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Synchronisation phenomenon in three blades rotor driven by regular or chaotic oscillations

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Abstract. The goal of the paper is to analysed the influence of the different types of excitation on the synchronisation phenomenon in case of the rotating system composed of a rigid hub and three flexible composite beams. In the model is assumed that two blades, due to structural differences, are de-tuned. Numerical calculation are divided on two parts, firstly the rotating system is exited by a torque given by regular harmonic function, than in the second part the torque is produced by chaotic Duffing oscillator. The synchronisation phenomenon between the beams is analysed both either for regular or chaotic motions. Partial differential equations of motion are solved numerically and resonance curves, time series and Poincaré maps are presented for selected excitation torques.

1 Introduction

The synchronisation phenomenon was discovered by Christiaan Huygens in the XVII centry. He hang two pendulums clocks on one beam and found out that after some transient time pendulums moved synchronously. Kapitaniak and et al. have recalled the same experiment in the paper [1]. Authors chose two pendulum clocks as identical as possible and observed the behavior of them, also they checked whether Huygens was able to observe all type of synchronisation in the XVII centry. Moreover, the computer simulations have been presented to check the influence of non identical clocks on synchronisation process. Although the synchronisation phenomenon is quite popular and well known issue, most of the scientist describe systems composed of different type of pendulums. The synchronisation of two self-excited double pendulums attached to the horizontal beam has been considered in [2]. Authors derived the synchronisation conditions for a model and explained types of synchronisation as well. The theoretical and experimental studies of synchronisation of two rotating in a vertical plane parametric pendulums attached to elastic support under harmonic excitation have been presented in [3]. In that model two types of synchronous states have been found by authors, complete and phase synchronisation. The experimental and numerical analysis of a set of double pendulums have been described in [4]. Authors was able to observe different type of synchronous configurations with a constant phase between pendulums. All previously mentioned papers, focused the vertical motion of pendulums. The synchronisation phenomenon as well as nonlinear dynamics for a model composed of two pendulums attached to a rotating hub in the horizontal plane have been studied in [5]. Authors found out that in case of non-symmetric pendulums the synchronisation with locked phase occurred. Furthermore, they proved that natural frequencies and vibration modes strongly depends on the mass of the hub. Similar conclusion about the influence of the mass hub on a rotating system has been presented in [6]. In that paper authors analysed beam with a heavy tip mass on one end fixed to a rotating beam. The nonlinear model included bending, tension and nonlinear curvature, authors proved that nonlinear terms in equation of motion cause softening effect on the system. While, dynamics of rotating structure with a thin-wall composite beams have been analysed in [7, 8], where authors derived equation of motion using Hamilton principle and Galerkin method to reduce the model to the ordinary differential equations. Moreover, the nonclassical effects as material anisotropy, transverse shear and rotary inertia have been considered in the hub-beam model. Most of studies on rotating structures have been motivated by mechanical and aerospace engineering; for instance helicopters rotors, jet engines and wind turbines are the typical examples of rotating structures in engineering. The equations of motion dedicated to helicopter blades have been presented in [9, 10].

The aim of this paper is to study synchronisation effect in the system composed of three composite beams attached to a hub rotating in the horizontal plane. The blades have been analysed as a Timoshenko beam. The first beam is considered as reference one while beam No. 2 is 10% thicker and beam No. 3 is 5% thicker compering to the reference one. Motivation for this study arises from observation of a production process of composite beams, in which it was not very easy to make three beams with the same geometric dimensions. Model with one de-tuned beam have been studied by the authors in their previous

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paper [11]. The synchronisation phenomenon in the preset paper is studied when the model is exited by harmonic torque or torque given by a chaotic oscillator. The influence of aerodynamic loads is not taken into account in the analysed model.

2 Model and equations of motion

Let us consider a rotor composed of three slender, straight and elastic composite beams fixed to a rigid hub, of radius R_0 and mass moment of inertia J_h, The hub is able to oscillate or rotate about vertical axis CZ_0 as shown in Figure 1.

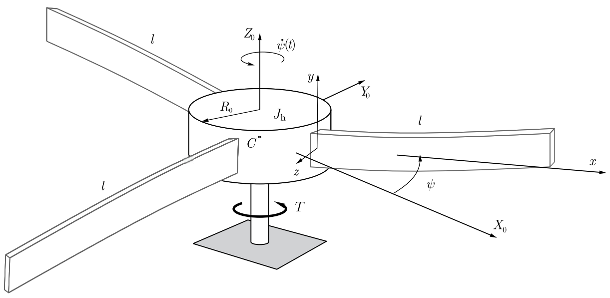

Figure 1. Model of the rotating hub with three elastic beams

The hub current position is represent by angle $\psi(t)$ with respect to the inertial reference frame (X_0, Y_0, Z_0). Angular velocity of the rotor $\dot{\psi}(t)$ is assumed to be not necessarily constant. The system is driven by external torque T, which is applied to the hub. Every single beam is described by the length l_i, width and thickness of the cross-section d_i and h_i $(i = 1, 2, 3)$, respectively. The model considers one nominal beam (beam No. 1) and two with 10% and 5% higher thickness, thus beam No.2 and No.3 are de-tuned. The system of partial differential equations (PDEs) for the hub and each beam has been derived by using the Hamilton's principle of least action as is presented in [7, 8]. Next, the Galerkin projection method has been applied to reduce system of PDEs to a set of ordinary equations of motions (ODEs). After conversion, the final set of governing equations of motion takes a form:

$$\left(J_h + 1 + \sum_{i=2}^{N} J_{bi} + \sum_{i=1}^{N} \alpha_{hi2} q_i^2\right)\ddot{\psi} + \sum_{i=1}^{N} \alpha_{hi1}\ddot{q}_i + \sum_{i=1}^{N} \alpha_{hi3} q_i \dot{q}_i \dot{\psi} + \zeta_h \dot{\psi} = \mu(t)$$
$$\ddot{q}_i + \alpha_{i2}\ddot{\psi} + \alpha_{i4} q_i \dot{q}_i \dot{\psi} + (\alpha_{i1} + \alpha_{i3}\dot{\psi}^2) q_i + \zeta_i \dot{q}_i = 0 \qquad (1)$$

The first of the set of Equation 1 represent the dynamics of the hub, where term J_h and J_{bi} are dimensionless mass moment of inertia of the hub and dimensionless mass moment of inertia of each beam, respectively. All these are expressed as a magnitude of the inertia of the first beam. Whereas, the dimensionless angular velocity of the hub is denoted as $\dot{\psi} = \frac{d\psi}{dt}$. Term $\mu(t)$ is considered as a external torque (excitation of the rotor). The excitation is investigated in two variants, firstly as a driving torque, expressed as analytical function $\mu = \mu_0 + \rho \sin(\omega t)$ and secondly as a chaotic Duffing oscillator, where $\mu = \alpha_d x$. The variable x is calculated from the Duffing's equation $\ddot{x} + k\dot{x} + x^3 = \beta + \rho \cos(\omega t)$ as have been presented by Ueda in [12]. The second equation of the set 1 corresponds to a dynamics of each beam, where q_i $(i = 1, 2, 3)$ is the generalized coordinate of the complex flexural-torsional specimen deformation [8]. Furthermore, the viscous damping coefficients for a each beam ζ_i and the hub ζ_h are arbitrary included.

3 Results of numerical studies

Firstly, all coefficients that are presented in equation of motion as α_{ij} have been calculated on a basis of the real glass-epoxy laminate beams samples, the values are given in the Table 1.

Table 1. Dimensionless coefficients, based on the real physical model

Beam No.1	Beam No. 2	Beam No. 3
$\alpha_{11} = 12.0388$	$\alpha_{21} = 14.5664$	$\alpha_{31} = 13.2728$
$\alpha_{12} = 1.97656$	$\alpha_{22} = 1.97681$	$\alpha_{32} = 1.97669$
$\alpha_{13} = 0.49465$	$\alpha_{23} = 0.49462$	$\alpha_{33} = 0.49464$
$\alpha_{14} = -1.5517$	$\alpha_{24} = -1.5514$	$\alpha_{34} = -1.5515$
$\alpha_{h11} = 0.01120$	$\alpha_{h21} = 0.01232$	$\alpha_{h31} = 0.01176$
$\alpha_{h12} = -0.0084$	$\alpha_{h22} = -0.0093$	$\alpha_{h32} = -0.0088$
$\alpha_{h13} = -0.0169$	$\alpha_{h23} = -0.0186$	$\alpha_{h33} = -0.0177$

All the numerical calculation have been performed by direct numerical simulation of the system represented by Eqs. 1. As it has been mentioned before, the beam No. 2 and beam No. 3 are considered as de-tuned, 10% and 5% thicker then the remaining reference beam No. 1. The resonance curves and time histories plot are obtained for a hub exited by a harmonic torque ($\mu(t)$).

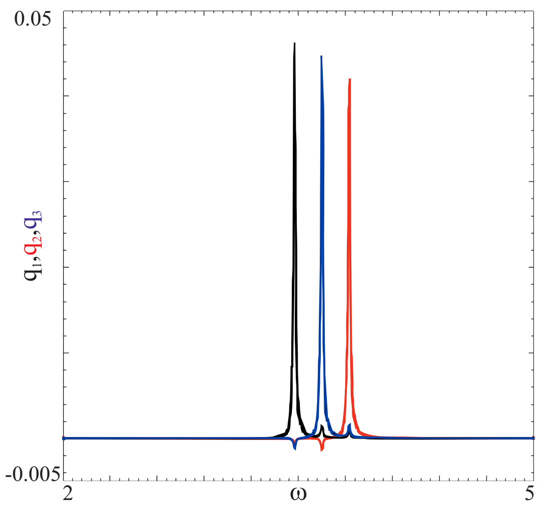

Figure 2. Resonance curves for amplitude of excitation $\rho = 0.01$ of displacements of each beam, beam No.1 - black line, beam No.2 - red line and beam No.3 - blue line

The solutions for a system with a harmonic function are sought near the resonance zones for a selected amplitude of excitation ($\rho = 0.01$), also the mass moment of inertia of the hub is assumed as $J_h = 5.0$.

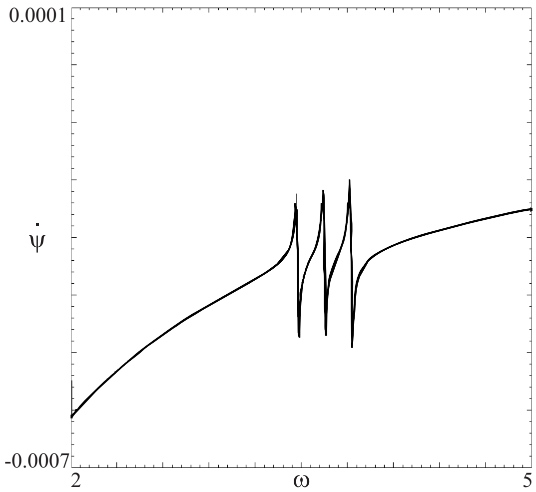

Figure 3. Resonance curves for amplitude of excitation $\rho = 0.01$ of angular velocity of the hub

The resonance curves for amplitude of displacement of each beam and angular velocity of the hub are shown in the Fig. 2 and 3, respectively. On both graphs three resonances can be noticed, first close to $\omega \approx 3.48$, second close to $\omega \approx 3.65$ and the third close to $\omega \approx 3.83$.

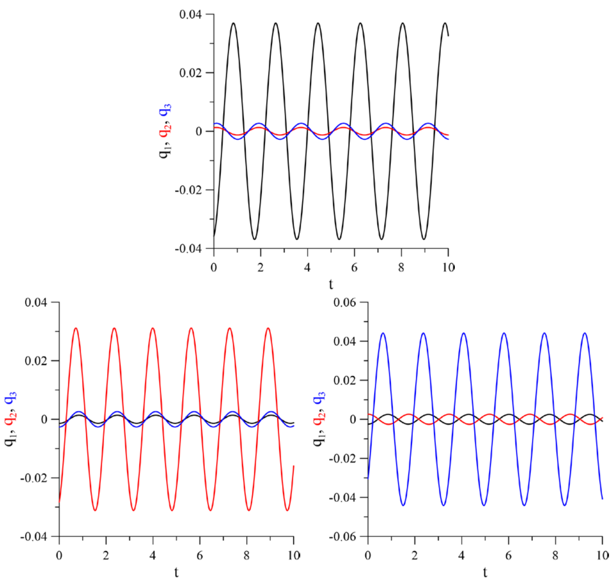

Figure 4. Time series of each beam (a) close to first resonance zone; (b) close to second resonance zone; (c) close to third resonance zone

The first resonance is observed for beam No. 1, with small oscillation of the second and third beam. In this resonance zone we observe anti-phase synchronisation with locked phase between beams No. 1 and No. 2 also between beams No. 1 and No. 3. Furthermore, second and third beam are completely synchronized close to the first resonance zone (see Fig.4a). Close to the second resonance zone large oscillations are localised in beam No.3, while motion of two other beams are very small. In this case the complete in-phase synchronisation can be observed for beam No. 1 and No. 3, while the motion of beam No. 2 compering to other beams is synchronized with locked phase (Fig. 4b). In the third resonance zone oscillations are found for a beam No. 2, also the synchronisation with locked phase can be observed between beam No. 2 and No. 3, as well as. between beam No. 1 and No. 3. While, motion of the first and the second beam is completely anti-phase synchronized (Fig. 4c).

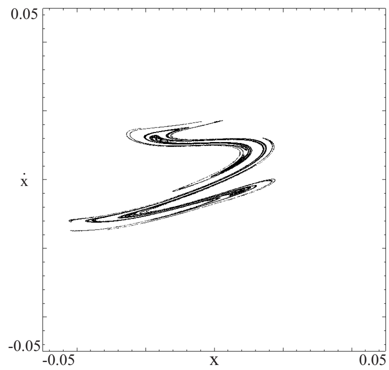

Figure 5. Portrait of a strange attractor obtained from the Duffing equation

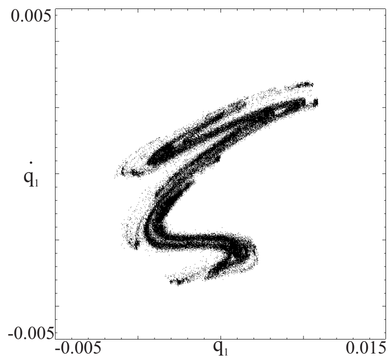

Figure 6. Portrait of a strange attractor for a beam No. 1

For the model excited by the chaotic Duffing oscillator, the Poincaré maps have been obtained. The parameters for a Duffing equation have been taken from [12], and $k = 0.05$, $\beta = 0.03$, $\rho = 0.16$. In Figure 5, the chaotic strange attractor for Duffing equation have been shown. The strange chaotic attractor have been observed for each beam, see Fig. 6, 7, 8. Furthermore, the time series are studied to analyse the synchronisation phenomenon. The complete in-phase synchronisation can be observed between every beam, when the rotating de-tuned structure is exited by Duffing oscillator (see Fig. 9).

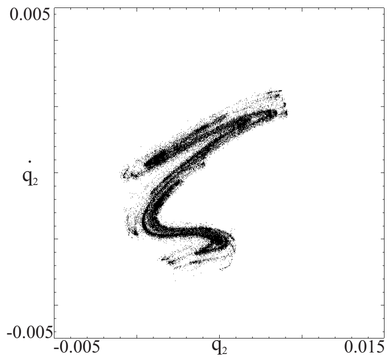

Figure 7. Portrait of a strange attractor for a beam No. 2

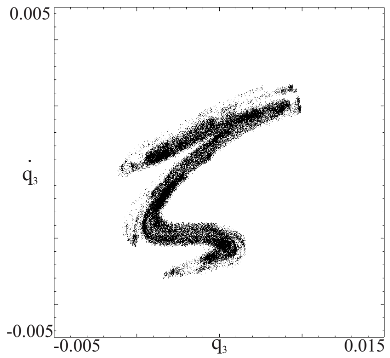

Figure 8. Portrait of a strange attractor for a beam No. 3

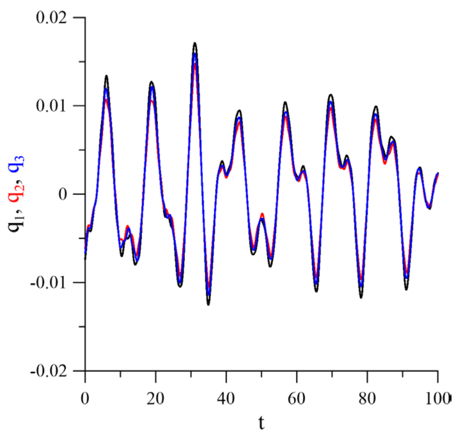

Figure 9. Time series of each beam in case of excitation by chaotic oscillator

4 Conclusions

In case of the rotating de-tuned structure excited by the harmonic torque three resonances have been observed, first close to $\omega \approx 3.48$, second around $\omega \approx 3.65$ and the third one close to $\omega \approx 3.83$. Resonance curves for amplitudes for every beam are different, due to complete de-tuning of the rotor. Moreover, different synchronisation scenario can be observed in each resonance zone, at the first resonance zone the synchronisation with locked phase and complete in-phase synchronisation have been found. Around the second resonance zone similar synchronisation scenario is noticed but for different beams. While close to third resonance the anti-phase synchronisation occurred between beams No. 1 and No. 2. Moreover, the portrait of strange chaotic attractor have been obtained for every single beam, in case of excitation by Duffing oscillator. The complete in-phase synchronisation of motion can be observed between every beam, when the rotor is excited by chaotic oscillations. To sum up the type of excitation completely change the process of synchronisation in the analysed de-tuned rotating structure.

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References

[1] M. Kapitaniak, K. Czolczynski, P. Perlikowski, A. Stefanski, T. Kapitaniak, Physics Reports **517**, 1 (2012)

[2] P. Koluda, P. Perlikowski, K. Czolczynski, T. Kapitaniak, Communications in Nonlinear Science and Numerical Simulation **19**, 977 (2014)

[3] A. Najdecka, T. Kapitaniak, M. Wiercigroch, International Journal of Non-Linear Mechanics **70**, 84 (2015)

[4] J. Strzalko, J. Grabski, J. Wojewoda, M. Wiercigroch, T. Kapitaniak, Chaos: An Interdisciplinary Journal of Nonlinear Science **22**, 047503 (2012)

[5] J. Warminski, Z. Szmit, J. Latalski, The European Physical Journal Special Topics **223**, 827 (2014)

[6] J. Warminski, J.M. Balthazar, 7th Conference on Dynamical Systems - Theory and Applications: proceedings **1** (2003)

[7] F. Georgiades, J. Latalski, J. Warminski, Meccanica **49**, 1833 (2014)

[8] J. Latalski, J. Warminski, G. Rega, Mathematics and Mechanics of Solids (2016)

[9] M.R.M. Crespo da Silva, C.C. Glynn, Journal of Structural Mechanics **6**, 437 (2007)

[10] M.R.M. Crespo da Silva, C.C. Glynn, Journal of Structural Mechanics **6**, 449 (2007)

[11] Z. Szmit, J. Warminski, **after reviews** (2017)

[12] Y. Ueda, Annals of the New York Academy of Sciences **357**, 422 (1980)