a linear controller,

Table 3. Attributes of the test rig set-up. (30 rpm).

Attributes	Ω_3 Exp.	Ω_3 Num.
Rise time [$\tau_r(s)$]	1.3207	1.4841
Settling time [$\tau_s(s)$]	N/A	1.8692
Peak time [$\tau_p(s)$]	1.9462	6.0147
Overshoot [% OS]	8.9790	0.4908

Table 4. Attributes of the test rig set-up. (50 rpm).

Attributes	Ω_3 Exp.	Ω_3 Num.
Rise time [$\tau_r(s)$]	1.2246	1.4793
Settling time [$\tau_s(s)$]	N/A	1.8646
Peak time [$\tau_p(s)$]	1.8598	6.0165
Overshoot [% OS]	12.0614	0.4919

but also has shown to restrain torsional vibrations for high nominal velocities.

In a first approach, this methodology is important to understand the test rig system and which controller (or combination of controllers) may be applied in experimental analysis.

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