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Influences of the Control on the Nonlinear Vibrations of a Variable Compression Ratio Mechanism

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Abstract. For the mechanism described in references the study of the nonlinear vibrations is performed considering a multibody approach for the elements of the mechanism and different laws of motion for the control element. A great attention is paid to the equilibrium of the motion. The influence of different parameters of control is highlighted in the paper. The results are numerically validated.

1 Introduction

The references were presented in our previous papers [12-14] and we will not repeat here.

The mechanism (Fig. 1) consists in the shaft OA which rotates uniformly around the point O, the triangular shell ABC, the levers CE and BD, and the piston situated at the point D.

The distance between the point E and the OY -axis is denoted by d, while the distance between the piston and the same axis is equal to e.

The triangular shell ABC and the levers BD and CE have plane-parallel motions.

The end E of the lever CE can move in vertical direction during the functioning of the engine, resulting different compression ratios and different points of extreme for the piston.

We denote by C_i, $i = \overline{1,5}$, the centers of mass for different elements of the mechanism, and by φ_i, $i = \overline{1,5}$, the rotational angles for the same elements.

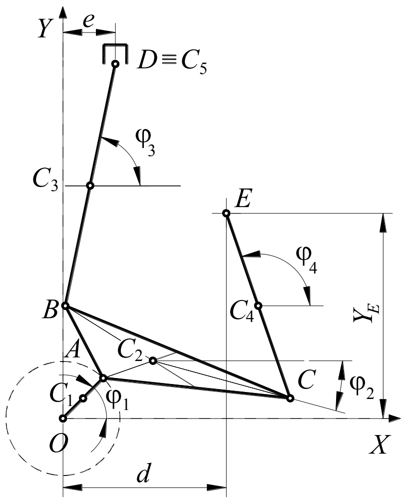

Fig. 1. The mechanism.

2 Multibody approach

The position of the mechanism is completely known if we know the position of the center of mass for each element (that is, the coordinates X_{C_i} and Y_{C_i}) and the corresponding rotational angles φ_i. It results a number of 15 unknowns.

Let us denote by $[\mathbf{A}_i]$, $i = \overline{1,5}$, the rotation matrix of an element,

$$[\mathbf{A}_i] = \begin{bmatrix} \cos\varphi_i & -\sin\varphi_i \\ \sin\varphi_i & \cos\varphi_i \end{bmatrix}, \tag{1}$$

and let us consider a generic point P that belongs to the elements i_1 and i_2.

If $x_P^{(i_1)}$ and $y_P^{(i_1)}$ are the coordinates of the point P relative to the mobile reference system $C_{i_1}x_{i_1}y_{i_1}$, while X_P and Y_P are the coordinates of the same point relative to the fixed reference system OXY, then one may write the relation

$$\begin{bmatrix} X_P \\ Y_P \end{bmatrix} = \begin{bmatrix} X_{C_{i_1}} \\ Y_{C_{i_1}} \end{bmatrix} + [\mathbf{A}_{i_1}] \begin{bmatrix} x_P^{(i_1)} \\ y_P^{(i_1)} \end{bmatrix} = \begin{bmatrix} X_{C_{i_1}} + x_P^{(i_1)}\cos\varphi_{i_1} - y_P^{(i_1)}\sin\varphi_{i_1} \\ Y_{C_{i_1}} + x_P^{(i_1)}\sin\varphi_{i_1} + y_P^{(i_1)}\cos\varphi_{i_1} \end{bmatrix}, \tag{2}$$

in which $X_{C_{i_1}}$ and $Y_{C_{i_1}}$ are the coordinates of the center of mass C_{i_1} relative to the fixed reference system.

Considering now the element i_2 for which the coordinates of the center of mass C_{i_2} relative to the fixed reference system are $X_{C_{i_2}}$ and $Y_{C_{i_2}}$, while the coordinates of the point P in the local reference system are $x_P^{(i_2)}$ and $y_P^{(i_2)}$ one may write a similar expression

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$$\begin{bmatrix} X_P \\ Y_P \end{bmatrix} = \begin{bmatrix} X_{C_{i_2}} \\ Y_{C_{i_2}} \end{bmatrix} + \left[\mathbf{A}_{i_2}\right] \begin{bmatrix} x_P^{(i_2)} \\ y_P^{(i_2)} \end{bmatrix} = \begin{bmatrix} X_{C_{i_2}} + x_P^{(i_2)} \cos\varphi_{i_2} - y_P^{(i_2)} \sin\varphi_{i_2} \\ Y_{C_{i_2}} + x_P^{(i_2)} \sin\varphi_{i_2} + y_P^{(i_2)} \cos\varphi_{i_2} \end{bmatrix}. \tag{3}$$

Equating the relations (2) and (3), it results

$$\begin{bmatrix} X_{C_{i_1}} + x_P^{(i_1)} \cos\varphi_{i_1} - y_P^{(i_1)} \sin\varphi_{i_1} \\ Y_{C_{i_1}} + x_P^{(i_1)} \sin\varphi_{i_1} + y_P^{(i_1)} \cos\varphi_{i_1} \end{bmatrix} = \begin{bmatrix} X_{C_{i_2}} + x_P^{(i_2)} \cos\varphi_{i_2} - y_P^{(i_2)} \sin\varphi_{i_2} \\ Y_{C_{i_2}} + x_P^{(i_2)} \sin\varphi_{i_2} + y_P^{(i_2)} \cos\varphi_{i_2} \end{bmatrix}, \tag{4}$$

wherefrom one gets two constraint functions

$$X_{C_{i_1}} + x_P^{(i_1)} \cos\varphi_{i_1} - y_P^{(i_1)} \sin\varphi_{i_1} = X_{C_{i_2}} + x_P^{(i_2)} \cos\varphi_{i_2} - y_P^{(i_2)} \sin\varphi_{i_2}, \tag{5}$$

$$Y_{C_{i_1}} + x_P^{(i_1)} \sin\varphi_{i_1} + y_P^{(i_1)} \cos\varphi_{i_1} = Y_{C_{i_2}} + x_P^{(i_2)} \sin\varphi_{i_2} + y_P^{(i_2)} \cos\varphi_{i_2}. \tag{6}$$

We denote by $\{\mathbf{q}\}$ the column matrix

$$\{\mathbf{q}\} = \left[X_{C_1} \;\; Y_{C_1} \;\; \varphi_1 \;\ldots\; X_{C_5} \;\; Y_{C_5} \;\; \varphi_5\right]^{\mathrm{T}}. \tag{7}$$

The coordinates of the point C_1 offer the first two constraint functions

$$f_1(\{\mathbf{q}\}) = X_{C_1} - OC_1 \cos\varphi_1 = 0, \quad f_2(\{\mathbf{q}\}) = Y_{C_1} - OC_1 \sin\varphi_1 = 0. \tag{8}$$

Assuming a homogeneous triangular shell, denote by θ_1 and θ_2 the angles BC_2C and AC_2C, and by m_a, m_b, m_c the lengths of the medians of the triangle ABC, one may write

$$m_a^2 = \frac{2\left[(AC)^2 + (AB)^2\right] - (BC)^2}{4}, \quad m_b^2 = \frac{2\left[(AB)^2 + (BC)^2\right] - (AC)^2}{4}, \quad m_c^2 = \frac{2\left[(BC)^2 + (AC)^2\right] - (AB)^2}{4}, \tag{9}$$

$$\theta_1 = \arccos\left(\frac{\left(\frac{2}{3}m_c\right)^2 + \left(\frac{2}{3}m_b\right)^2 - (BC)^2}{2 \times \frac{2}{3}m_c \times \frac{2}{3}m_b}\right), \quad \theta_2 = \arccos\left(\frac{\left(\frac{2}{3}m_a\right)^2 + \left(\frac{2}{3}m_c\right)^2 - (AC)^2}{2 \times \frac{2}{3}m_a \times \frac{2}{3}m_c}\right), \tag{10}$$

$$x_A^{(2)} = \frac{2}{3}m_a \cos\theta_2, \; y_A^{(2)} = \frac{2}{3}m_a \sin\theta_2, \; x_B^{(2)} = \frac{2}{3}m_b \cos\theta_1, \; y_B^{(2)} = \frac{2}{3}m_b \sin\theta_1, \; x_C^{(2)} = \frac{2}{3}m_c, \; y_C^{(2)} = 0. \tag{11}$$

Since point A belongs to the elements 1 and 2, point B belongs to the elements 2 and 3, point C belongs to the elements 2 and 4, and the point D belongs to the elements 3 and 5, one may write the relations (5), resulting the following constraint functions

$$f_3(\{\mathbf{q}\}) = X_{C_1} + x_A^{(1)} \cos\varphi_1 - y_A^{(1)} \sin\varphi_1 - X_{C_2} - x_A^{(2)} \cos\varphi_2 + y_A^{(2)} \sin\varphi_2 = 0, \tag{12}$$

$$f_4(\{\mathbf{q}\}) = Y_{C_1} + x_A^{(1)} \sin\varphi_1 + y_A^{(1)} \cos\varphi_1 - Y_{C_1} - x_A^{(2)} \sin\varphi_2 - y_A^{(2)} \cos\varphi_2 = 0, \tag{13}$$

$$f_5(\{\mathbf{q}\}) = X_{C_2} + x_B^{(2)} \cos\varphi_2 - y_B^{(2)} \sin\varphi_2 - X_{C_3} - x_B^{(3)} \cos\varphi_3 + y_B^{(3)} \sin\varphi_3 = 0, \tag{14}$$

$$f_6(\{\mathbf{q}\}) = Y_{C_2} + x_B^{(2)} \sin\varphi_2 + y_B^{(2)} \cos\varphi_2 - Y_{C_3} - x_B^{(3)} \sin\varphi_3 - y_B^{(3)} \cos\varphi_3 = 0, \tag{15}$$

$$f_7(\{\mathbf{q}\}) = X_{C_2} + x_C^{(2)} \cos\varphi_2 - y_C^{(2)} \sin\varphi_2 - X_{C_4} - x_C^{(4)} \cos\varphi_4 + y_C^{(4)} \sin\varphi_4 = 0, \tag{16}$$

$$f_8(\{\mathbf{q}\}) = Y_{C_2} + x_C^{(2)} \sin\varphi_2 + y_C^{(2)} \cos\varphi_2 - Y_{C_4} - x_C^{(4)} \sin\varphi_4 - y_C^{(4)} \cos\varphi_4 = 0, \tag{17}$$

$$f_9(\{\mathbf{q}\}) = X_{C_3} + x_D^{(3)} \cos\varphi_3 - y_d^{(3)} \sin\varphi_3 - X_{C_5} - x_D^{(5)} \cos\varphi_5 + y_D^{(5)} \sin\varphi_5 = 0, \tag{18}$$

$$f_{10}(\{\mathbf{q}\}) = Y_{C_3} + x_D^{(3)} \sin\varphi_3 + y_D^{(3)} \cos\varphi_3 - Y_{C_5} - x_D^{(5)} \sin\varphi_5 - y_D^{(5)} \cos\varphi_5 = 0. \tag{19}$$

Keeping into account that the point C_5 moves only in the vertical direction, and the element 5 (the piston) does not rotate, one gets another two constraint functions

$$f_{11}(\{\mathbf{q}\}) = X_{C_5} - e = 0, \tag{20}$$

$$f_{12}(\{\mathbf{q}\}) = \varphi_5 = 0. \tag{21}$$

In this paper we assume that the point E has a fixed position, that is, another two constraint functions are obtained

$$f_{13}(\{\mathbf{q}\}) = X_{C_4} + C_4E \cos\varphi_4 - d = 0, \tag{22}$$

$$f_{14}(\{\mathbf{q}\}) = Y_{C_4} + C_4E \sin\varphi_4 - (Y_E)_0 = 0, \tag{23}$$

where $(Y_E)_0$ is the fixed coordinate of the point E in the vertical direction.

Since we obtained 14 constraint functions $f_i(\{\mathbf{q}\}) = 0$, $i = \overline{1,14}$, it results that the mechanism has one degree of freedom.

We apply the theory of the extremes with constraints resulting the Lagrange function

$$F(\{\mathbf{q}\}, \lambda_1, \ldots, \lambda_{14}) = Y_{C_5} + \sum_{i=1}^{14} \lambda_i f_i(\{\mathbf{q}\}). \tag{24}$$

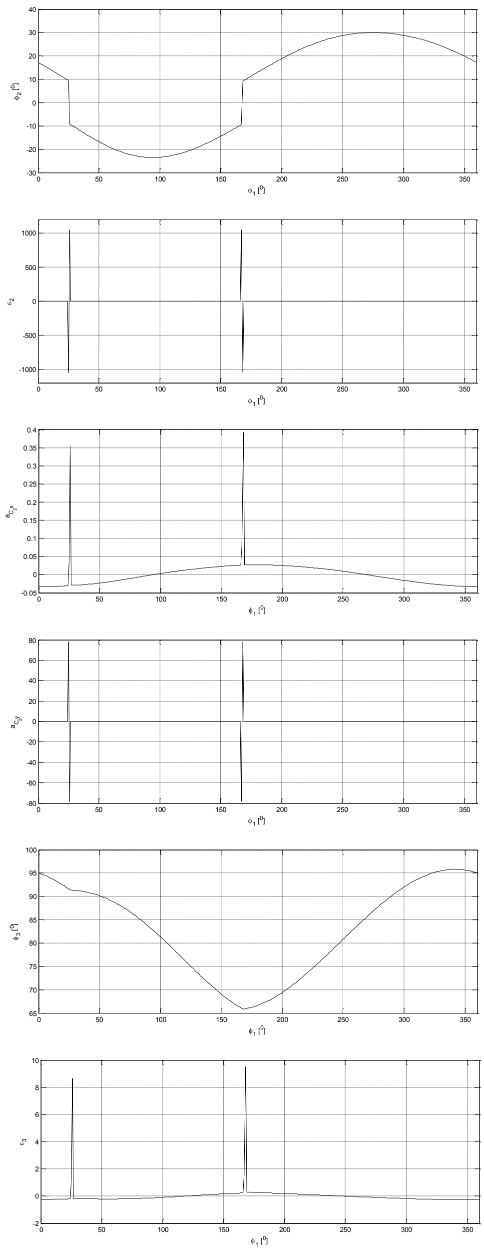
One has to solve the following nonlinear system of 29 equations with 29 unknowns (X_{C_1}, Y_{C_1}, φ_1, ..., X_{C_5}, Y_{C_5}, φ_5, λ_1, ..., λ_{14})

$$\frac{\partial F}{\partial q_j} = 0, \; \frac{\partial F}{\partial \lambda_i} = 0, \; i = \overline{1,14}, \; j = \overline{1,15}. \tag{25}$$

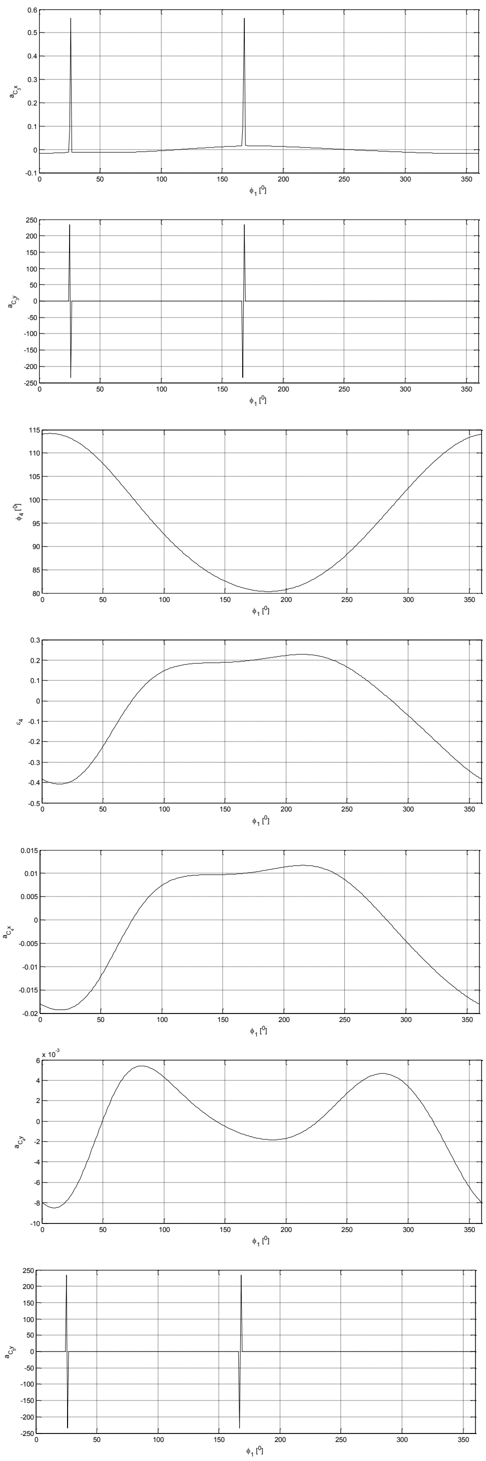
If the coordinate Y_E is not a fixed one, that is, the end E of the lever may move in the vertical direction, then the mechanism may have one degree of freedom (if the law of motion $Y_E = Y_E(t)$ is known), or two degrees of freedom (if the law is not known). The discussion is made in [12].

3 Numerical simulation

We selected the following standard values for different parameters of the mechanism: $(AB)_0 = 0.043$ m, $(BC)_0 = 0.128$ m, $(CA)_0 = 0.099$ m, $(CE)_0 = 0.103$ m, $(OA)_0 = 0.030$ m, $(BD)_0 = 0.130$ m, $(d)_0 = 0.086$ m, $(e)_0 = 0$ m, $(Y_E)_0 = 0.108$ m. For each parameter a maximum deviation of ± 0.002 m is used during the numerical simulation.



a) *b*) *c*) *d*) *e*) *f*)

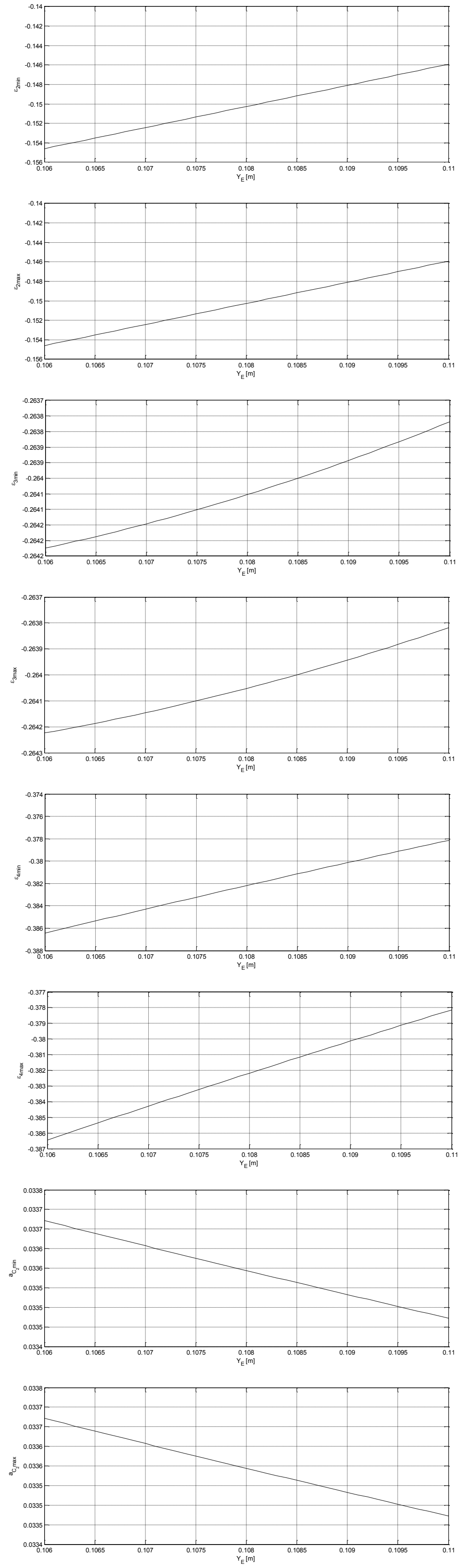


g) *h*) *i*) *j*) *k*) *l*) *m*)

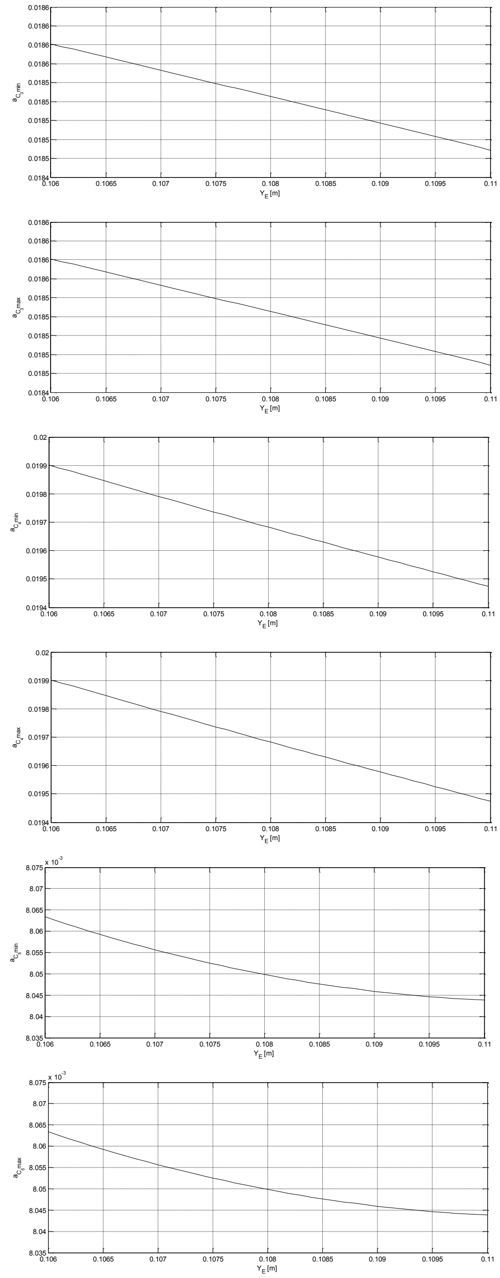
Fig. 2. Different variations in the standard case: *a*) $\varphi_2 = \varphi_2(\varphi_1)$; *b*) $\varepsilon_2 = \varepsilon_2(\varphi_1)$; *c*) $a_{C_2x} = a_{C_2x}(\varphi_1)$; *d*) $a_{C_2y} = a_{C_2y}(\varphi_1)$; *e*) $\varphi_3 = \varphi_3(\varphi_1)$; *f*) $\varepsilon_3 = \varepsilon_3(\varphi_1)$; *g*) $a_{C_3x} = a_{C_3x}(\varphi_1)$; *h*) $a_{C_3y} = a_{C_3y}(\varphi_1)$; *i*) $\varphi_4 = \varphi_4(\varphi_1)$; *j*) $\varepsilon_4 = \varepsilon_4(\varphi_1)$; *k*) $a_{C_4x} = a_{C_4x}(\varphi_1)$; *l*) $a_{C_4y} = a_{C_4y}(\varphi_1)$; *m*) $a_{C_5y} = a_{C_5y}(\varphi_1)$.

We are interesting in the reduced linear and angular accelerations of different elements. Some diagrams are captured in the Figs. 2 (for the standard values), 3 (by varying Y_E), and 4 (by varying CE).

One may observe that when Y_E increases the linear acceleration decrease and angular accelerations increase. This statement holds true for all elements; consequently, when the inertial forces decrease, the inertial torques increase.



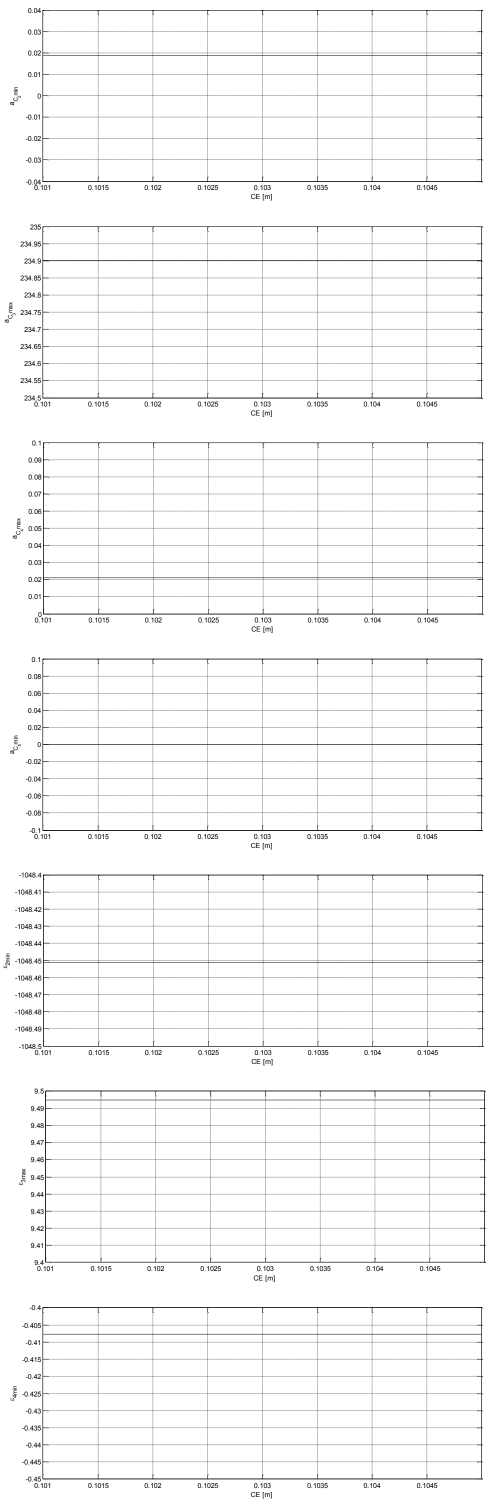
a) *b*) *c*) *d*) *e*) *f*) *g*) *h*)



i) *j*) *k*) *l*) *m*) *n*)

Fig. 3. Different variations of parameters when Y_E varies: *a*) $\varepsilon_{2\,\min} = \varepsilon_{2\,\min}(\varphi_1)$; *b*) $\varepsilon_{2\,\max} = \varepsilon_{2\,\max}(\varphi_1)$; *c*) $\varepsilon_{3\,\min} = \varepsilon_{3\,\min}(\varphi_1)$; *d*) $\varepsilon_{3\,\max} = \varepsilon_{3\,\max}(\varphi_1)$; *e*) $\varepsilon_{4\,\min} = \varepsilon_{4\,\min}(\varphi_1)$; *f*) $\varepsilon_{4\,\max} = \varepsilon_{4\,\max}(\varphi_1)$; *g*) $a_{C_2\,\min} = a_{C_2\,\min}(\varphi_1)$; *h*) $a_{C_2\,\max} = a_{C_2\,\max}(\varphi_1)$; *i*) $a_{C_3\,\min} = a_{C_3\,\min}(\varphi_1)$; *j*) $a_{C_3\,\max} = a_{C_3\,\max}(\varphi_1)$; *k*) $a_{C_4\,\min} = a_{C_4\,\min}(\varphi_1)$; *l*) $a_{C_4\,\max} = a_{C_4\,\max}(\varphi_1)$; *m*) $a_{C_5\,\min} = a_{C_5\,\min}(\varphi_1)$; *n*) $a_{C_5\,\max} = a_{C_5\,\max}(\varphi_1)$.

We may state that the contribution of the variation of CE for the variations of the extreme values of linear and angular accelerations does not exist, as it results from Fig. 4. Moreover, the contribution of the variation of CE for the variations of the instantaneous values of linear and angular accelerations is not significant.



a) *b*) *c*) *d*) *e*) *f*) *g*)

Fig. 4. Different variations of parameters when CE varies: *a*) $a_{C_2\,\min} = a_{C_2\,\min}(\varphi_1)$; *b*) $a_{C_3\,\max} = a_{C_3\,\max}(\varphi_1)$; *c*) $a_{C_4\,\max} = a_{C_4\,\max}(\varphi_1)$; *d*) $a_{C_5\,\min} = a_{C_5\,\min}(\varphi_1)$; *e*) $\varepsilon_{2\,\min} = \varepsilon_{2\,\min}(\varphi_1)$; *f*) $\varepsilon_{3\,\max} = \varepsilon_{3\,\max}(\varphi_1)$; *g*) $\varepsilon_{4\,\min} = \varepsilon_{4\,\min}(\varphi_1)$.

It results that the only element with a significant contribution is the coordinate Y_E of the control lever.

One may easily observe that the motion is always stable. The stability is a consequence of the previous diagrams which show that the variations of different parameters are small when the parameters Y_E and CE vary. It results the stability with respect these two parameters. Because the variations of the extreme values of the linear and angular accelerations are bounded, it also results that the variations of the instantaneous linear and angular accelerations are bounded and they can be made as small as we want, resulting the stability of the motion.

4 Conclusions

From the previous discussion one may conclude:

– the existence of jumps for linear and angular accelerations, which lead to greater inertial forces and torques;

– a very complicate synthesis of mechanism.

Particular values for the dimensions of particular elements [25] may lead to an useless mechanism, that is, the mechanism works like a simple crank-shaft one, no matter which is the position of the point E .

The previous discussion implies to involve more elements in the study. The classical synthesis of the mechanisms, that is, the establishing of certain dimensions of the elements, must be continued with the study of the resulted linear and angular accelerations in order to determine the inertial forces and torques, and the study of angular velocities in order to determine the wear of the joints.

Keeping into account the previous remark [25], it is possible to determine a variable compression ratio mechanism which satisfies the constructive and dynamical requirement.

References

1. F. Freudenstein, E.R. Maki, Journal of Mechanisms, Transmissions and Automation in Design **105**, 259 (1983)
2. T. Hoeltgebaum, R. Simoni, D. Martins, Mechanism and Machine Theory **96**, 308 (2016)
3. V. Hara, N. Pandrea, D. Popa, M. Stan, S. Boncea, 1995, *Adaptive thermal engines* (Publishing House of the University of Pitești, 1995)
4. A. Clenci, *Automobile equipped with engine with variable compression ratio* (Research report 2003)
5. I. Miller, www.me.udel.edu/eeg425 / SaabVarComp.doc, 2001.
6. P.J. Mahlesh, V.K. Aparna, International Journal of Mechanical Engineering Research and Development (IJMERD) **2**, 9 (2012)
7. A. Teodorczyk, published on Institute of Heat Engineering Warsaw (1995)
8. http://freerepublic.com/ focus/f-news/1818771/posts by Red Badger (2007)
9. SAE article http://articles.sae.org/6043/ published

by John Kendall (2009)

10. SAE article http://articles.sae.org/15040/ published by John Kendall (2016)
11. S. Nagarajaa, K. Sooryaprakashb, R. Sudhakaran, *Proceedia Earth and Planetary Science* (2015)
12. B. Mănescu, I. Dragomir, N.-D. Stănescu, *AVMS 2017*, Timișoara (2017)
13. B. Mănescu, I. Dragomir, N.-D. Stănescu, N. Pandrea, *ICMSAV XLI*, Cluj-Napoca (2017)
14. B. Mănescu, I. Dragomir, N.-D. Stănescu, N. Pandrea, A. Clenci, D. Popa, Aspects in the Synthesis of a Variable Compression Ratio Mechanism, *Car 2017*, Pitești (2017)
15. K. Sivaramakrishnan, 2017, Egyptian Journal of Petroleum in press (2017)
16. K. Satyanarayana, A.K. Viswanath, T.V. Hanumanta Rao, S.V. Uma Maheswara Rao, Procedia Engineering **173**, 1672 (2017)
17. B. Wu, Q. Zhan, X. Yu, G. Lv, X. Nie, S. Liu, Applied Thermal Engineering **118**, 621 (2017)
18. J.A.A. Yamin, M.H. Dado, Applied Energy **77**, 447 (2004)
19. L. Eriksson, I. Nielsen 2014, *Modeling and control of engines and drivelines* (John Wiley & Sons, London 2014)
20. L.W. Tsai, *Mechanism design: Enumeration of kinematic structures according to function* (CRC Press, London 2000)
21. H. S. Yan, *Creative design of mechanical devices* (Springer, Berlin 1998)
22. Y. Nilsson, *Modelling for fuel optimal control of a variable compression engine* (Linköping University 2007)
23. P. Podevin, A. Clenci, 2008, *Moteurs à taux de compression variable* (BM 2525, Editions Techniques de l'Ingénieur, Paris 2008)
24. A. Shaik, N. Shenbaga Vinayaga Moorthi, R Rudramoorthy, Proc. IMechE Part D: J. Automobile Engineering **221**, 1159 (2007)
25. Pandrea N, *Private communications* (2017)