Dynamic responses of vehicle ballasted track interaction system for heavy haul trains

Yingjie Wang[1,*], *Zuzana* Dimitrovová[2, 3], and *Jong-Dar* Yau[4]

[1]School of Civil Engineering, Beijing Jiaotong University, Beijing, China.
[2]Department of Civil Engineering, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Caparica, Portugal
[3]IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, Portugal
[4]Department of Architecture, Tamkang University, New Taipei City, Taiwan.

Abstract. In this study, the dynamic responses of ballasted tracks subjected to heavy haul trains are investigated. The vehicle ballasted track interaction (VTI) model can be divided into three parts, which are vehicle model, ballasted track model and wheel/rail contact model. The vehicle is modelled as a multi-body system with 10 degrees of freedom. The ballasted track consisting of rail, sleeper, ballast, sub-ballast and sub-grade is modelled as four elastic components. The vehicle model and ballasted track model is linked by the wheel/rail contact model, which is determined by the nonlinear Hertzian theory. The VTI model is solved by an iterative procedure and validated with some field experiments. The dynamic responses of the vehicle/track system are compared with those from the moving load model. Moreover, the wheel/rail contact force, the force from rail to sleeper and the force from sub-ballast to subgrade are computed for different axle loads of 25t, 30t, 35t and 40t. It is deduced that maximum values of these forces increase in a linear form with the increasing of axle load.

1 Introduction

Railway has been playing a crucial role in the economic and social development during last decades[1]. Especially in China, several heavy haul railways such as Datong-Qinghuangdao railway, Shuozhou-Huanghua railway, and Watang-Rizhao railway were constructed and operated for raw coal transport from Inner Mongolia, Shanxi, and Shaanxi providences to east of China. Nowadays, the 30t per axle freight wagons have started operation in these three lines, which may cause much more damage for railway infrastructures.

To study the dynamic response of ballasted track under moving train, Oscarsson et al built a vertical train/track system model, where the ballast was modelled as rigid mass, and with this model the deflections, accelerations and forces in various track components are calculated[2]. Lei et al established a three layer beam model for ballasted track structures and studied the track vibration subjected to mixed passenger and freight cars by Fourier transform technique[3]. To model the ballast vibration, Zhai et al proposed a five-parameter model based on the assumption of pyramid distribution, which is validated using the experimental results[4]. Mosayebi et al built a locomotive car and ballasted track interaction model and the effect of some track parameters including sleeper type, track modulus on the displacements of track components was investigated[5]. From the available technical literatures, it is indicated that the sub-ballast layer is not considered in the railway track model. For this reason, this paper is focused on this topic.

In this paper, a vertical vehicle ballasted track interaction (VTI) model is developed, where the vehicle model is built using multi-body dynamics and the ballasted track model is developed with finite element method. The vehicle model and the ballasted track model are linked by the wheel/rail contact model, and the nonlinear Hertzian theory is used herein. In continuation, an iterative procedure is performed to compute the dynamic responses of the vehicle/track system, and the VTI model is validated by field experiments. Then the dynamic responses of ballasted track are computed through the numerical studies and some conclusions are drawn at the end.

2 Vehicle Ballasted Track Interaction Model

As shown in Figure 1, the heavy haul train is passing on the ballasted track, and each axle load of the freight vehicle is 30 tons. In this paper, the vehicle ballasted track interaction (VTI) model is developed, which consist of vehicle model, ballasted track model and wheel/rail contact model.

2.1 Vehicle model

For the vehicle, each freight vehicle is composed of one car body, two identical bogies and four identical wheel

* Corresponding author: wyj_bjtu@126.com

sets, which are regarded as rigid bodies as shown in Figure 2. The car body is supported at each side by a double-axle bogie through secondary suspension systems and each bogie is linked with wheel sets through the primary suspension systems. Both the primary and the secondary suspension systems are modelled as linear spring-dashpot units.

Fig. 1 Freight vehicle passing on ballasted track

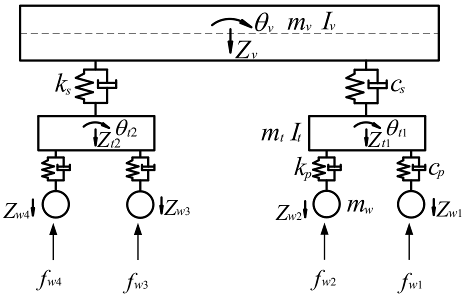


Fig. 2. Freight vehicle model

In the vehicle model, the vertical and pitching motion of the car body, front bogie and rear bogie are considered. For each wheel set, only the vertical displacement is considered. Therefore, the heavy haul vehicle is modelled as a multi-body system with 10 degrees of freedom.

Based on D'Alembert principle, the equations of motion for the vehicle system can be given as:

$$[M_V]\{\ddot{\mathbf{U}}_V\}+[C_V]\{\dot{\mathbf{U}}_V\}+[K_V]\{\mathbf{U}_V\}=\{\mathbf{F}_V\} \tag{1}$$

where $[M_V]$, $[C_V]$ and $[K_V]$ denote the mass, damping and stiffness matrices, respectively; $\{\ddot{\mathbf{U}}_V\}$, $\{\dot{\mathbf{U}}_V\}$ and $\{\mathbf{U}_V\}$ denote the accelerations, velocities and displacements, respectively; $\{\mathbf{F}_V\}$ is the external force vector acting on the vehicle system, which includes the gravities from the vehicle components and wheel/rail contact forces.

2.2 Ballasted track model

The ballasted track is modelled as four-layer structure consisting of rail, sleeper, ballast, sub-ballast and subgrade, which are connected one by one with spring-damping elements, as shown in Figure 3. For the ballasted track model, the rail is treated as an Euler-Bernoulli beam supported by discrete fasteners and sleepers, the ballast and sub-ballast layers are regarded as a series of lumped masses, and the spring-damping elements are used to link the fastening system, the elastic layer between ballast and sub-ballast, and the visco-elastic layer of the subgrade.

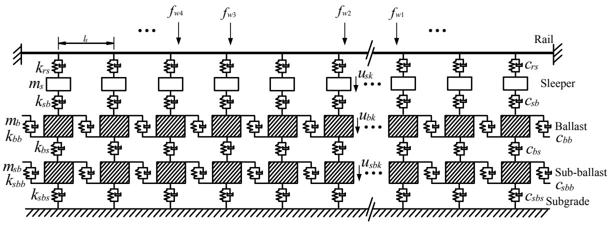


Fig. 3. Ballasted Track model

In this paper, the ballasted track is modelled by finite element method. To save the computing time and ensure the accuracy of the results at the same time, each rail element length is set as one sleeper space and supported by two adjacent sleepers[6]. For rail element, there are four degrees of freedom (DOFs): two vertical displacements u_j and u_{j+1} and two rotations θ_j and θ_{j+1} at two nodes. And each sleeper, ballast mass and sub-ballast mass only have one degree of freedom (DOF): vertical displacement u_{sk}, u_{bk}, u_{sbk}, respectively.

By assembling rail, sleeper, ballast mass and sub-ballast mass equations together, the equation of motion of the ballast track system under moving vehicle can be expressed as

$$[M_T]\{\ddot{\mathbf{U}}_T\}+[C_T]\{\dot{\mathbf{U}}_T\}+[K_T]\{\mathbf{U}_T\}=\{\mathbf{F}_T\} \tag{2}$$

where $[M_T]$, $[C_T]$ and $[K_T]$ denote the mass, damping and stiffness matrices of the track system, respectively, $\{\ddot{\mathbf{U}}_T\}$, $\{\dot{\mathbf{U}}_T\}$ and $\{\mathbf{U}_T\}$ denote the accelerations, velocities and displacements of the rail nodes, sleepers and ballast masses, respectively, $\{\mathbf{F}_T\}$ is the external force vector acting on the track system, that is the wheel/rail contact forces from the freight vehicle.

2.3 Wheel/rail contact model

In the vehicle ballasted track interaction (VTI) model, the upper vehicle and the substructure are linked by the assumed wheel/rail interaction relationship. In this study, the wheel/rail contact force $f_{wi}(t)$ is described by the nonlinear Hertzian theory as below,

$$f_{wi}(t)=\begin{cases} C_H\left[Z_{wi}(t)-u_{ri}(x,t)\right]^{3/2} & Z_{wi}(t)-u_{ri}(x,t)>0 \\ 0 & Z_{wi}(t)-u_{ri}(x,t)\le 0 \end{cases} \tag{3}$$

where $Z_{wi}(t)$ is the vertical displacement of the ith wheel set, $u_{ri}(x,t)$ is the vertical displacement of the rail at the point where the ith wheel set is located; C_H is the Hertzian constant and $C_H = 9.37\times10^{10}$N/m$^{3/2}$.

3 Solution and model verification

Due to the nonlinear feature of the wheel/rail contact force, as indicated in Section 2.3, the vehicle ballasted track interaction (VTI) model is solved by an iterative procedure, which proceeds as follows[7]:

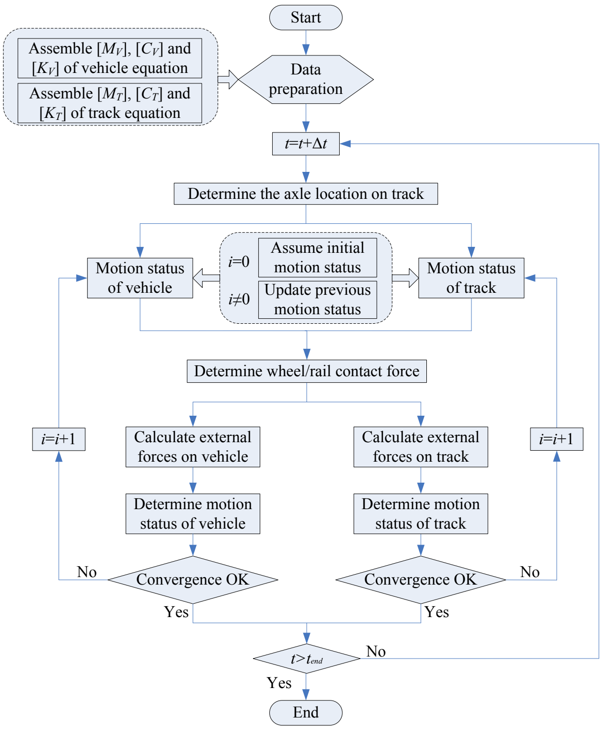


Fig. 4. Flowchart of the solution procedure

(a) Data preparation: Assemble [M_V], [C_V] and [K_V] that are defined in Eq. (1) and [M_T], [C_T] and [K_T] from Eq. (2).

(b) Determine the time duration t_{end} and the time interval Δt, required for the dynamic response time histories. In this study, the time interval Δt is set as 2×10^{-5} s.

(c) Determine the axle location on the track at time step $t + \Delta t$. For the first iteration step ($i = 0$), take the dynamic responses of the vehicle and track at the last time step t as the initial values for the iteration for the current time step $t + \Delta t$.

(d) Use the dynamic responses of the vehicle and the track at the last iteration step i to compute the wheel/rail contact forces in Eq. (3) for the current iteration step i + 1.

(e) Calculate the external forces acting on the vehicle and track, respectively, and solve the equation of motion of the vehicle and the track in Eqs. (1) and (2) using a step-by-step integration method such as the Newmark-β method, to find the vehicle and track responses at the iteration step $i + 1$.

(f) Use the vehicle and track responses determined under (e) for the next iteration step until the following convergence criterion is reached for both the vehicle and the track responses from Eqs. (1) and (2):

$$\left\| {}^{t+\Delta t}_{i+1}\mathbf{U}_T - {}^{t+\Delta t}_{i}\mathbf{U}_T \right\| \le CC, \quad \left\| {}^{t+\Delta t}_{i+1}\mathbf{U}_V - {}^{t+\Delta t}_{i}\mathbf{U}_V \right\| \le CC \quad (4)$$

The allowable tolerance for the convergence criterion is set as 10^{-6} m in this paper, which are found to be sufficient to determine the solution with acceptable accuracy from numerical comparison. If the convergence criterions in Eq. (4) are not satisfied, a new iteration step (i + 1) +1 can be started from (d). If the criterions are satisfied, a new time step can be started from (c).

To validate the vehicle ballasted track interaction (VTI) model, some experiment is performed. During the field testing, the freight train is composed of 100 wagons with 30 t per axle and the running speed of the train is 80 km/h. In Figure 5, the displacements of rail midpoint between two adjacent sleepers from the VTI model and field testing data are compared.

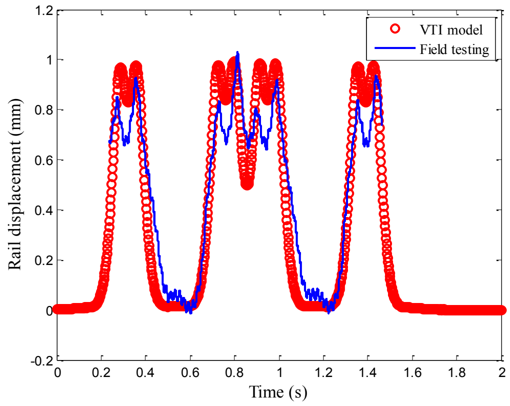


Fig. 5. Model validation

In the VTI model, there are only two vehicles considered due to the computation cost. Figure 5 indicates that the response obtained by the VTI model agrees reasonably well with the field testing data. Eight peaks, which are induced by the eight wheel sets from the two vehicles, are clearly shown in this figure.

4 Numerical studies

In these numerical investigations, the parameters of freight vehicle and ballasted track model are listed in Table 1 and Table 2, respectively[8]. In daily operation of the heavy haul train, the maximum running speed of the train is set as 80 km/h, which is also used in the following numerical examples.

Table 1. The vehicle model parameters

Notation	Parameters	Value	Unit
m_v	Mass of car body	55800	kg
m_t	Mass of bogie	900	kg
m_w	Mass of wheel set	600	kg
I_v	Mass moment of inertia of car body	2.7×10^6	kg m^2
I_t	Mass moment of inertia of bogie	855	kg m^2
k_p	Stiffness of primary suspension	1.3×10^7	N/m
k_s	Stiffness of secondary suspension	3.1×10^6	N/m
c_p	Damping of primary suspension	9.0×10^4	N s/m
c_s	Damping of secondary suspension	0.5×10^5	N s/m
l_v	Freight car length	14	m
l_t	Half of distance between two bogie centers	4.9	m
l_w	Half of distance between two axles in each bogie	0.89	m

Table 2. The ballasted track model parameters

Notation	Parameters	Value	Unit
m_r	Rail mass per unit length	74.414	kg/m
E_r	Elastic modulus of rail	2.06×10^{11}	N/m^2
I_r	Moment of inertia of rail	4.489×10^{-5}	m^4
m_s	Sleeper mass(half)	192.5	kg
m_b	Ballast mass	360	kg
m_{sb}	Sub-ballast mass	422	kg
l_s	Spacing of rail pad	0.6	m
k_{rs}	Rail pad stiffness	2.24×10^8	N/m
k_{sb}	Ballast stiffness	1.84×10^8	N/m
k_{bb}	Ballast shear stiffness	5.52×10^7	N/m
k_{bs}	Sub-ballast stiffness	9.66×10^8	N/m
k_{sbb}	Sub-ballast shear stiffness	2.90×10^8	N/m
k_{sbs}	Subgrade stiffness	1.18×10^8	N/m
c_{rs}	Rail pad damping	5×10^4	N s/m
c_{sb}	Ballast damping	3.61×10^4	N s/m
c_{bb}	Ballast shear damping	1.08×10^4	N s/m
c_{bs}	Sub-ballast damping	8.94×10^4	N s/m
c_{sbb}	Sub-ballast shear damping	2.68×10^4	N s/m
c_{sbs}	Subgrade damping	3.1×10^4	N s/m

The wheel/rail contact force is the connection of the vehicle model and the ballasted track model, and the contact forces for the four wheel sets of the first vehicle against rail location are computed, as shown in Figure 6.

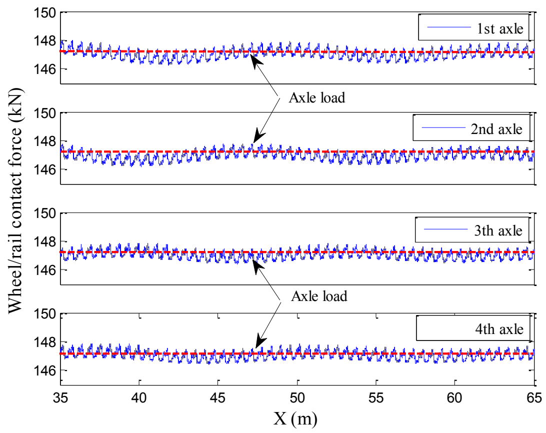


Fig. 6. Wheel/rail contact force

It can be concluded from Figure 6, that when the vehicle travels over the ballasted track, the wheel/rail contact force fluctuates around the vehicle axle load (147.15 kN), with the wavelength of 0.6 m, which equals to the distance of the two adjacent sleepers. It means that the wheel/rail contact force is a periodic force, which is dominated by the axle load and sleeper spacing.

The dynamic displacements of the ballasted track, including rail, sleeper, ballast mass and sub-ballast mass under the moving freight vehicle can be computed as a function of time, as shown in Figure 7.

Figure 7 indicates that the maximum displacements of ballasted track for rail, sleeper, ballast mass and sub-ballast mass are 1.00, 0.75, 0.43 and 0.37mm, respectively. It can be seen that the vibration amplitudes of the ballasted track decrease rapidly from rail to sub-ballast mass.

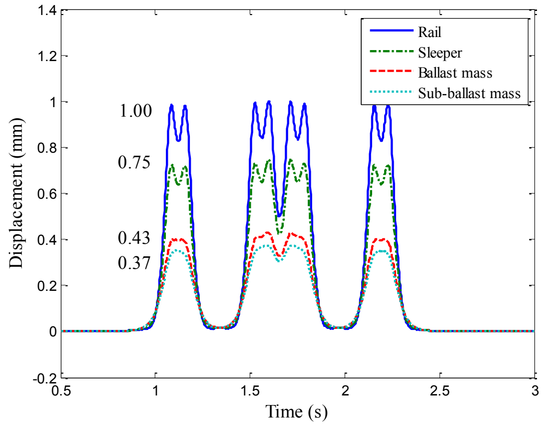


Fig. 7. Displacement of ballasted track

4.1. Effect of running speed

To investigate the effect of vehicle running speed on the dynamic responses of the vehicle ballasted interaction (VTI) model, the maximum values of wheel/rail contact force against different running speed of 20, 40, 60, 80, 100 km/h are calculated, as shown in Figure 8.

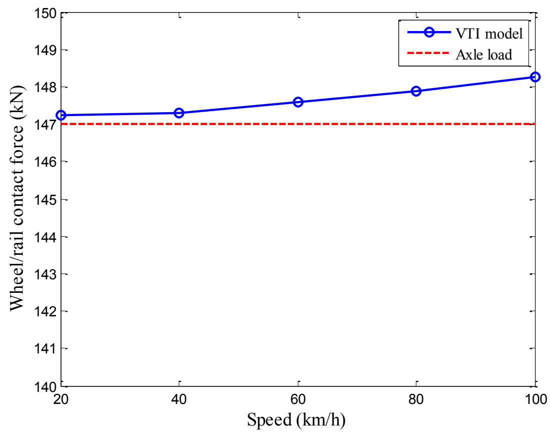
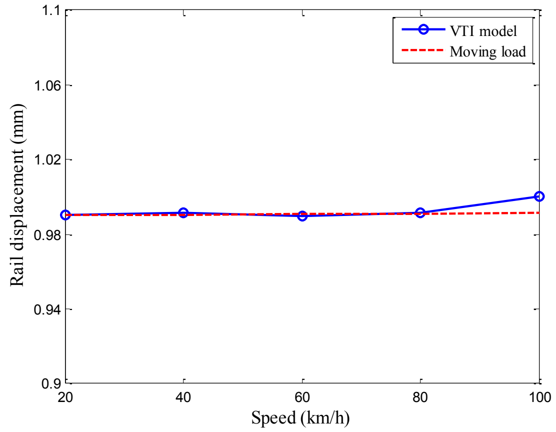


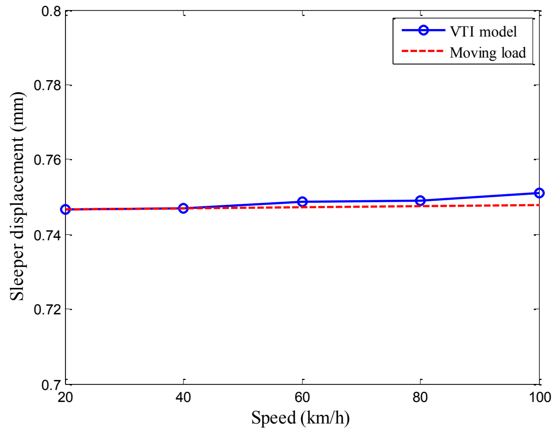
Fig. 8. Wheel/rail contact force against running speed

It is indicated that the maximum value of wheel/rail contact force for different running speed is a little higher than the value of axle load (147.15 kN), which is the dashed line in Figure 8.

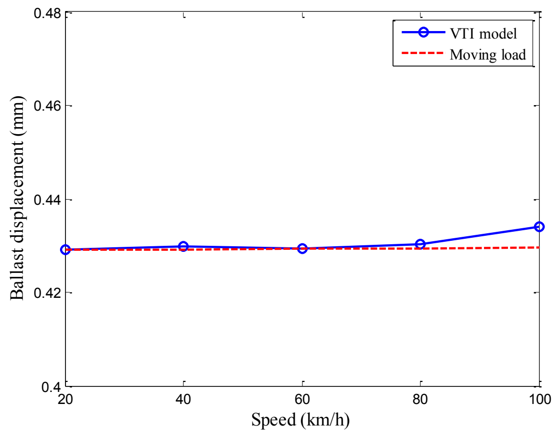
The maximum displacements of ballasted track for rail, sleeper, ballast mass and sub-ballast mass are also computed against different running speed of 20, 40, 60, 80, 100 km/h, as shown in Figure 9. For comparison, the dynamic responses of ballasted track under moving load model are also computed.



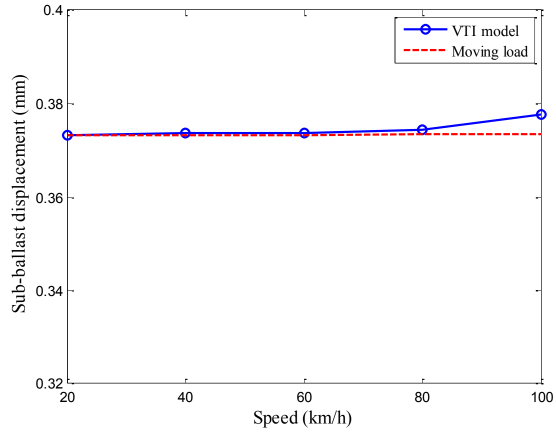
(a) Rail



(b) Sleeper



(c) Ballast



(d) Sub-ballast

Fig. 9. Displacement of ballasted track against running speed

As shown in Figure 9, for the lower running speed, especially less than 40 km/h, the maximum displacements of ballasted track computed from the VTI model are very close to these from the moving load model. However, when the running speed is higher, the results from VTI model are larger than those from the moving load model. In conclusion, the moving load model may underestimate the dynamic responses of ballasted tracks.

4.2 Effect of axle load

To investigate the effect of different axle load on the dynamic responses of the vehicle ballasted track interaction system, the maximum values of wheel/rail contact force against axle loads of 25, 30, 35 and 40 tones are computed, as shown in Figure 10.

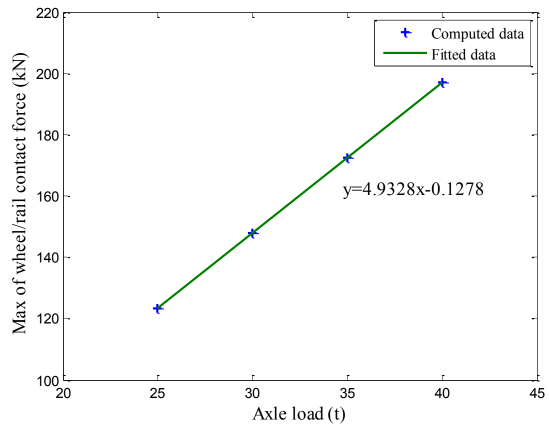
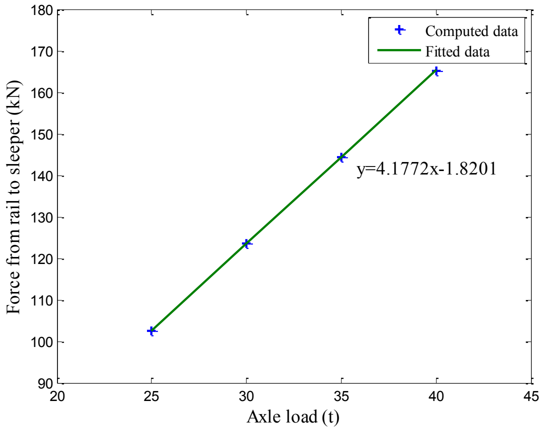


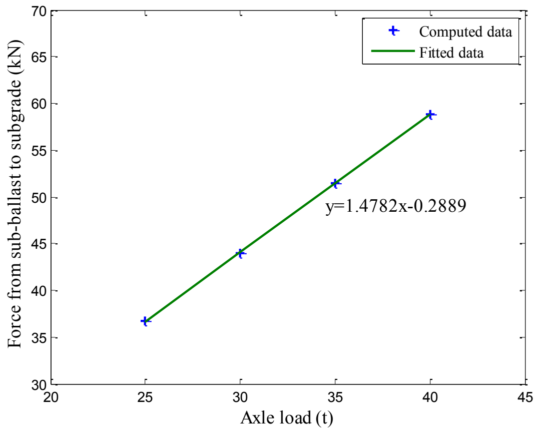
Fig. 10. Wheel/rail contact force against axle load

Figure 10 shows that with the increase of axle load, the maximum values of wheel/rail contact force increase. The regression equation of the wheel/rail contact force according to the axle load is presented in Table 3. It can be deduced that the wheel/rail contact force as a function of the axle load changes linearly according to y=a*x+b, where x stands for the axle load, y for the maximum value of wheel/rail contact force and a, b are fitted coefficients.

For the heavy haul railway, the rubber pad is used between the rail and sleeper to provide the elastic support. However, if the force passing from the rail to the sleeper is very high, the rubber may be worn out quickly. Another fact that should be carefully considered is the force transmitted from the sub-ballast to the subgrade, which may cause the failure of the subgrade. Therefore, the maximum values of these forces were extracted against axle loads of 25, 30, 35 and 40 tones and are shown in Figure 11.



(a) Force from rail to sleeper



(b) Force from sub-ballast to subgrade

Fig. 11. Forces against axle load

Figure 11 indicates that when the axle load of the freight vehicle increases, the maximum values of the force from rail to sleeper and the force from sub-ballast to subgrade will also increase. The regression equation coefficients of the variation of these two forces with respect to the axle load are presented in Table 3.

Table 3. Regression equations

Items	Linear Eq 'y=a*x+b'		R^2 value
	a	b	
Wheel/rail contact force	4.9328	-0.1278	1
Force from rail to sleeper	4.1772	-1.8201	1
Force from sub-ballast to the subgrade	1.4782	-0.2889	1

From the results listed in Table 3, it can also be deduced that the dependences of the force from rail to sleeper and the force from sub-ballast to subgrade as a function of the axle load are linear (y=a*x+b). Moreover, is can be concluded that the coefficient "a" of these three regression equations, *i.e.* the slope of the corresponding lines, decreases from the top to the bottom of the ballasted track. Also, for the same axle load, the wheel/rail contact force is the largest one among these three, the force from rail to sleeper is next and the force from sub-ballast to subgrade is the lowest one.

5 Conclusions

In this paper, a vertical vehicle ballasted track interaction (VTI) model is developed and validated using the field test data. From the numerical studies, the following conclusions are drawn:
1) The wheel/rail contact force is a periodic force, which is dominated by the axle load and sleeper spacing.
2) For higher running speeds, the moving load model may underestimate the dynamic responses of ballasted tracks, because the wheel/rail contact force from the VTI model is a slightly larger than the axle load.
3) With an increase of the axle load, the maximum values of the wheel/rail contact force, of the force from rail to sleeper and of the force from sub-ballast to subgrade increase linearly. This point can be adopted for engineering analysis of wheel/track contact in railway design phase.

This work was supported in part by the National Natural Science Foundation of China under Grant No. 51408036 and Scientific Project of China Railway Group Limited under Grant No. 2014-重点-55.

References

1. S. Ponnuswamy. *Railway Transportation Engineering, Operation and Management* (Alpha Science International Ltd, Oxford, 2012).
2. J. Oscarsson, T. Dahlberg, Veh Syst Dyn, **29**, 73-84 (1998).
3. X. Lei, J.G. Rose, P I Mech Eng F-J Rai, **222**, 413-421 (2008).
4. W.M. Zhai, K.Y. Wang, J.H. Lin, J Sound Vib, **270**, 673-683 (2004).
5. S. Mosayebi, J. Zakeri, M. Esmaeili, J Mech Sci Technol, **30**, 1065-1069 (2016).
6. P. Lou, X.G. Zhong, J.F. Tang, Q.Y. Zeng, P I Mech Eng F-J Rai, **220**, 305-315 (2006).
7. Y.J. Wang, Y.X. Mao, A.Q. Hu, J Vibroeng, **17**, 1488-1498 (2015).
8. W.M. Zhai. *Vehicle-track coupled dynamics* (Science Press Ltd, Beijing, 2015).