

Reflection and Transmission of Acoustic Waves through the Layer of Multifractional Bubbly Liquid

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Abstract. The mathematical model that determines reflection and transmission of acoustic wave through a medium containing multifractional bubbly liquid is presented. For the water-water with bubbles-water model the wave reflection and transmission coefficients are calculated. The influence of the bubble layer thickness on the investigated coefficients is shown. The theory compared with the experiment. It is shown that the theoretical results describe and explain well the available experimental data. It is revealed that the special dispersion and dissipative properties of the layer of bubbly liquid can significantly influence on the reflection and transmission of acoustic waves in multilayer medium.

1 Introduction

The investigations of wave dynamics and acoustics of disperse media are of significant interest. Currently, the basics of mechanics and thermophysics of bubbly liquids, as well as the most significant results in the study of wave processes in these environments are presented in monographs [1-5]. Hao and Prosperetti [6] obtained the results of study of the vapor bubble dynamics in liquid. Commander and Prosperetti [7] presented a model of the propagation of plane waves with small amplitude in a mixture of liquid and of gas bubbles. This model is adequate for the frequencies far from resonance at the volume content of a disperse phase of 1-2%. Nigmatulin et al. [8] showed the need of allowing for the effect of liquid compressibility for problems of acoustics of bubbly liquids. Shagapov [9] considered a problem of propagation of small disturbances in liquids with polydisperse bubbles. Nigmatulin et al. [10] studied the small radial oscillations of vapor-gas bubbles in liquid under the influence of an acoustic field. They showed that capillary effects and phase transitions lead to the new resonant frequency of small bubbles. This frequency is different from the Minnaert frequency.

The propagation of acoustic waves in a two-fractional mixture of liquid with bubbles is investigated in [11-13]. A mathematical model of the propagation of sound waves in the two-fractional mixture of liquid with polydisperse vapor-gas and gas bubbles with phase transitions is shown in [14-16]. The propagation of acoustic waves in multifractional mixtures of liquid with gas and vapor-gas bubbles of different sizes and different compositions is studied in [17-19]. The problem of reflection of an acoustic wave from a two-layer medium containing a layer of bubble liquid is considered in [20-22].

In this study the acoustic wave reflection and transmission through a multilayer object containing a multifractional bubbly fluid layer is investigated.

2 Dispersion relation

Linearized equations for one-dimensional disturbances in a multifractional mixture of liquid with bubbles are obtained from the general equations of motion for bubbly mixtures [1]. The dispersed phase consists of $N+M$ fractions having various gases in bubbles and different in the bubbles radii. Phase transitions accounted for N fractions. The total bubble volume concentration is small (less than 1%). Solving this system the dispersion relation is obtained:

$$\left(\frac{K_*}{\omega}\right)^2 = \frac{1}{C_f^2} + \frac{\rho_{10}}{p_0} \left(\frac{(S_{1i} + S_{1j})(S_{3i} + S_{3j})}{S_{2i} + S_{2j} - i\omega} + S_{Ri} + S_{Rj} \right) \quad (1)$$

where C_f is the frozen speed of sound ($C_f = C_1 / \alpha_{10}$) and also the following notations are adopted:

$$S_{1i} = \sum_{i=1}^M H_{1i}, \quad S_{1j} = \sum_{j=1}^N H_{1j}, \quad S_{3i} = \sum_{i=1}^M H_{3i},$$

$$S_{3j} = \sum_{j=1}^N H_{3j}, \quad S_{2i} = \sum_{i=1}^M \left(\frac{m_i}{\tau_{T1i}} + H_{2i} \right),$$

$$S_{2j} = \sum_{j=1}^N \left(\frac{m_j}{\tau_{T1j}} + H_{2j} \right), \quad S_{Ri} = \sum_{i=1}^M \left(\frac{\alpha_{20i}}{N_{Ri}} \left(1 - \frac{M_{4i}}{M_{3i}} \right) \right),$$

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$$S_{Rj} = \sum_{j=1}^N \left(\frac{\alpha_{20j}}{N_{Rj}} \left(1 - \frac{M_{4j}}{M_{3j}} \right) \right),$$

$$H_{1j} = \frac{m_j}{\tau_{T1j}} \left(\frac{M_{1j}M_{4j}}{M_{3j}} + M_{2j} \right),$$

$$H_{li} = \frac{m_i}{\tau_{Tli}} \left(\frac{M_{li}M_{4i}}{M_{3i}} + M_{2i} \right), \quad H_{2j} = \frac{m_j}{\tau_{T1j}} \frac{M_{1j}b_j}{M_{3j}},$$

$$H_{2i} = \frac{m_i}{\tau_{Tli}} \frac{M_{li}b_i}{M_{3i}}, \quad H_{3j} = \frac{\alpha_{j0}}{N_{Rj}} \frac{b_j}{M_{3j}}, \quad H_{3i} = \frac{\alpha_{i0}}{N_{Ri}} \frac{b_i}{M_{3i}},$$

$$M_{1j} = G_j - M_{2j} - \frac{L_{1j}N_{3j}}{L_{4j} - \delta N_{2j}}, \quad M_{2j} = \frac{N_{2j}L_{1j}}{N_{Rj}(L_{4j} - \delta N_{2j})},$$

$$M_{3j} = \frac{N_{3j}\delta - L_{2j}N_{3j}}{L_{4j} - \delta N_{2j}} + L_{3j} + M_{4j},$$

$$M_{4j} = \frac{L_{4j} - L_{2j}N_{2j}}{N_{Rj}(L_{4j} - \delta N_{2j})}, \quad M_{li} = -(N_{3i} + M_{2i}),$$

$$M_{2i} = \frac{N_{2i}}{N_{Ri}}, \quad M_{3i} = 1 + N_{3i}(1 + b_i) + M_{4i},$$

$$M_{4i} = \frac{1}{N_{Ri}}(1 + N_{2i}(1 + b_i)), \quad N_{1j} = \frac{i\omega\tau_{T1j}}{m_j} - 1, \quad N_{li} = 1,$$

$$N_{2j} = i\omega\tau_{Tj} - 1, \quad N_{2i} = i\omega\tau_{Ti} - 1,$$

$$N_{3j} = k_{2j}(c_j - R_j) - 1 + G_j, \quad N_{3i} = k_{2i}(c_i - R_i) - 1,$$

$$L_{1j} = E_j(1 - i\omega\tau_j), \quad E_j = \frac{R_V}{R_j} \frac{p_0}{l_0\rho_{V0}^\circ} (1 - k_{Vj0}),$$

$$L_{2j} = -\frac{l_0k_{2j}}{(1 - k_{Vj0})T_0} + \Delta R_j - L_{1j}(1 + b_j),$$

$$L_{3j} = 1 - G_j(1 + b_j), \quad L_{4j} = L_{1j} + \Delta R_j N_{2j}, \quad k_{2j} = \frac{i\omega\tau_{Tj}}{c_j},$$

$$k_{2i} = \frac{i\omega\tau_{Ti}}{c_i}, \quad b_j = \frac{c_1\tau_{Tj}}{c_j\tau_{T1j}}, \quad b_i = \frac{c_1\tau_{Ti}}{c_i\tau_{T1i}},$$

$$N_{Rj} = \frac{-(i\omega)(a_{j0})^2 G_{Rj}\rho_{10}^\circ}{3(t_{Aj}G_{Rj} + 1)p_0}, \quad N_{Ri} = \frac{-(i\omega)(a_{i0})^2 G_{Ri}\rho_{10}^\circ}{3(t_{Ai}G_{Ri} + 1)p_0},$$

$$G_{Rj} = \frac{1}{t_{Rj}} - i\omega, \quad G_{Ri} = \frac{1}{t_{Ri}} - i\omega, \quad t_{Rj} = \frac{(a_{j0})^2}{4\nu_1},$$

$$t_{Ri} = \frac{(a_{i0})^2}{4\nu_1}, \quad t_{Aj} = \frac{a_{j0}}{C_1(\alpha_{j0})^{1/6}}, \quad t_{Ai} = \frac{a_{i0}}{C_1(\alpha_{i0})^{1/6}},$$

$$m_{j0} = \frac{\rho_{j0}^\circ}{\rho_{10}^\circ}, \quad m_{i0} = \frac{\rho_{i0}^\circ}{\rho_{10}^\circ}, \quad m_j = \frac{\rho_{j0}}{\rho_{10}}, \quad m_i = \frac{\rho_{i0}}{\rho_{10}},$$

$$i = 1, 2, \dots, M, \quad j = 1, 2, \dots, N.$$

Here, a – the radius of the bubbles, α – the volume concentration, ρ°, ρ – the true and average densities, p – the pressure, c – the specific heat capacity, ν – the kinematic viscosity, R – the gas constant, T – the temperature, l – the specific heat of evaporation, λ – the thermal conductivity, k – the weight concentration, τ – the relaxation time. Subscript 1 refers to the parameters of the liquid phase, 2 – disperse phase, C_1 – the velocity of sound in pure liquid.

The dispersion relation (1) considered as the dependence of the complex wavenumber K^* on the vibration frequency ω determines propagation of acoustic perturbations in multifractional mixtures of a liquid with vapor-gas and gas bubbles (of various initial radii, various initial volume contents, and various thermophysical properties of gases in the bubbles).

3 Mathematical model

In analysing the interaction between an acoustic signal and a multilayer object (Fig. 1), the following method of calculations is used. According to [23], the result of the reflection and transmission of a plane monochromatic wave $\exp(iK_*x - \omega t)$ from a multilayer object is the plane waves $R\exp(iK_*x + \omega t)$ and $T\exp(iK_*x - \omega t)$, where R and T are the wave reflection and transmission coefficients, respectively, determined in terms of the layer impedances Z_i and the entry impedances of the layer boundaries Z_i^{in} . For a three-layer medium the coefficients R and T are as follows:

$$R = \frac{Z_2(Z_1 - Z_3) - i(Z_2^2 - Z_1Z_3)\text{tg}(K_2d_2)}{Z_2(Z_1 + Z_3) - i(Z_2^2 + Z_1Z_3)\text{tg}(K_2d_2)}$$

$$T = \prod_{j=1}^2 \left(\frac{Z_j^{in} + Z_j}{Z_j^{in} + Z_{j+1}} \exp[iK_jd_j] \right) \quad (2)$$

$$Z_j = \rho_j \frac{\omega}{K_j}, \quad Z_2^{in} = \frac{Z_1 - iZ_2\text{tg}(K_2d_2)}{Z_2 - iZ_1\text{tg}(K_2d_2)} Z_2, \quad j = 1, 2, 3.$$

Here, d_2 is the thickness of the layer with bubbles, K is the wave number, ω is the perturbation frequency, and ρ is the layer density. For a homogeneous layer, the wave number is determined as $K_j = \omega/C_j$, where C_j is the speed of sound in the j -th layer.

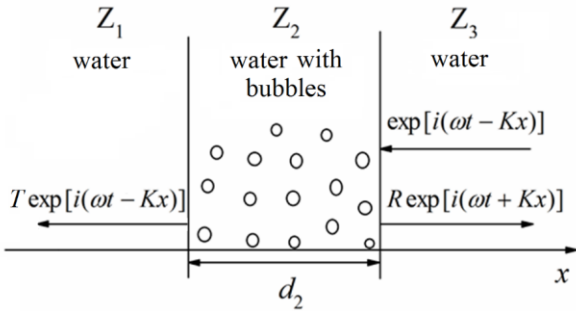


Fig. 1. Scheme of the medium.

4 Results

We will consider the acoustic signal transmission through the three-layer medium, namely, water-water with bubbles-water. Disperse phase of bubbly liquid contain vapor-air bubbles, bubbles of carbon dioxide with water vapor and bubbles of helium. Let the bubble layer thickness be $d_2=4$ mm, the bubble radii – 2mm (vapor-air bubbles), 1mm (bubbles of carbon dioxide with water vapor), 1.5mm (bubbles of helium), $f_0=1630$ Hz (resonance frequency of 2mm bubbles) and the volume content $\alpha_{20}=0.01$. The mixture pressure $p_0=0.1$ MPa and the temperature $T_0=288$ K. The calculations were performed according equations (1), (2). The dependences of the coefficients of the wave reflection and transmission through the given bubble layer on the dimensionless disturbance frequency are presented in the figure 2. Clearly that a minimum of the transmission coefficient and a maximum of the reflection coefficient are observable in the region of the resonance frequency of the bubbles. This means that at the given frequency the bubble layer almost completely reflects the incident acoustic wave. In addition, three fractions with different initial radii of the disperse phase leads to the appearance of three local minima and maxima on this coefficients. This is due to the difference in the values of the resonance frequencies of the intrinsic vibrations of bubbles of each fraction.

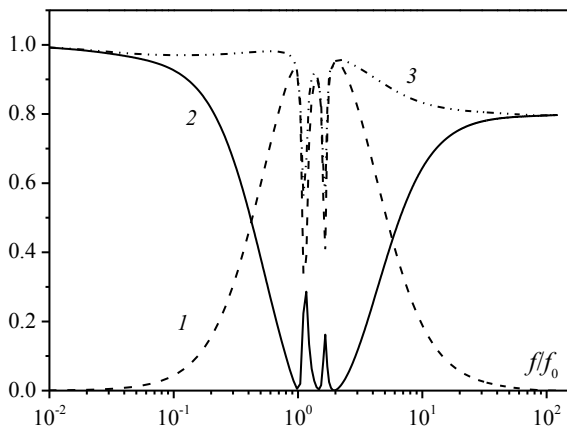


Fig. 2. Coefficients of sound reflection and transmission through a bubbly screen, 1 – R^2 , 2 – T^2 , 3 – R^2+T^2 .

On the figures 3, 4 the influence of the bubble layer thickness on the acoustic wave transmission and

reflection is illustrated. Parameters of bubbly liquid are the same as on figure 2, curves 1 – $d_2=4$ mm, curves 2 – $d_2=8$ mm, curves 3 – $d_2=12$ mm, curves 4 – $d_2=16$ mm. An increase in the bubble layer thickness leads to a decrease in the transmission coefficient and, correspondingly, to an increase in the reflection coefficient on the entire frequency range, except from a vicinity of the resonance frequency of the bubbles. With increase in the bubble layer thickness the range near the resonance frequency ($f \approx f_0$), where the wave transmission coefficient takes near-zero values (opposite effect on the reflection coefficient), also expands.

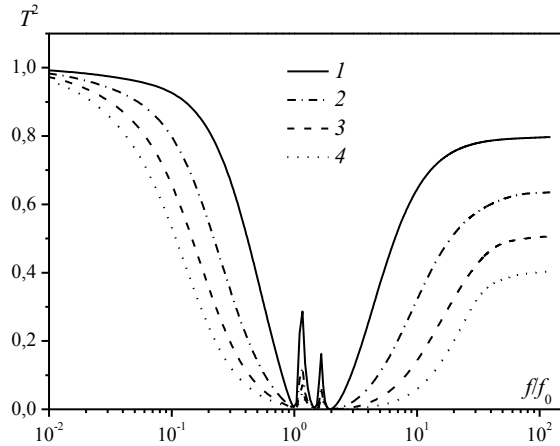


Fig. 3. Coefficient of sound transmission through a bubbly screen at different layer thickness.

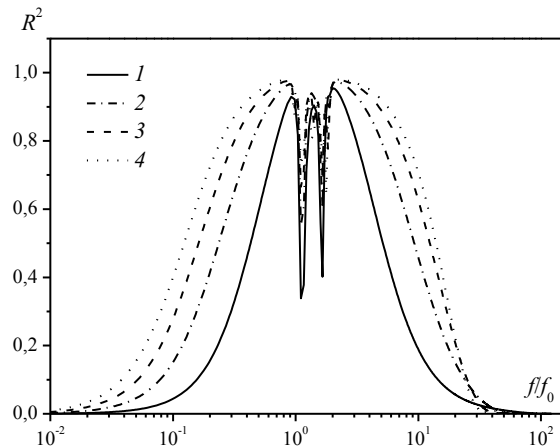


Fig. 4. Coefficient of sound reflection through a bubbly screen at different layer thickness.

On the figure 5 we have compared the theory with the experimental data [24] for coefficients of sound reflection and transmission through a bubbly screen. In this paper, heat and mass transfer are taken into account in our model, in contrast to [24], where only heat exchange is taken into account. Parameters of dispersed medium: $\alpha_{20}=0.0146131$, $a_0=2$ mm, $d_2=1.5$ sm, $f_0=1630$ Hz. Curves R and T for the mixture of water with air bubbles are plotted on the basis of the dispersion equation [25], which, in the absence of phase transitions ($k_V \rightarrow 0$), is consistent with Eq. (1). We note that for the transmission coefficient agreement with the experiment

is obtained. Our model, in a particular case, describes the experimental data well.

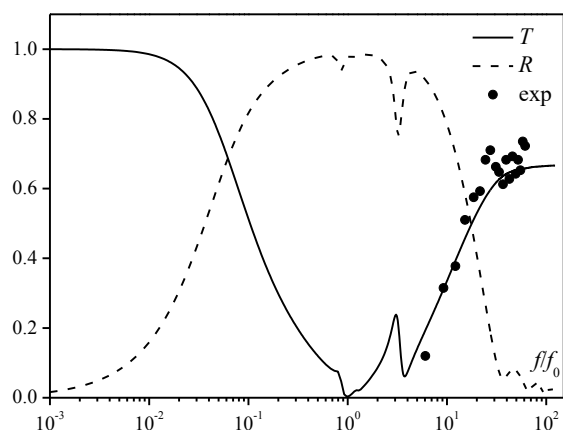


Fig. 5. Comparison of the coefficients of sound reflection and transmission through a bubbly screen with the experimental data [24].

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References

1. R.I. Nigmatulin, *Dynamics of multiphase media* (New York: Hemisphere, 1991)
2. S. Temkin, *Suspension acoustics: an introduction to the physics of suspensions* (Cambridge: Cambridge University Press, 2005)
3. T.G. Leighton, *The acoustic bubble* (London: Academic Press, 1994)
4. V.E. Nakoryakov, B.G. Pokusaev, I.R. Shreiber, *Wave propagation in gas-liquid media* (New York: CRC Press, 1993)
5. D.A. Gubaidullin, *Dynamics of two-phase gas-vapor-droplet media* (Kazan: Kazan mathematical society, 1998)
6. Y. Hao, A. Prosperetti, *Physics of Fluids* **11**, 2008 (1999)
7. K.W. Commander, A. Prosperetti, *J. Acoustical Society of America* **85**, 732 (1989)
8. R.I. Nigmatulin, V.Sh. Shagapov, N.K. Vakhitova, *USSR Academy of Sciences Doklady* **304**, 1077 (1989)
9. V.Sh. Shagapov, *J. of Applied Mechanics and Technical Physics* **18**, 77 (1977)
10. R.I. Nigmatulin, N.S. Khabeev, F.B. Nagiev, *International Journal Heat and Mass Transfer* **24**, 1033 (1981)
11. D.A. Gubaidullin, A.A. Nikiforov, R.N. Gafiyatov, *High Temperature* **50**, 250 (2012)
12. R.N. Gafiyatov, D.A. Gubaidullin, A.A. Nikiforov, *Fluid Dynamics* **48**, 366 (2013)
13. D.A. Gubaidullin, R.N. Gafiyatov, *J. Phys.: Conf. Ser.* **567**, 012020 (2014)
14. D.A. Gubaidullin, D.D. Gubaidullina, Yu.V. Fedorov, *J. Phys.: Conf. Ser.* **567**, 012011 (2014)
15. D.A. Gubaidullin, Yu.V. Fedorov, *Acoustical Physics* **62**, 179 (2016)
16. D.A. Gubaidullin, D.D. Gubaidullina, Yu.V. Fedorov, *J. Phys.: Conf. Ser.* **669**, 012011 (2016)
17. D.A. Gubaidullin, A.A. Nikiforov, R.N. Gafiyatov, *High Temperature* **53**, 240 (2015)
18. D.A. Gubaidullin, A.A. Nikiforov, R.N. Gafiyatov, *J. Phys.: Conf. Ser.* **669**, 012019 (2016)
19. D.A. Gubaidullin, R.N. Gafiyatov, *J. Phys.: Conf. Ser.* **789**, 012019 (2017)
20. D.A. Gubaidullin, Yu.V. Fedorov, *Doklady Physics* **61**, 499 (2016)
21. D.A. Gubaidullin, Yu.V. Fedorov, *Fluid Dynamics* **52**, 107 (2017)
22. D.A. Gubaidullin, D.D. Gubaidullina, Yu.V. Fedorov, *J. Phys.: Conf. Ser.* **789**, 012016 (2017)
23. L.M. Brekhovskikh, O.A. Godin, *Acoustics of layered media* (Moscow: Nauka, 1989)
24. T.M. Tien, *Sound propagation through a bubble screen at finite gas-volume fraction* (Taiwan: National Cheng Kung University, 2001)
25. D.A. Gubaidullin, A.A. Nikiforov, R.N. Gafiyatov, *Izv. Vuzov. Problemy Energetiki* **3-4**, 3 (2009)