New approach to decision making by multiple criteria analysis

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Abstract. Many decision processes in technical and economical sciences require multiple criteria decision making. The most widely applied methods for multiple criteria evaluation of alternatives are based on the evaluation of alternatives in terms of an additive preference function. All of them require the estimation of weights of usually conflicting criteria. There are several methods how to find the weights of the criteria and how to find the evaluation of each solution in each criterion. The decision process based on simple weighted sum of values may not be the best approach in all situations. This paper contains a new approach of the evaluation of measured value set by different mathematical operators than the usually used multiple criteria evaluation methods. The approach was applied in a case study for multiple criteria evaluation. Generally, this new decision-support tool can help in various situations where different types of effects caused by a construction or reconstruction can occur. This is a very frequent situation in dealing with building defects, too.

1 Averaging operators

In different situations, various aggregating operators can be used to obtain a value we call an average.

Let R be the set of all real numbers and m the number of measurement of the excavated value. Then, the general averaging operator can be defined as a mapping Avg from R^m into R, satisfying the following conditions:

1. A is a continuous mapping of R^m into R.
2. A is monotonic in each of its m coordinates, that is: if $p_j \geq q_j$ for each $j=1,\dots,m$, then $A(p_1,p_2,\dots,p_m) \geq A(q_1,q_2,\dots,q_m)$.
3. A is idempotent, that is: $A(p, p, \dots , p) = p$ for any real number p.
4. A is internal, that is: $min(p_1,p_2,\dots,p_m) \leq A(p_1,p_2,\dots,p_m) \leq max(p_1,p_2,\dots,p_m)$

Special types of averaging operators called quasiarithmetic means are most frequently used. These operators can be expressed by an arithmetic formula.

Let $a_1,a_2,\dots,a_m$ be real numbers representing some measured values.

Such an α-quasiarithmetic mean is defined for positive values $a_1,a_2,\dots,a_m$ by the following formula:

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$$Avg_{\alpha}(a_1, a_2, \ldots, a_m) = \left(\frac{1}{m} \sum_{j=1}^{m} a_j^{\alpha} \right)^{1/\alpha} \tag{1}$$

For $\alpha=1$, we obtain the commonly known arithmetic mean, for $\alpha=2$, the harmonic mean.

In the limit $\alpha \to 0$, we obtain a geometric mean, for $\alpha \to \infty$ we obtain maximum and for $\alpha \to -\infty$ we obtain the minimum.

1.1 Level of Consensus

Of course, the measured values are not the only important piece of information for decision making. Information about the level of the consensus among the values can also be important.

Consensus level concept can be introduced for cases when the answer is *YES/NO* or for cases when the answer uses a discrete scale.

Consensus is a value from the interval $[0,1]$ that determines the level of agreement of the measured values.

Consensus is equal to *1* when all the values are same.

Consensus is equal to *0* when half of the values is *YES* and the other half have is *NO*.

Consensus can be viewed as a truth value of the proposition, “The measurements proof this situation”.

This concept of consensus can be supplemented to express an agreement of the measurements results with a specified target value τ as follows:

Let X be a discrete random variable of size n with probability distribution $p(X)$ on the previously given closed interval $[X_{min}, X_{max}]$ so that $p_i = P(X_i)$ for $i=1,\ldots,n$, and let $d_X = X_{max} - X_{min}$ be the width of X. Then, the τ-agreement $Agr(X|\tau)$ of the distribution is a number defined by the following formula:

$$Agr(X|\tau) = 1 + \frac{1}{n} \sum_{i=1}^{n} p_i . \log_2 (1 - \frac{|x_i - \tau|}{2d_x}) \tag{2}$$

Further generalization can be made to abandon the restriction that implies that the values express only a choice of some milestone in the discrete scale. Commonly the results of measurements can be described as a choice of any real number on the closed interval $[X_{min}, X_{max}]$. Then, the formula for τ -agreement changes to

$$Agr(X|\tau) = 1 + \frac{1}{N} \sum_{i=1}^{N} p_i . \log_2 (1 - \frac{|x_i - \tau|}{2d_x}) \tag{3}$$

where N is the number of the measurements. The main difference between these formulas is that in the first formula n is the number of levels for the discrete scale (for the Likert scale, usually $n=5$, or $n=9$), and p_i is the number of measurements according to the corresponding level, while in the second formula, N is the number of measurements, and the value of each of them is evaluated separately.

Regardless of the choice of different discrete scales or different intervals for attribute value estimation, the value of τ -agreement is a number from the closed interval $[0,1]$. The value of τ -agreement equals *1* when all the measured values match value τ .

The value of τ -agreement is *0* when the values from all measurements match one of the boundary points of interval the $[X_{min},X_{max}]$, while τ is the opposite boundary. $Agr(X|\ \tau)$ can be considered a fuzzy truth value of the proposition "The measurements proof that the measured value has the quantity of τ ".

1.2 Consensus optimization

Let us look to a value $Agr(X|\ \tau)$ as to a function of an argument τ and the set of values be an closed interval *[0,1]*. The range of the argument τ can be also considered as a closed interval $[\tau_{min}, \tau_{max}]$ where τ_{min} and τ_{max} are such values that the measured quantities are clearly between them. If we do not have any information about the foundations of measured values, we can put τ_{min} to be the lowest measured value and τ_{max} to be the highest.

The function $Cons_X(\tau) = Agr(X|\ \tau)$ is continuous because it is defined as a combination of continuous elementary functions. So it is a continuous mapping from a closed interval $[\tau_{min}, \tau_{max}]$ into a continuous interval *[0,1]*. Such a function must reach a maximum. This maximum is depended only on the set of measurements *X*. We can denote the value of τ for which this maximum is reached as *Mavg(X)*. This value *Mavg(X)* holds four conditions for general averaging operator:

1. *A* is a continuous mapping of R^m into *R*. Clearly *Mavg(X)* is a mapping of R^m into *R*. And it is continuous because it was formed as a combination of continuous elementary functions.
2. *A* is clearly monotonic in each of its *m* coordinates, that is: if $p_j \geq q_j$ for each $j=1,...,m$, then $A(p_1,p_2,...,p_m) \geq A(q_1,q_2,...q_m)$.
3. *A* is idempotent, that is: $A(p, p, ... , p) = p$ for all real numbers *p*. For the situation of an idempotent measurement the value $Agr(X|\ \tau)$ is equal to 0 for τ not equal to *p* and it is equal to *1* for τ equal to *p*. So the maximum of this function is reached in the point $\tau=p$ and it is equal to *1*. The value *Mavg(X)* is equal to *p*.
4. *A* is internal, that is: $min(p_1,p_2,...,p_m) \leq A(p_1,p_2,...p_m) \leq max(p_1,p_2,...,p_m)$. Clearly the maximum of the function $Cons_X(\tau)$ is reached between the points $min(p_1,p_2,...,p_m)$ and $max(p_1,p_2,...,p_m)$. So the last condition is fulfilled.

The value *Mavg(X)* can be used as other way how to compute an average of measured or estimated values. One big disadvantage of such averaging is the fact that it is very complicated to express such a value especially in the situation when the basic set *X* is extremely large. Unfortunately this is usually the situation of technical measurements. Much more useful can be this way of averaging in situations when the dimensions of set *X* are quite small. This is usually the situation of expert estimate of values used for example for multicriterial evaluation and multicriterial optimization. In such a situation the number of estimated values is not large, usually tens maximally hunderts of expert estimates. The *Mavg* can be a very useful way to deal with such sets of estimations.

2 Case Study

In the following text we introduce a small case study which shows the benefits of the above defined average operator *Mavg*. The input data describe the lifetime of concrete construction elements. Data obtained by classical computation of an average value from the expert estimations are given in the following table:

Table 1. Construction element.

Construction element	Lifetime (years)	Average lifetime (years)
Concrete Foundation	80-150	100
Concrete Outdoor Walls	60-80	70
Concrete Indoor Walls	100-150	120
Concrete Outdoor Steps	60-80	70
Concrete Roof	80-150	100
Outdoor Walls with Surface Finish	100-150	120
Ventilation Shafts	40-70	60
Window Sills	60-80	70
Concrete Fence	60-80	70
Concrete Sewer Shaft	60-80	70
Concrete Sidewalk	20-30	25

For closer research the distribution of input values schould be known. For the second element – the concrete outdoor walls – the expert estimations were gradually 60,60,65,70,70,70,75,75,75 and 80 years. The arithmetic mean of these values is 70 years. By computing the value $Agr(X|\tau)$ for different values of estimation τ we obtain subsequently:

$$Agr(X|60) = 0,9865,$$

$$Agr(X|65) = 0,9913,$$

$$Agr(X|70) = 0,9939,$$

$$Agr(X|75) = 0,9923,$$

$$Agr(X|80) = 0,9864.$$

Even with closer estimations of agreement value τ we obtain

$$Agr(X|69) = 0,9934,$$

$$Agr(X|71) = 0,9936.$$

So the value $Mavg(X)$ is equal to 70 years (or very close to it) and the estimation obtained by simple arithmetic mean computation agrees with the computation by the new method.

Another situation will occur in the estimation of first construction element – concrete foundation. The expert estimations were gradually 80,80,80,80,80,80,100,120,150 and 150 years. The arithmetic mean of these values is 100 years. Computing the values $Agr(X|\tau)$ we obtain:

$$Agr(X|80) = 0,9918486,$$

$$Agr(X|90) = 0,9918389,$$

$$Agr(X|100) = 0,9914760,$$

$$Agr(X|110) = 0{,}990149,$$

and even smaller values of $Agr(X|\tau)$ for higher values of τ.

More detail exploring leads to values:

$$Agr(X|84) = 0{,}9918887,$$

$$Agr(X|85) = 0{,}9918894,$$

$$Agr(X|86) = 0{,}9918886.$$

So the maximum agreement value $Mavg(X)$ is somewhere near 85 years and this should be the lifetime of concrete foundation estimated by the expert group. Such a computation describes the situation better then the simple arithmetic mean computation with the resulting value 100 years.

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