Proportional-Derivative Control of Stick-Slip Oscillations in Drill-Strings

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Abstract. Stick-slip oscillation in drill-string is a universal phenomenon in oil and gas drilling. It could lead to the wear of drill bit, even cause catastrophic failure of drill-strings and measurement equipment. Therefore, it is crucial to study drilling parameters and develop appropriate control method to suppress such oscillation. This paper studies a discrete model of the drill-string system taking into account torsional degree-of-freedom, drill-string damping, and highly nonlinear friction of rock-bit interaction. In order to suppress the stick-slip oscillation, a new proportional-derivative controller, which can maintain drill bit's rotation at a constant speed, is developed. Numerical results are given to demonstrate its efficacy and robustness.

1 Introduction

For conventional rotary drilling, drill-string is a principal component for the entire drilling system. As shown in Fig.1 (a), a typical drilling system is composed of a rotary table, a slender drill pipe, a drill collar, and a drill bit. The drill-string could run several kilometres deep, but the diameter of the drill pipe does not exceed 0.3 metres. When drill pipes transmit the drive torque from rotary table to drill bit, deformations in drill pipes are very complex, especially considering the constraints of the borehole. The slenderness of the drill pipe makes it prone to exhibit undesired oscillations, including bit bouncing, lateral oscillation, torsional oscillation, and whirling motion. In oil and gas drilling, stick-slip behavior exists in 50% of drilling time [1], and the whipping and high rotation of the drill bit may cause bit bouncing and whirling motion [2]. Moreover, formation change is also very complex during drilling procedure, so the interactive model between rock and bit is highly nonlinear [3]. In addition, the interaction between drill-strings and borehole also makes drilling system uncertain [4].
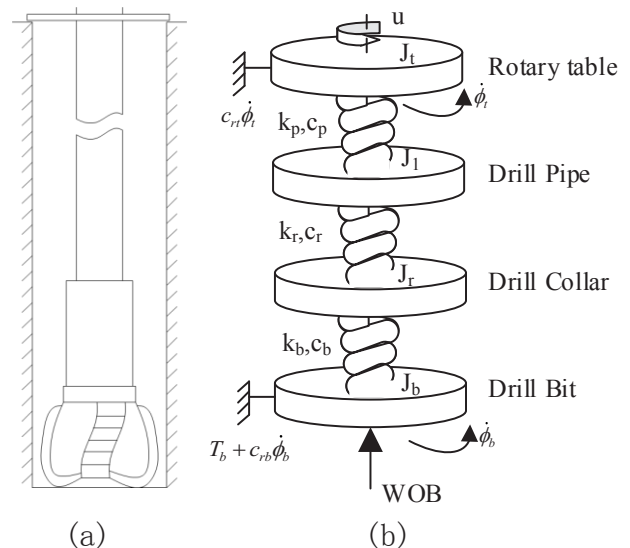

Fig. 1. The schematics of (a) the drill-string and (b) the simplified physical model of the drill-string system

There are vast of existing literatures using lumped-parameters model to study the drill-string dynamics, e.g.

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[5-7]. Balakumar [5] developed a lumped-parameter model to study the coupled axial, torsion, and lateral dynamics of a drill-string dynamics, including stick-slip and delay effect. Canudas-de-Wit et al. [6] proposed a lumped parameter model to use the weight on bit as an additional variable for suppress stick-slip oscillation. Liping Tang [7] proposed a lumped torsional pendulum model of the drilling system, and studied the stick-slip dynamics and negative damping effect. However, most of the dynamics model could not be guaranteed to suppress the stick-slip oscillation steadily and appropriate to the parameters changing. Since this paper proposed a proportional-derivative (PD) controller to suppress stick-slip oscillations using the lumped-mass model.

The remaining sections of this paper are organized as follows. In Section 2, a discrete dynamic model is developed for the drill-string system, and a new PD controller is studied. In Section 3, simulation results are presented to validate the effectiveness of the controller. Finally, conclusions are drawn in Section 4.

2 Drill-String Model and Proportional-Derivative Controller

The drilling system, which is illustrated in Fig.1 (a), is comprised of rotary table, along with drill pipe, drill collar and drill bit. In order to simplify the system, a discrete drill-string model is developed for analytical and numerical study. The mechanical schematic of drill-string system shown in Fig.1 (b) is considered as four disks, each with rotational inertia properties. The drill-string is connected by massless springs, which have torsional stiffness and torsional damping. The rotary table is driven by a motor fixed on the rotary table with control torque output. The interaction between drill bit and rock is a highly nonlinear friction.

Making use of the dynamic theory, the government equation of motion for the discrete system [8] can be written as

$$J\ddot{\phi} + C\dot{\phi} + K\phi + T = U \qquad (1)$$

Where Φ is the rotational position of lumped mass, J is the rotational inertia matrix, C is the torsional damping matrix, and K is the torsional stiffness matrix, T is the torque of friction between drill bit and rock, and U is the control torque applied on the rotary table. According to

the finite element method, matrixed (J, C, and K) are comprised as follows.

$$J = \begin{bmatrix} J_t & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 \\ 0 & 0 & J_r & 0 \\ 0 & 0 & 0 & J_b \end{bmatrix}$$

$$C = \begin{bmatrix} c_{rt}+c_p & -c_p & 0 & 0 \\ -c_p & c_p+c_r & -c_r & 0 \\ 0 & -c_r & c_r+c_b & -c_b \\ 0 & 0 & -c_b & c_{rb}+c_b \end{bmatrix}$$

$$K = \begin{bmatrix} k_p & -k_p & 0 & 0 \\ -k_p & k_p+k_r & -k_r & 0 \\ 0 & -k_r & k_r+k_b & -k_b \\ 0 & 0 & -k_b & k_b \end{bmatrix}$$

The frictional torque for the rock-bit contact is modelled as a combination of Stribeck [9] and the Karnopp's [10] model described as

$$T_b = \begin{cases} T_r & if \; |\dot{\phi}_b| < \zeta \; and \; |T_r| \le T_s, \\ T_s \, \mathrm{sgn}(T_r) & if \; |\dot{\phi}_b| < \zeta \; and \; |T_r| > T_s, \\ \mu_d R_b W_{ob} \, \mathrm{sgn}(\dot{\phi}_b) & if \; |\dot{\phi}_b| > \zeta. \end{cases} \qquad (2)$$

In the sticking phase $(|\dot{\Phi}_b| < \zeta$ and $|T_r| < T_s)$, the bit velocity is less than a small positive constant ξ, and the torque T_r is less or equals to the static friction torque T_s, where $T_r = c_b(\dot{\Phi}_t - \dot{\Phi}_b) + k_b(\Phi_t - \Phi_b)$ and $T_s = \mu_s W_{ob} R_b$, where μ_s and μ_d are the static friction coefficient and dynamic friction coefficient, respectively. W_{ob} is the weight on bit (WOB), R_b is the bit radius.

In the process of stick to slip $(|\dot{\Phi}_b| < \zeta$ and $|T_r| > T_s)$, the drill bit velocity is still less than the small constant ξ, but the reaction torque T_r is greater than the static friction torque T_s and the drill bit start to move.

In the slip phase $(|\dot{\Phi}_b| > \zeta)$, the drill bit starts to rotate, and the friction torque includes the effects of drill bit radius, the WOB, and the dynamic friction coefficient [11].

For the purpose of control stick-slip oscillation, a new proportion-differential controller is designed.

$$u = k_1(\phi_t - \phi_b) + k_2(\dot{\phi}_d - \dot{\phi}_t) + k_3(\dot{\phi}_d - \dot{\phi}_b) \qquad (3)$$

Where u is the control torque applied on rotary table, k_1 is proportion coefficient, k_2 and k_3 is the differential coefficient, $\dot{\Phi}_d$ is the desired angular velocity, $\dot{\Phi}_t$ is the rotary table velocity, and $\dot{\Phi}_b$ is the drill bit velocity.

3 Simulation Results

Numerical studies were conducted with the earlier described discrete system model by applying the PD controller. The geometry and physical parameters for the drill-string are given in Table 1. In the follow subsections, the stick-slip oscillation, control of stick-slip, and the respond to WOB oscillation are brought forth.

Table 1. Geometry and physical parameters for the drill-string

Parameter	Value
J_t	910 kg $\cdot$ m^2
J_1	2800 kg $\cdot$ m^2
J_r	750 kg $\cdot$ m^2
J_b	450 kg $\cdot$ m^2
c_p	150 N $\cdot$ m $\cdot$ s/rad
c_r	190 N $\cdot$ m $\cdot$ s/rad
c_b	180 N $\cdot$ m $\cdot$ s/rad
c_{rt}	410 N $\cdot$ m $\cdot$ s/rad
c_{rb}	80 N $\cdot$ m $\cdot$ s/rad
k_p	700 N $\cdot$ m $\cdot$ s/rad
k_r	1080 N $\cdot$ m/rad
k_b	910 N $\cdot$ m/rad
μ_s	0.8
μ_d	0.45
WOB	30 kN
R_b	0.15 m
v_d	6 rad/s
ζ	10^{-6}
k_1	500
k_2	300
k_3	200

Fig.2 shows the simulation result without controller. The figure presents that both angular velocities of the rotary table and the drill bit change repeatedly with the constant torque. Moreover, in Fig.2 (a), the peak speed of the drill bit is about 3.5 rad/s, which is much greater than the rotary table velocity. As is show in Fig.2 (b), the angular acceleration of drill bit also changes between positive and negative. In this rapidly changes of angular speed and angular acceleration, the tooth of the drill bit will be more likely to wear out than it is in constant speed.

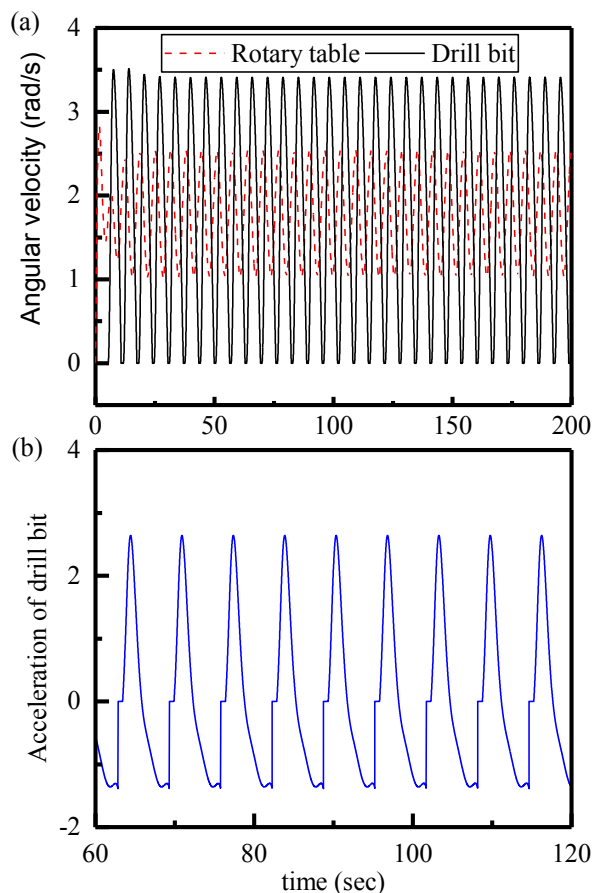Therefore, it is necessary to study the stick-slip oscillation and suppress it.

Fig. 2. Time histories of (a) angular velocities of the rotary table (red dash line) and the drill bit (black solid line) with WOB = 30 kN, and u = 3450 Nm.

The simulation result using the PD controller is shown in Fig.3. It can be observed from Fig.3 (a) that the stick-slip is suppressed when the controller is switched on at t = 33 seconds, and both the angular velocities of rotary table and the drill bit are stabilized at about t = 200 seconds. At the same time, the maximum amplitude control torque is about 8.5 kN $\cdot$ m, as seen in Fig.3 (b). In Fig.4, we can see that drill bit velocity converges at 5.26 rad/s, which is a little less than the desired velocity 6 rad/s. The result for torsional oscillation in Fig.3 and Fig.4 indicate that the PD controller can suppress stick-slip oscillation and keep the velocity on a constant desired velocity.
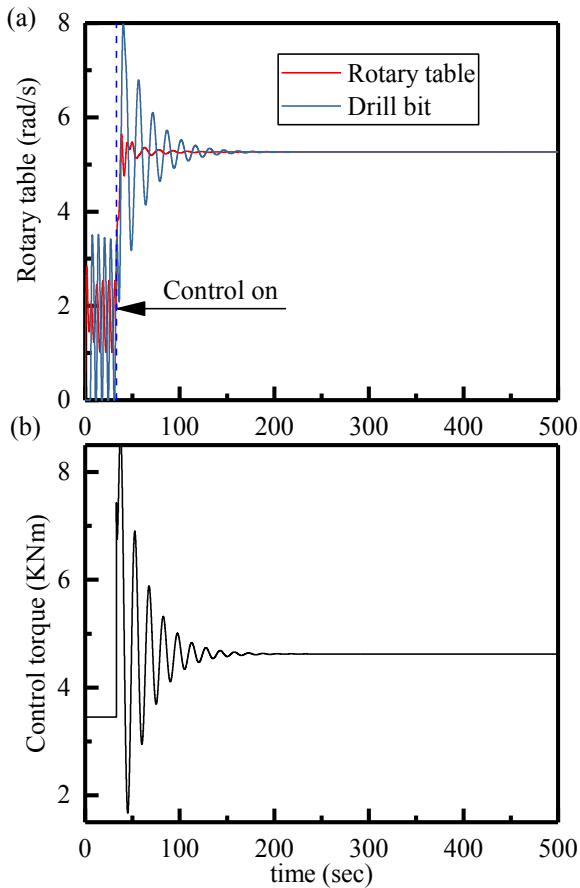
(a)

(b)

Fig. 3. Time histories of (a) angular velocities of rotary table (red solid line) and drill bit (black solid line), and (b) the control torque using the PD controller with WOB = 30 kN.
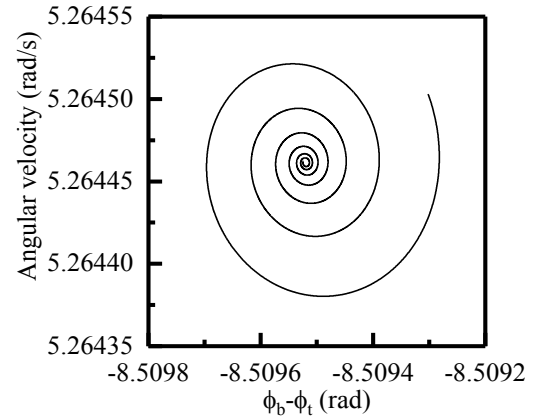

Fig.4 Trajectory of the drill-string on the phase plane (Φ_b - Φ_t) by using the PD controller with WOB = 30 kN

To study the stability of PD controller to parameter sensitive, three different WOB values are applied on the drill bit, and the results are shown in Fig. 5 and Fig. 6. It can be observed that the angular velocity can be able to stabilize the system with WOB changing. When WOB decreases from 30 kN to 20 kN, and 20 kN to 10 kN, the angular velocity has a small fluctuation, and then stay at a constant speed. Trajectory of the drill-string on the phase plane (Φ_b - Φ_t) by using the PD controller with different WOB are presented in Fig.6. It is clearly to describe the changing of the velocity with the angular displacement difference between drill bit and rotary table. For the PD controller, it has a proper estimation of unknown physical parameters for the switching control.
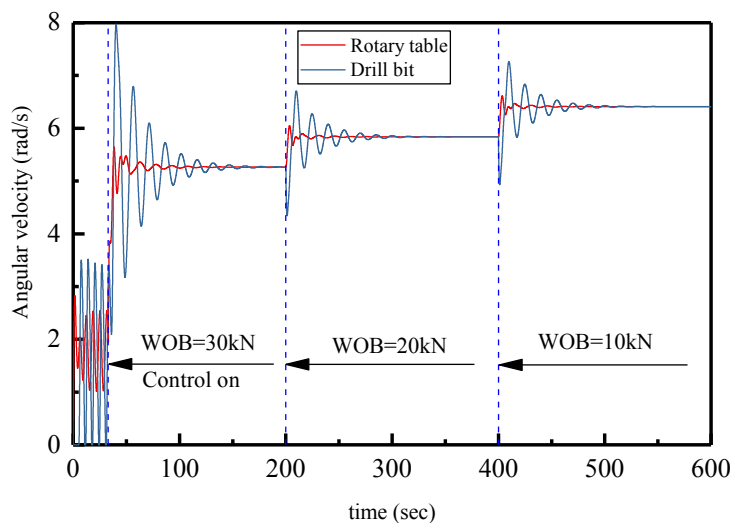

Fig.5 Angular velocities of rotary table (red solid line) and drill bit (blue solid line) by using PD controller with WOB = 30 kN, 20 kN, and 10 kN.
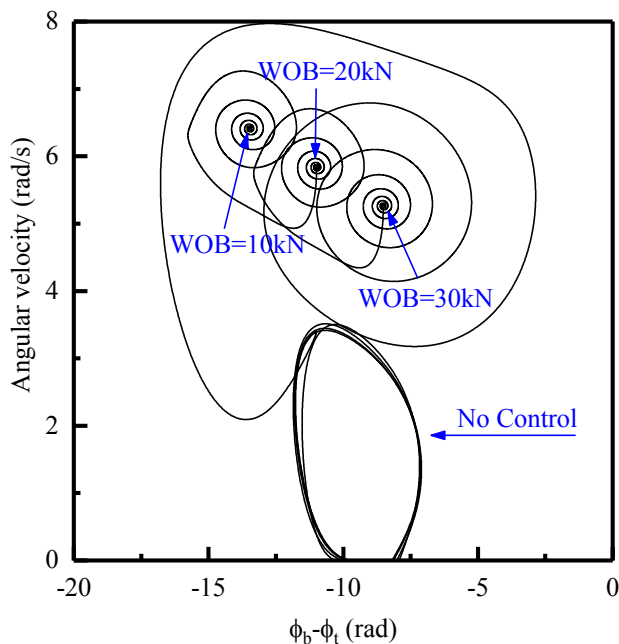
Fig.6 Trajectory of the drill-string on the phase plane (Φ_b - Φ_t) by using the PD controller with WOB = 30 kN, 20 kN, and 10 kN

4 Conclusions

This paper studied the stick-slip oscillations in drill-strings by using a discrete model with torsional degree-of-freedom from a control point of view. The model has one control input and multi-degree-of-freedom to be controlled, considering a series of drill pipes, drill collars, and the highly nonlinear friction for rock-bit interaction. A new PD controller was developed to suppress stick-slip oscillations and track desired rotary speed under variations of other drilling parameters, such as WOB.

Future work will focus on improving controller's accuracy and verify through experimentation. Borehole constraints and axial bit movement will also be considered in the discrete model.

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