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Research on modulation transfer function of the linear-array CCD for hyperspectral imager

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Abstract. As an image-spectrum merging technology, hyperspectral imaging has been used in battlefield reconnaissance rapidly. Based on response of CCD to the sine input signal, the period and extremal positions of output signal were analyzed and simulated. According to the definition of MTF based on contrast, three expressions of the average MTF of line-array CCD were presented and compared with those based on Fourier transform method. Finally, the average MTF of line-array CCD is confirmed. Accordingly, the characteristics of modulation transfer function of line-array CCD are studied. The simulation results indicate that the frequency increase, while at Nuquist frequency, the effect of the initial position, which becomes much stronger with frequency increase, while at other frequency, the effet of the initial position is weak. In the frequency range of |ζ |>0.1%,the effect of the initial position on MTF can be ignored.

Key words. Hyperspectral Imager; Discrete imaging; Line-array CCD; MTF; Initial position

1 Introduction

With the rapid development of space technology and computer technology, the application of charge coupled device (CCD) in fields like hyper-spectral imaging is becoming more and more broad. As an internationally general method for evaluating the quality of imaging systems, modulation transfer function (MTF) can subjectively reflect the spatial frequency characteristics of an imaging system [1]. Thus, research on MTF characteristics of CCD and its influence on the imaging quality of the whole system are quite essential [2].

CCD is a discrete imaging device whose sampling of input signals is a linear process but not a complete spatial invariant process [3]. So far, Fourier method and contrast method are mainly applied to research MTF of CCD. Based on assumption of continuous scanning of a single image element, with application of contrast method, Kai M. Hock deduced the expression of average MTF of line-array CCD that is consistent with MTF of line-array CCD by Fourier method. Subsequently[4], O. Hadar proved the correctness that sinc function is used as the sampling function for arbitrary direction of two-dimensional sampling array[5]. The modulation transfer function is affected by the initial position. S.B.Campana pointed out in using contrast method (streak target) to measure MTF of CCD that if the phases of streak target and CCD change when the input signal frequency is near Nyquist frequency[6], MTF values of CCD fluctuate. S. K. Park proposed to use average MTF to evaluate the imaging quality of discrete imaging systems based on point spread function Fourier transform[7]. J.Feltz used a line-array CCD signal in the sinusoidal input signal period to output the maximum value and the minimum value according to the contrast definition of MTF so as to offer the line-array CCD and expression of phase-related MTF[8]. Song min experiments verified that the experiment results agree with Feltz's theoretical calculation[9]. And Jyrki uses the sinusoidal signal to measure the modulation transfer function of CCD (steps of 0.1*(pixel size) in the object plane). The simulation results show that the relative position has a strong effect on the modulation transfer function at the near Nuquist frequency. It has less influence on other frequencies. however, no theoretical analysis was made[10].

In this paper, based on specific analysis on response characteristics of line-array CCD to sinusoidal input signal, the mathematical model of MTF of line-array CCD is established according to the contrast definition of MTF so as to discuss the expression forms of average MTF of line-array CCD, based on which the influence of the initial position on MTF of line-array CCD. The research content in this paper can provide certain theoretical evidences for measurement of MTF of CCD.

2 Theoretical Analysis

2.1 The response characteristics of line-array CCD to sinusoidal input signal

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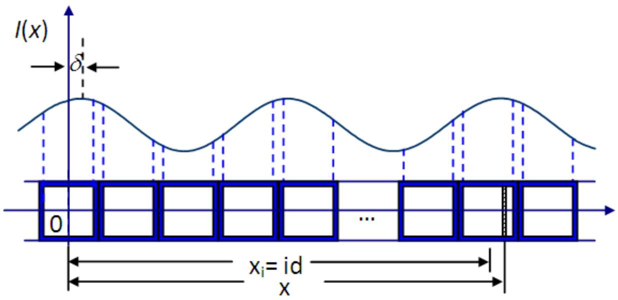

Figure1. Sketch map of line-array CCD sampling

As shown in Fig.1, the image element photosensitive unit width of line-array CCD is w and the spacing between image elements is d. The center of the first CCD image element is taken as the origin of coordinate and its serial number i is 0. $x_i=id$ refers to the coordinate of the center of No.i image element. If the signals that are irradiated on the line-array CCD are distributed as

$$I(x) = 1 + C_{in}(f)\cos(2\pi f(x+\delta)) \tag{1}$$

In the formula, f, $C_{in}(f)$ and δ separately refers to the spatial frequency, modulation degree and relative location (the relative spatial location of the input signal and No.0 CCD image element) of the input signal. During the sampling process, the signal output on each image element is obtained by accumulation of $I(x)$ from the interval between (x_i-w/2) and (x_i+w/2). thus, the signal output of No.i CCD image element is

$$I_i(f,\delta) = 2\int_{x_i-w/2}^{x_i+w/2} w(1+\cos(2\pi f(x+\delta)))dx$$
$$= 2w^2(1+\mathrm{sin}\,c(fw)\cos(2\pi f(x_i+\delta))) \tag{2}$$

The following conclusion can be drawn from formula (2):

1.the position of signal output extremum

When $2\pi f(x_{i\,\max}+\delta)=2k\pi$ (k=0, ±1, ±2…), that is, $x_{i\,max}=k/f-\delta$, there are maximal values in continuously-sampling signal output. If $x_{i\,max}=i'_{\max}d$ and $round(x)$ refers to x after rounding. When the serial number of CCD image elements $i_{max}=round(i'_{\max})$, the maximal value in signal output $I_{max\,i}$. $|i'_{max}-i_{max}|$ reaches the minimum and there is the maximum value I_{max}. $|i'_{max}-i_{\max}|=0$ in signal output, the position of the maximum value of signal output coincides with the position of the minimum value of input signal and $I_{\max}=w^2(1+sinc(wf))$.

When $2\pi f(x_{i\,min}+\delta)=(2k+1)\pi$ (k=0, ±1, ±2…), that is, $x_{i\,min}=i'_{\min}d=(k+1/2)/f-\delta$, there are minimal values in continuously-sampling signal output. Assuming $x_{i\,min}=i'_{\min}d$, when the serial number of CCD image elements $i_{\min}=round(i'_{\min})$ and the minimal value in signal output $I_{max\,i}$. $|i'_{\max}-i_{\max}|$ reaches the minimum, there is minimum value $I_{\min}$. $|i'_{\min}-i_{\min}|=0$ in signal output. The position of the minimum value of signal output coincides with the position of the minimum value of input signal and $I_{\min}=w^2(1\text{-sinc}(wf))$.

2. the period of signal output

Because of periodic distribution of input signals, the output signal size of subsequent sampling points following a certain image element will repeat the result of its previous sampling point. Such as signal output unit is named as a signal output period. The period of input sinusoidal signal is $T=1/f$ and the period of corresponding signal output image element number is $T_i=1/(fd)$. $T'_i=rats(T_i)$ and $rats(x)$ numerically rationalize x. Because of discrete sampling of CCD image elements, the period of line-array CCD signal output image element number refers to the numerator part of numerically-rationalized T. Thus, the period of image element number is related to the value of f and d but not related to δ and w.

2.2 Average MTF of line-array CCD

It can be seen from formula (2) that when the input signal is a sinusoidal signal, the amplitude of output signal changes with the relative location δ whose changing period is d. Thus, MTF of line-array CCD is related to the relative location δ. The average MTF of line-array CCD is defined as:

$$\overline{MTF}(f) = \frac{\sum_{j=1}^{m} MTF(f,\delta_j)}{m} \tag{3}$$

Where $MTF(f, \delta_j)$ refers to MTF whose initial position is δ_j and m refers to the number of initial positions. According to definition of MTF, MTF refers to the ratio of the system's output modulation degree $C_{out}(f)$ and input modulation degree $C_{in}(f)$. When the input modulation degree $C_{in}(f)$ is 1,

$$MTF(f,\delta_j) \equiv \frac{C_{out}(f)}{C_{in}(f)} = \frac{I_{\max}(f,\delta_j) - I_{\min}(f,\delta_j)}{I_{\max}(f,\delta_j) + I_{\min}(f,\delta_j)}$$

On account of discrete sampling of CCD image elements, when the spatial frequency is f and the initial position is δ_j, there will be several maximal values $I_{max\,i}$ and minimal values $I_{min\,i}$ in signal output within the period of a CCD image element signal output and there are three expressions of MTF shown as below:

2.2.1 The method of maximum and minimum values of signal output

Through the maximum and minimum values, $I_{\max}$ and $I_{\min}$, of signal output within a signal output period of line-array CCD, $MTF''(f, \delta_j)$ whose initial position is δ_j can be directly obtained, that is:

$$MTF''(f,\delta_j) = \frac{I_{\max}(f,\delta_j) - I_{\min}(f,\delta_j)}{I_{\max}(f,\delta_j) + I_{\min}(f,\delta_j)} \tag{4}$$

Through substituting formula (4) into formula (3), $\overline{MTF''}(f)$ is obtained.

2.2.2 the averaging method of MTF

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n' MTFs that are obtained through maximal value $I_{max\,i}$ and minimal value $I_{min\,i}$ of signal output within No.i semi period of input sinusoidal signal are averaged to act as $MTF'(f,\delta_j)$ whose initial position is δ_j, that is:

$$MTF'(f,\delta_j)=\frac{1}{n'}\sum_{i=1}^{n'}\frac{I_{max\,i}(f,\delta_j)-I_{min\,i}(f,\delta_j)}{I_{max\,i}(f,\delta_j)+I_{min\,i}(f,\delta_j)} \quad (5)$$

Through substituting formula (5) into formula (3), $\overline{MTF'(f)}$ is obtained.

2.2.3 the averaging method of maximal and minimal values of signal output

n maximal and minimal values of signal output within a signal output period are separately averaged to act as maximal value $I_{max\,i}$ and minimal value $I_{min\,i}$, through which $MTF(f,\delta_j)$ whose initial position is δ_j is obtained, that is:

$$MTF(f,\delta_j)=\frac{1}{n}\frac{\sum_{i=1}^{n}I_{max\,i}(f,\delta_j)-\sum_{i=1}^{n}I_{min\,i}(f,\delta_j)}{\sum_{i=1}^{n}I_{max\,i}(f,\delta_j)+\sum_{i=1}^{n}I_{min\,i}(f,\delta_j)} \quad (6)$$

Through substituting formula (6) into formula (3), $\overline{MTF}(f)$ is obtained.

$$\overline{MTF}(f)=\frac{1}{mn}\sum_{j=1}^{m}\frac{\sum_{i=1}^{n}I_{max\,i}(f,\delta_j)-\sum_{i=1}^{n}I_{min\,i}(f,\delta_j)}{\sum_{i=1}^{n}I_{max\,i}(f,\delta_j)+\sum_{i=1}^{n}I_{min\,i}(f,\delta_j)} \quad (7)$$

3 Simulation Analysis

Simulation calculation will be conducted on signal output characteristics of line-array CCD, expressions of average MTF and the influence of initial positions on MTF next. In detailed calculation, the spacing between image elements is taken as d=18μm and photosensitive unit width of image elements is w=16μm. Corresponding Nyquist frequency is f_N=1/(2d)=27.78 lp/mm.

3.1 Signal output characteristics

If f=18lp/mm, δ=(3d)/20=2.7μm, the detailed calculation is shown as below.

3.1.1 signal output period

T_i=1/(fd)=250/81, that is, the signal output period is 250 CCD image elements.

3.1.2 position of extremums

Position of maximal values: according to $i'_{max}d=k/f-\delta$, i'_{max}=(k−0.0486)/0.324, k=0, ±1, ±2, ±3⋯, That is, i'_{max}=2, 6.02, 9.11, 12.20… and the serial number of corresponding CCD image element is i_{max}=$round$(i'_{max}) =2, 6, 9, 12⋯.

Position of minimal values: according to $i'_{min}d=(k+1/2)/f-\delta$, i'_{min}=(2k+0.9514)/0.648, k=0, ±1, ±2, ±3⋯, That is, i'_{min} =1.47, 4.55, 7.64, 10.73… and the serial number of corresponding CCD image element is i_{min}=$round$(i'_{min})=1, 5, 8, 11⋯.

3.1.3 computer simulation

The signal output curve of No.0~100 CCD image elements is shown as Fig.2.

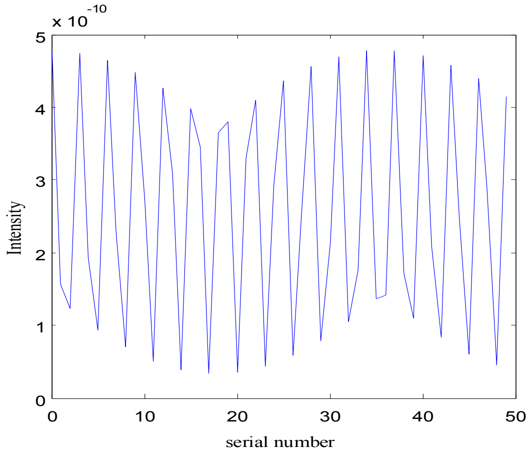

Figure2. Intensity distribution of output signal

3.2 Comparison between average MTF and MTF by Fourier method

Through Fourier method, MTF of line-array CCD is the product of Fourier transform of rectangle function $rect(x/w)$ and Fourier transform of sampling function $rect(x/d)$, that is:

$$MTF=\left|\sin c(fw)\sin c(fd)\right|$$

Values of input signal frequency are uniformly taken in Nyquist frequency. m=50, that is, step length is 18/50μm. $\overline{MTF''(f)}$ curve, $\overline{MTF'(f)}$ curve, $\overline{MTF}(f)$ curve and MTF obtained through Fourier transform are shown as Fig.3, in which Nyquist frequency is normalized as 1. It can be seen from Fig.3 that there is greater deviation between $\overline{MTF''(f)}$ curve and MTF curve, smaller deviation between $\overline{MTF'(f)}$ curve and MTF curve and $\overline{MTF}(f)$ curve coincides with MTF curve. It can be known from simulation comparison results that the expression formula of average MTF through contrast method is formula (7).

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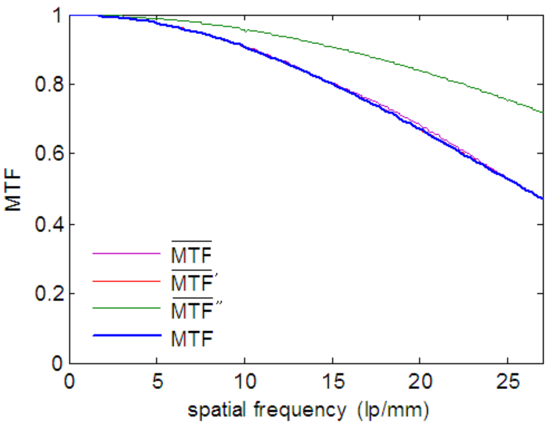

Figure3. Simulation and comparison of MTF

3.3 MTF characteristics in f_N frequency domain

In engineering design and tests, people usually pay more attention to the system's MTF of line-array CCD at Nyquist frequency. Thus, according to formula (7), we focally calculate Nyquist frequency and the changing principle of MTF near Nyquist frequency division with positions, during which CCD image element number n=1000, *m*=50.

f_N refers to Nyquist frequency. f_N / l （l=1, 2, 3, 4 ⋯） refers to Nyquist frequency division. l refers to Nyquist frequency division number. ζ refers to input signal frequency. f refers to the percentage deviation relative to Nyquist frequency division, that is,

$$\zeta = \frac{f - f_N/l}{f_N/l}$$

The deviation percentage ζ used in calculation is ±0.005%, ±0.02%, ±0.04%, ±0.06%, ±0.08%, ±0.1%.

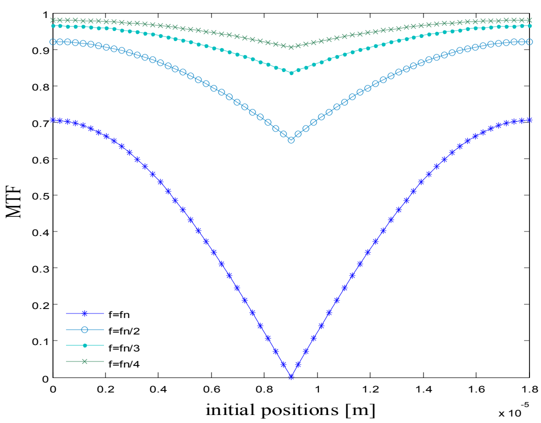

Figure4. Curve of MTF with initial positions

When input signal frequency f=f_N / l （l=1, 2, 3, 4）, the curve of MTF with initial positions is shown as Fig.4. When l=1 and l=2, under the deviation percentage mentioned above, the curve of MTF with initial positions (ζ<0) is shown as Fig.5and Fig.6.

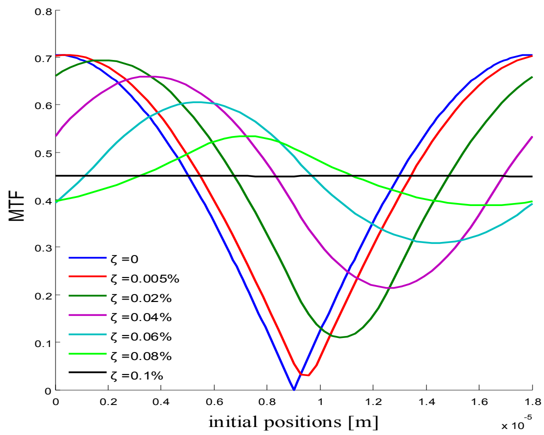

Figure5. Curve of MTF with initial positions at near Nyquist frequency division（l=1）

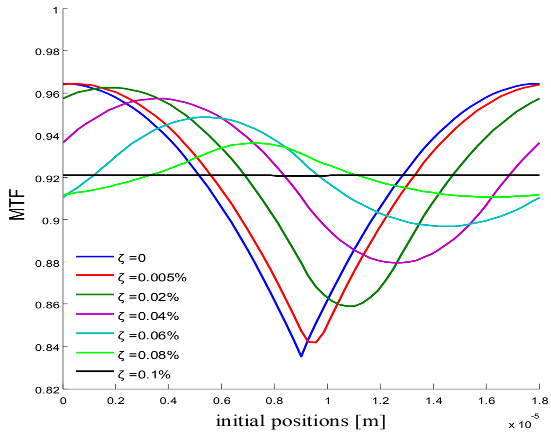

Figure6. Curve of MTF with initial positions at near Nyquist frequency division（l=3）

It can be indicated from comparison between Fig.4, Fig.5and Fig.6 that, at the Nyquist frequency division and near Nyquist frequency division, the influence of the initial position on MTF decreases as l increases. The maximum amplitude difference of MTF at $f_N / 4$ is 0.052 and MTF almost does not change with initial positions. Near Nyquist frequency division, the influence of the initial position on MTF decreases as the deviation percentage ζ increases. When ζ=±0.1%, the maximum amplitude difference of MTF is 0.007 and MTF no longer changes with initial positions.

4 Conclusion

◆ In this paper, the response characteristics of line-array CCD to sinusoidal input signal are analyzed and the theoretical analysis and numerical value simulation results of signal output period and extremum positions are provided.

◆ In this paper, starting from definition of MTF, MTF analytic expression formula of line-array CCD is deduced and the expression formula of average MTF of line-array CCD is obtained through simulation comparison, based on which MTF characteristics of line-

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array CCD are researched. Conclusion is described as below: the simulation results indicate that the frequency increase, while at Nuquist frequency, the effect of the initial position, which becomes much stronger with frequency increase, while at other frequency, the effet of the initial position is weak. In the frequency range of ∣ ζ ∣>0.1%,the effect of the initial position on MTF can be ignored.

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