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Exploring Topsnut-Graphical Passwords by Twin Odd-elegant Trees

Hong-yu WANG[1], *Jin* XU[1,⋆], and *Bing* Yao[2,3]

[1] *School of Electronics Engineering and Computer Science, Peking University, 100871, China*
[2] *College of Mathematics and Statistics, Northwest Normal University, Lanzhou, 730070, China*
[3] *College of Electronic and Information Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China*

Abstract. Graphical passwords are facing a good opportunity as 2-dimension codes are accepted by many people, since it has been applied in mobile devices, electronic equipments with touch screen, and so on. QR codes can be considered as a type of graphical passwords. Topsnut-graphical password differs from the existing graphical passwords, and has been investigated and developed. In this article, a new type of Topsnut-graphical passwords has been designed by technique of graph theory, called twin odd-elegant labelling. We make the twin odd-elegant graphs for one-key vs two or more locks (conversely, one-lock vs two or more keys). These Topsnut-GPWs show perfect matching characteristics of locks (TOE-lock-models) and keys (TOE-key-models). We show examples for testing our methods which can be easily transformed into effective algorithms.

1 Introduction

Since its introduction in 1994, the QR Code (Quick Response codes, QR-codes, they are referred as a *printable computer language*) has gained wide acceptance in such diverse industries as manufacturing, warehousing and logistics, retailing, healthcare, life sciences, transportation and office automation [2]. QR codes can be considered as a type of graphical passwords that are used widely, since it has been applied in mobile devices, electronic equipments with touch screen, and so on. QR code (abbreviated from Quick Response Code) is the trademark for a type of matrix barcode (or two-dimensional barcode) first designed for the automotive industry in Japan. A barcode is a machine-readable optical label that contains information about the item to which it is attached. A QR code uses four standardized encoding modes (numeric, alphanumeric, byte/binary, and kanji) to efficiently store data; extensions may also be used [1]. However, QR codes can not be used to those departments required with high-level security. We have known that passwords in the security community should be easy to remember, and the user authentication protocol should be executable quickly and easily by humans. And passwords should be secure, i.e. they should look random and should be hard to guess; they should be changed frequently, and should be different on different accounts of the same user; they should not be written down or stored in plain text. On the other hands, no report tells us that other graphical passwords were applied to business and were accepted by people (Ref. [7], [3], [5]). Wiedenbeck *et al.* [12] asked for two principal research questions:

⋆e-mail: jxu@pku.edu.cn

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Question 1: Are graphical passwords a viable alternative to alphanumeric passwords in terms of security, as well as password creation, learning, performance, and retention?

Question 2: Are users' perceptions of graphical passwords different from those of alphanumeric passwords?

An idea of "graphical structure plus number theory" (Topsnut) for creating new type of graphical passwords was proposed firstly by Wang *et al.* in [9, 10]. And in making new Topsnut graphical passwords (Topsnut-GPWs), Wang *et al.* [11] have designed some Topsnut-GPWs by knowledge of graph theory for answering the following problem of number theory:

Problem of odd-graceful KL-matching pairs: Let $[0,2q] = \{0,1,2,\dots,2q\}$ and $[1,2q-1]^o = \{1,3,5,\dots,2q-1\}$ with $q \geq 2$. If there are two subsets X_1, X_2 of $[0,2q]$ hold: (i) $X_1 \cup X_2 = [0,2q]$ and $\{0\} \subseteq X_1 \cap X_2 \subseteq [0,2q]$; (2) $\{|a-b| : a \in X_i^{odd}, b \in X_i^{even}\} = [1,2q-1]^o$ where X_i^{odd} is an odd-number set and X_i^{even} is an even-number set of X_i such that $X_i^{odd} \cup \in X_i^{even} = X_i$ and $X_i^{odd} \cap \in X_i^{even} = \emptyset$ with $i = 1,2$. We say X_1 and X_2 to be an odd-graceful KL-matching pair. Find all odd-graceful KL-matching pairs for any given set $[0,2q]$ with $q \geq 2$.

Clearly, one can use odd-graceful KL-matching pairs to design more complex Topsnut-GPWs. We, in Section 2, will introduce several techniques by knowledge of graph theory. Our main works are to show new Topsnut-GPWs made by the techniques, which also are the part solutions of the problem of odd-elegant KL-matching pairs proposed in the last section. On the other hands, our methods for dealing with the new Topsnut-GPWs can be easily transformed into algorithms.

2 Definitions for techniques

We use standard notation and terminology of graph theory [4]. Graphs mentioned are loopless, no multiple edges, undirected, connected and finite, unless otherwise specified. A (p,q)-graph G is one with p vertices and q edges. The shorthand symbol $[m,n]$ stands for an integer set $\{m, m+1, \dots, n\}$, where m and n are integers with $0 \leq m < n$; the notation $[s,t]^o$ indicates an odd-set $\{s, s+2, \dots, t\}$, where s and t both are odd integers with $1 \leq s < t$; and the notation $[k,\ell]^e$ indicates an even-set $\{k, k+2, \dots, \ell\}$, where k and ℓ both are even integers with $0 \leq k < \ell$.

Definition 1 *[14] Suppose that a (p,q)-graph G admits a mapping $f : V(G) \to [0,2q-1]$ such that $f(u) \neq f(v)$ for distinct $u,v \in V(G)$, and the label $f(uv)$ of every edge $uv \in E(G)$ is defined as $f(uv) = f(u) + f(v) \pmod{2q}$ and the set of all edge labels is equal to $[1,2q-1]^o$, we call f an* odd-elegant labelling *and G to be* odd-elegant.

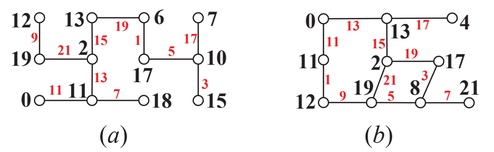

Figure 1. (a) An odd-elegant tree; (b) An odd-elegant graph.

Definition 2 *[13] Suppose that a bipartite graph G admits a labelling f such that $\max\{f(x) : x \in X\} < \min\{f(y) : y \in Y\}$, where (X,Y) is the bipartition of vertex set $V(G)$ of G. We call f a set-ordered labelling (an So-labelling for short).*

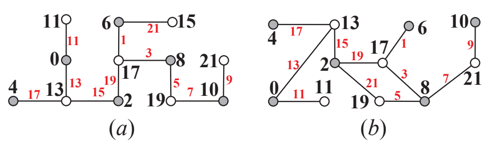

Figure 2. (a) A set-ordered odd-elegant tree; (b) A set-ordered odd-elegant graph.

Let G_j be a (p_j, q_j)-graph with $j = 1,2$. The graph G obtained by identifying two certain vertices $x_{i,1}$ of G_1 with two certain vertices $x_{i,2}$ of G_2 into one vertex x_i, where i=1,2, x_1 and x_2 is called the *identified-vertex*, respectively. is denoted as $G = G_1 \circ_2 G_2$, called an *double identifying graph*. So, the graph $G = G_1 \circ_2 G_2$ has $p_1 + p_2 - 2$ vertices and $q_1 + q_2$ edges.

Suppose that two vertices w_1 and w_2 of G are two identified-vertices of identifying $x_{i,1}$ with

$x_{i,2}$, where $i = 1, 2$. Then we can split the identified-vertices w_i into two vertices $x_{i,1}$ and $x_{i,2}$ (called the *splitting-vertices*), so G can be split into two parts G_1 and G_2.

The above process of producing G from G_1 and G_2 is called an *identifying operation*, another process of splitting G into two parts G_1 and G_2 is called a *splitting operation*.

Definition 3 *Let two connected (p_i, q)-graphs G_i with $i = 1, 2$, and let $p = p_1 + p_2 - 2$. If the identifying (p, q)-graph $G = G_1 \circ_2 G_2$ has a mapping f: $V(G) \rightarrow [0, q-1]$ such that (i) $f(x) \neq f(y)$ for any pair of vertices $,y \in V(G)$; (ii) f is an odd-elegant labelling of G_i with $i = 1, 2$. Then we say G to be a* twin odd-elegant graph *(a TOE-graph), f a* TOE-labelling. *G_1 a* TOE-source graph, *and G_2 a* TOE-associated graph, *vice versa.*

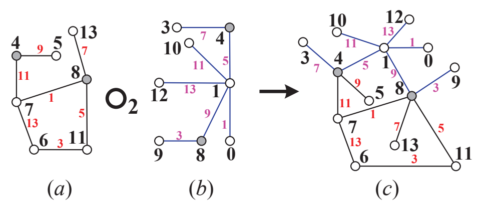

Figure 3. A twin odd-elegant graph $G = G_1 \circ_2 G_2$ with its OE-source graph G_1 and OE-associated graph G_2 by an identifying operation.

Furthermore, if each G_i with $i = 1, 2$ is a connected graph in Definition 3, and exist TOE-source G_1 is a bipartite connected graph having its own bipartition (X_1, Y_1) satisfy Definition 2, we say that the identifying (p, q)-graph $G = G_1 \circ_2 G_2$ is a *set-ordered twin odd-elegant graph*, f a *set-ordered twin odd-elegant labelling*. Here, the source graph G_1 is a set-ordered odd-elegant graph by Definition 1 and 2.

Similarly, we present the definition of a twin odd-elegant tree as follows.

Definition 4 *Let T_i be a tree of order n with $i = 1, 2$. If the identifying tree $G_T = T_1 \circ_2 T_2$ has a mapping $f : V(G_T) \rightarrow [0, 2n-3]$ such that (i) $f(x) \neq f(y)$ for any pair of vertices $x, y \in V(G_T)$; (ii) f is an odd-elegant labelling of T_i, where $i = 1, 2$. Then G_T is called a* twin odd-elegant tree *(a TOE-tree), f a* TOE-labelling, *and T_1 a* TOE-source tree, *and T_2 a* TOE-associated tree *of T_1. Furthermore, we say T_1 a* TOE-source self-associated tree *if T_1 is isomorphic to T_2.*

Two examples for illustrating Definition 4 are show in Figure 4 and Figure 10. Let T_1 has its bipartition (X_1, Y_1) in Definition 2; and if the TOE-source tree T_1 has a set-ordered odd-elegant labelling, then we call G_T a *set-ordered twin odd-elegant tree*(an So-TOE tree for short), and f is *set-ordered twin odd-elegant labelling*(an So-TOE labelling for short).

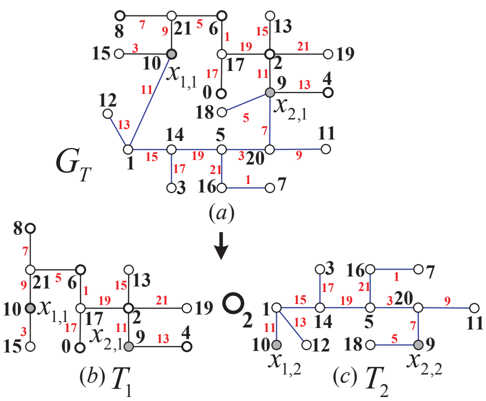

Figure 4. A TOE tree G_T has its TOE-source tree T_1 and TOE-associated tree T_2, and two split-vertices $x_{i,1}$ and $x_{i,2}$ labelled with numbers 9 and 10, where $i = 1, 2$, by a splitting operation.

In our design of the new Topsnut-GPWs, a source graph/tree and its associated graph/tree in Definition 3 or Definition 4 are called a *TOE-lock-model* and a *TOE-key-model*, respectively.

3 Theoretical analysis and guarantee

Lemma 1 *Any star $K_{1,n}$ is a TOE-source tree of a So-TOE-tree G_T.*

Proof. Let $V(K_{1,n}) = \{x, y_1, y_2, \ldots, y_n\}$ and $E(K_{1,n}) = \{xy_i : i \in [1, n]\}$ be the sets of vertices and edges of $K_{1,n}$, respectively. We can define an odd-elegant labelling f with $f(x) = 0$,

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$f(y_i) = 2i - 1$ with $i \in [1,n]$, which induce $f(xy_i) = f(x) + f(y_i) \pmod{2n}$ for $i \in [1,n]$. So, $f(E(K_{1,n}) = [1, 2n-1]^o$. i.e. $K_{1,n} = T_1$

Let T_2 be a copy of $K_{1,n}$. We have the sets of vertices and edges of T_2 as $V(T_2) = \{x', y'_1, y'_2, \dots, y'_n\}$ and $E(T_2) = \{x'y'_i : i \in [1,n]\}$, respectively. Next, we define a labelling f' in the way $f'(x') = 1$, $f'(y'_i) = 2i-2$ with $i \in [1,n]$. Clearly, $f'(x'y'_i) = f'(x') + f'(y'_i) \pmod{2n}$ for $i \in [1,n]$. We have that $f'(E(T_2) = [1, 2n-1]^o$.

Since $f(y_1) = f'(x') = 1$ and $f(x) = f'(y'_1) = 0$, we identify the vertex y_1 of $K_{1,n}$ with the vertex x' of T_2 into one w_1 and the vertex x of $K_{1,n}$ with the vertex y'_1 of T_2 into one w_2 for forming the identifying tree $G_T = K_{1,n} \circ_2 T_2$. It is not hard to provide a labelling f_{G_T} of G_T as: $f_{G_T}(u) = f(u)$ for $u \in V(K_{1,n})$, and $f_{G_T}(v) = f'(v)$ for $v \in V(T_2)$. Immediately, $\{f(xy_i) : xy_i \in E(K_{1,n})\} = [1, 2n-1]^o$ and $\{f(x'y'_i) : x'y'_i \in E(T_2)\} = [1, 2n-1]^o$. Therefor, we claim that G_T is a TOE-tree since f_{G_T} is a TOE-labelling of G_T by Definition 4, which means that $K_{1,n}$ is an So-TOE source tree of G_T, and T_2 is a TOE-associated tree of $K_{1,n}$.

It is noticeable, $T_2 = K_{1,n}$ means that the TOE-source tree $K_{1,n}$ is isomorphic to its associated tree T_2 in T. So, $K_{1,n}$ is a *TOE-source self-associated tree*.

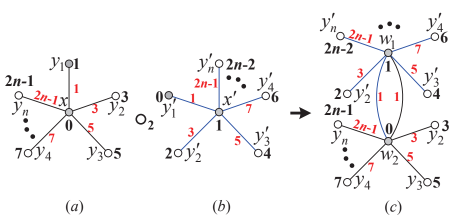

Figure 5. An illustration for Lemma 1.

Theorem 1 *Any set-ordered odd-elegant tree being not a star is an So-TOE-source tree of at least two So-TOE trees.*

Proof. Suppose that T_1 is a tree with n vertices, its vertex bipartition is (X, Y), where $X = \{x_i : i \in [1,s]\}$, $Y = \{y_j : j \in [1,t]\}$, $s + t = n$ and $\min\{s,t\} \geq 2$. By the hypothesis of the theorem, T_1 has a set-ordered odd-elegant labelling f_1 defined by $f_1(x_i) = 2(i-1)$, $i \in [1,s]$; $f_1(y_j) = 2(j+s-1)-1$, $i \in [1,t]$; and $f_1(x_iy_j) = f_1(y_j) + f_1(x_i) = 2(s+i+j-2) - 1 \pmod{2(n-1)}$ for each edge $x_iy_j \in E(T_1)$.

We can observe $f_1(V(T_1)) = \{0, 2, \dots, 2s-2, 2s-1, 2s+1, \dots, 2n-3\}$ and $f_1(E(T_1)) = [1, 2n-3]^o$.

Case 1. We construct a labelling f_2 of another tree T_2 by the labelling f_1 such that $f_2(V(T_2)) = \{1, 3, \dots, 2s-3, 2s-2, 2s-1, 2s, \dots, 2(n-2)\}$, and $f_2(E(T_2)) = \{f_2(uv) = f_2(u) + f_2(v) \pmod{2(n-1)} : uv \in E(T_2)\} = [1, 2n-3]^o$, and furthermore, $f_2(u) \neq f_2(v)$ for any $u, v \in V(T_2)$. This tree T_2 can be built up in the way: A bipartition (U_1, V_1) with $U_1 = \{u_i : i \in [1,s]\}$ and $V_1 = \{v_j : j \in [1,t]\}$, such that $f_2(u_i) = 2i - 1$, $i \in [1,s]$; $f_2(v_j) = 2(s-2+j)$, $j \in [1,t]$. Any edge $u_iv_j \in E(T_2)$ satisfy $f_2(u_iv_j) = f_2(u_i) + f_2(v_j) \pmod{2(n-1)}$ with $i \in [1,s]$ and $j \in [1,t]$. We construct the edge set of T_2 as $\{u_1v_j, u_iv_t : i \in [1,s], j \in [1,t-1]\}$ such that the edge labels are $f_2(u_iv_t) = 2i-3$, $f_2(u_1v_j) = 2j+2s-3 \pmod{2(n-1)}$ for $i \in [1,s]$ and $j \in [1,t-1]$. Observe that $f_2(E(T_2)) = [1, 2n-3]^o$, $f_1(y_1) = f_2(u_s)$ and $f_1(x_s) = f_2(v_1)$.

Now, we can identify the vertex x_s and y_1 of T_1 with the vertex v_1 and u_s of T_2 into one (the identified vertex) w_1 and w_2, respectively, so we obtain the desired tree $G_T = T_1 \circ_2 T_2$. And G_T has a labelling f defined as: $f(x_i) = f_1(x_i)$, $i \in [1, s-1]$; $f(y_j) = f_1(y_j)$, $i \in [2,t]$; $f(u_k) = f_2(u_k)$, $k \in [1, s-1]$, $f(v_l) = f_2(v_l)$, $l \in [2,t]$, $f(w_1) = 2s-2$ and $f(w_2) = 2s-1$. Clearly, any two vertices of G_T are assigned different numbers. According to Definition 4, G_T is an So TOE-tree having the source tree T_1.

Case 2. Similarly to Case 1, we can get the following results: Let $f_2(V(T'_2)) = \{1, 3, \dots, 2s-3, 2s-2, 2s, 2(s+1), \dots, 2n-4\} \cup \{0\}$, and $f_2(E(T'_2)) = [1, 2n-3]^o$, and furthermore $f_2(u) \neq f_2(v)$ for $u, v \in V(T'_2)$. This tree T'_2 can be built up in the way: A bipartition (U_2, V_2) with $U_2 = \{u_i : i \in [1, s-1]\}$ and $V_2 = \{v_j : j \in [1, t+1]\}$, such that $f_2(u_i) = 2i - 1$, $i \in [1, s-1]$; $f_2(v_j) = 2(s-2+j)$, $j \in [1,t]$, $f_2(v_{t+1}) = 0$. Any edge $u_iv_j \in E(T'_2)$ satisfies $f_2(u_iv_j) = f_2(u_i) + f_2(v_j) \pmod{2(n-1)}$ with $i \in [1, s-1]$ and $j \in [1, t+1]$. We construct the edge set of T'_2 as $\{u_1v_j, u_iv_{t+1} : i \in [2,s], j \in [1,t]\}$

such that the edge labels are $f_2(u_i v_{t+1}) = 2i - 1$, $f_2(u_1 v_j) = 2(s + j) - 3$ for $i \in [1, s - 1]$ and $j \in [1, t]$. Observe that $f_2(E(T_2')) = [1, 2n - 3]^o$, $f_1(x_1) = f_2(v_{t+1})$ and $f_1(x_s) = f_2(v_1)$.

Now, we can identify the vertex x_1 and x_s of T_1 with the vertex v_{t+1} and v_1 of T_2' into one (the identified vertex) w_1 and w_2, so we obtain the desired tree $G_{T'} = T_1 \circ_2 T_2'$. And $G_{T'}$ has a labelling f defined as: $f(x_i) = f_1(x_i)$, $i \in [2, s-1]$; $f(y_j) = f_1(y_j)$, $i \in [1, t]$; $f(u_k) = f_2(u_k)$, $k \in [1, s - 1]$, $f(v_l) = f_2(v_l)$, $l \in [2, t]$, $f(w_1) = 0$ and $f(w_2) = 2s - 2$. Clearly, any two vertices of $G_{T'}$ are assigned different numbers. According to Definition 4, $G_{T'}$ is an So-TOE tree having the source tree T_1.

Corollary 1 *(1) Every So-TOE-source tree is a TOE-source self-associated tree.*

(2) Not every TOE-tree consisting of TOE-source self-associated tree has an odd-elegant labelling.

Theorem 2 *If the TOE-source tree T_1 is a set-ordered odd-elegant tree, then the simple So-TOE graph $G_T = T_1 \circ_2 T_2$ has an odd-elegant labelling.*

4 Test and simultaneous

Experiment 1 *We show two examples for illustrating the results of Theorem 1. Our methods appeared in the proof of Theorem 1 are easily applied the design of Topsnut-GPws, since the methods are constructive. We illustrate theorem 2 with Figure 6(c) and Figure 9.*

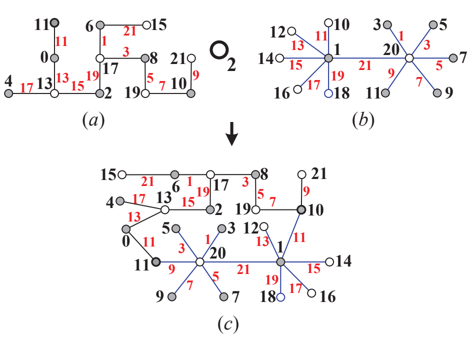

Figure 6. An example for illustrating Case 1 of Theorem 1.

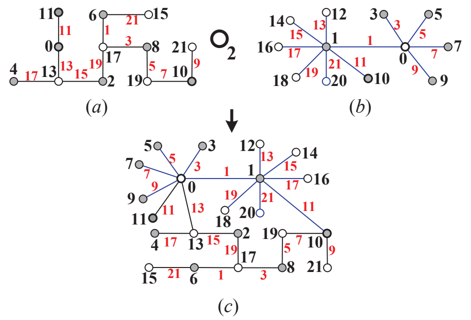

Figure 7. An example for illustrating Case 2 of Theorem 1.

Experiment 2 *Based on the technique provided by Corollary 1, we make a Topsnut-GPW shown in Figure 8(c) for the purpose of simultaneous.*

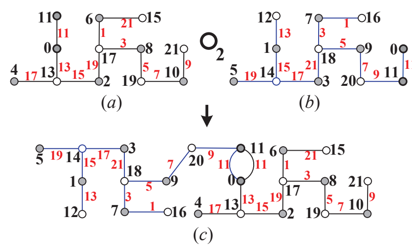

Figure 8. An example for illustrating the technique of Corollary 1.

Experiment 3 *It is is very important to get the property P of large scale of topological structure for smaller topological structures having a property P. We test the results of Theorem 2 by the scheme shown in Figure 9. Here, we can confirm that our method for proving Theorem 2 can be transformed into an effective algorithm.*

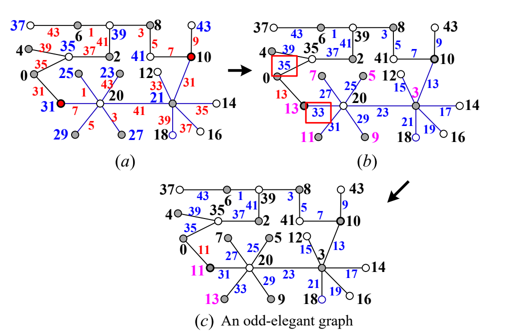

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Figure 9. A conversion process from an So-TOE-tree to odd-elegant graph for illustrating Theorem 2.

Experiment 4 *We have the following results* (1) A TOE-source graphs may have different TOE-associated graphs, and vice versa.

(2) A TOE-graph may contain two or more groups of TOE-source and TOE-associated graphs.

(3) Two groups of TOE-source and TOE-associated graphs may produce a TOE-graph.

As shown in Figure 11, there are eight different TOE-key-models for Figure 10 (b).

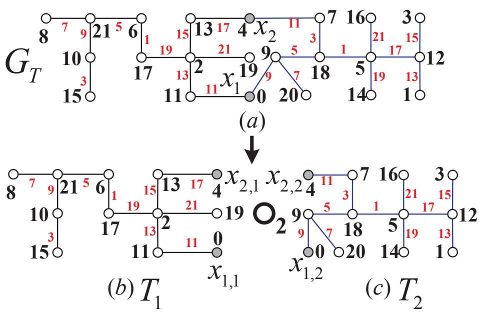

Figure 10. An So-TOE tree G_T has its TOE-source tree T_1 and TOE-associated tree T_2, and two splitting-vertices $x_{i,1}$ and $x_{i,2}$ labelled with numbers 0 and 4, where $i = 1, 2$, by a splitting operation.

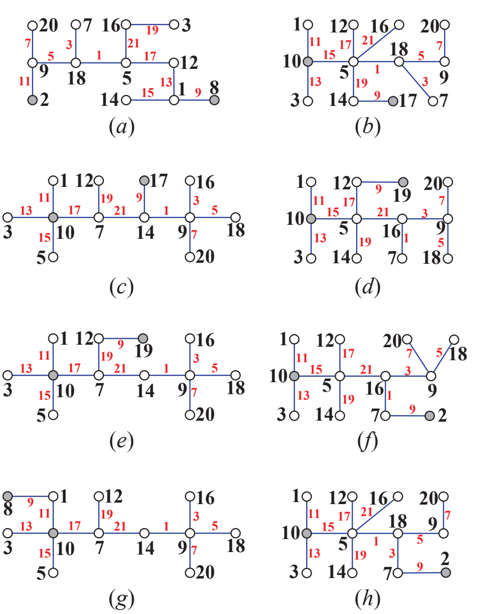

Figure 11.(a)-(h) Eight TOE-key-models of the TOE-lock-model for Figure 10(b) (eight TOE-associated trees of the TOE-source tree T_1).

5 Conclusion and researching problems

We have designed a new technique, called *(set-ordered) twin odd-elegant labelling*, by the idea of graph labellings. Furthermore, we have found some themods/algorithms to realize Topsnut-GPWs based our new graph labellings. These Topsnut-GPWs show perfect matching characteristics of locks (TOE-lock-models) and keys (TOE-key-models), which enable us to produce one-key vs two or more locks (conversely, one-lock vs two or more keys). We have many graph colorings were introduced and investigated by algorithmic methods [4], and have more graph labellings shown in a large survey due to Gallian [6], the survey collected over 1400 papers. Thereby, Topsnut-GPWs are supported by graph colorings/labellings. Every thing can become a password, and t is just beginning for Topsnut-GPWs. By our experience, we propose:

Conjecture 1 *Any tree is the TOE-source tree of a certain twin odd-elegant tree.*

Conjecture 2 *Any twin odd-elegant tree is simple having an odd-elegant labelling.*

Furthermore, we, motivated from problem of odd-graceful KL-matching pairs, propose the following problem of number theory:

Odd-elegant KL-matching pairs' problem. Let $[0, 2q-1] = \{0, 1, 2, \dots, 2q-1\}$ and $[1, 2q-1]^o = \{1, 3, 5, \dots, 2q-1\}$ with integer $q \geq 2$. If there are two subsets Y_1 and Y_2 of $[0, 2q-1]$ such that

(I) $Y_1 \cup Y_2 = [0, 2q-1]$ and $\{0, k\} = Y_1 \cap Y_2 \subseteq [0, 2q-1]$ for some $k \in [1, 2q-1]$; and

(II) the set $\{a + b \pmod{2q} : a \in Y_i^{odd}, b \in Y_i^{even}\} = [1, 2q-1]^o$, where Y_i^{odd} is an odd-number set and Y_i^{even} is an even-number set of Y_i holding $Y_i^{odd} \cup Y_i^{even} = Y_i$ and $Y_i^{odd} \cap Y_i^{even} = \emptyset$ with $i = 1, 2$.

We call these two sets Y_1 and Y_2 to be an odd-elegant KL-matching pair, and want to find all possible odd-elegant KL-matching pairs.

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