An approved frequency tracking method in connected element interferometry

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Abstract. Carrier tracking loop is a basic method of frequency tracking in CEI (connected element interferometry), but the tracking error is very large or even worse at high dynamic. To solve this problem, a frequency tracking method assisted by open loop frequency estimation is proposed in this paper. First, an open-loop frequency estimation algorithm is used to estimate the frequency of signals at different moments, then, the least squares fitting method is used to estimate the corresponding dynamic parameters of different orders. Next, a local signal is generated to compensate the received signal. After eliminating most of the dynamic characteristics, the signal is fed into PLL (Frequency locked loop) and high-precision frequency tracking results can be obtained. Simulation results show that, in the condition of low SNR, the new method has higher accuracy than the method assisted by FLL (Frequency locked loop), and the processing time is shorter. When there is a two order Doppler frequency change, the open-loop assisted method can overcome the steady phase error of the PLL, and is of great significance in the applications of connected elements interferometry.

1 Introduction

CEI is a method of target azimuth measurement, which has the advantages of high accuracy, short baseline and flexible station arrangement [1]. However, it has a very strict requirement for the measurement accuracy of signal frequency and phase [2]. Frequency tracking loop is a common method, but its accuracy is difficult to meet the requirements under high dynamic condition [3-5]. How to balance the contradiction between dynamic stress and tracking accuracy is a difficult and hot issue. At present, the carrier tracking method under high dynamic condition including the method based on kalman filter, the method assisted by FLL, and the method assisted by maximum likelihood estimation. Among them, kalman filter is used to estimate carrier phase difference, Doppler frequency and its change rate, and can adaptively adjust the loop parameters, but the uncertainty of the environment factors on the stability of the system greatly, and the computation is high, not easy to be implemented [6,7]. FLL has good resistance to dynamic stress, is responsible for eliminating most of the dynamic characteristics, but its tracking accuracy is low. The principle of this method is simple, but because of the large bandwidth of FLL loop, more noise is introduced into the tracking loop [8,9]. Maximum likelihood estimation is a kind of asymptotically unbiased estimation. The variance of parameter estimation can be close to the Cramer-rao lower bound (CRLB), but the storage and computation are relatively large, and the real-time application is greatly restricted [10, 11]. This paper introduces the least square fitting

method [12, 13], to predict continuous change of signal frequency by estimating signal frequency of partial period. And then the dynamic characteristic parameters of Doppler can be calculated, on this basis, generating a local signal to compensate Doppler frequency and eliminate most of the dynamic characteristics of the received signal. Finally accurate synchronization is finished by PLL.

2 Traditional frequency tracking loop in CEI

PLL is the common frequency tracking loop in CEI. The tracking error of PLL loop caused by thermal noise is [14]:

$$\sigma_T = \frac{1}{2\sqrt{2\pi\tau}} \sqrt{\frac{N_0}{P_s} B_L} \quad (1)$$

In formula (1), τ is integration time, N_0 is noise power spectral density, P_s is signal power and B_L is bandwidth of loop filter.

It can be seen from the formula (1) that the loop bandwidth determines the tracking accuracy when integration time is fixed. In high dynamic conditions, in order to ensure that the loop does not mismatch, a larger loop bandwidth is generally set, which to some extent sacrifice the tracking accuracy. On the other hand, when there is a two order Doppler frequency change rate, the three order PLL is usually used. At this point, there will be a steady phase error. Assuming the two order Doppler

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$$f_d = 1/3 R_1 t^2 + 1/2 R_2 t + f_{d0} \quad (2)$$
$$\begin{aligned}\theta_e(\infty) &= \lim_{s \rightarrow 0} s \bullet \theta_e(s) = \lim_{s \rightarrow 0} s \bullet H_e(s) \bullet \theta_1(s) \\ &= \lim_{s \rightarrow 0} s \bullet \frac{s^3}{s^3 + s^2 Ka + s Kb + Kc} \bullet \frac{2R_2}{s^4} \\ &= \frac{2R_2}{Kc}\end{aligned}\quad (3)$$

In order to improve the frequency tracking precision under high dynamic conditions, the PLL tracking method assisted by FLL is more widely used in space TT & C, and its principle is shown in figure 1.

FLL is used to eliminate most of the dynamic structure the PLL determines the final tracking accuracy in Figure 1. The structure of this method is relatively simple, but because the FLL loop bandwidth is larger, PLL will also introduce more noise power. When SNR is low the tracking accuracy is low or even lock will be lost.

In this paper, a high dynamic frequency tracking algorithm based on Rife open loop frequency estimation algorithm and least squares fitting is designed.

Rife algorithm is a classical open-loop frequency estimation algorithm for sinusoidal signals [15, 16], which effectively overcomes the fence effect of the

$$\hat{f}_c = \Delta f \cdot \left(k_0 - 1 + \alpha \cdot \frac{|G_{k_0+\alpha}|}{|G_{k_0+\alpha}| + |G_{k_0}|} \right) \quad (4)$$
$$\min_{\varphi} \sum_{i=1}^m \delta_i^2 = \sum_{i=1}^m (\varphi(x_i) - y)^2 \quad (5)$$
$$y = a_0 + a_1x + \cdots + a_kx^k \quad (6)$$
$$\begin{bmatrix} 1 & x_1 & \cdots & x_1^k \\ 1 & x_2 & \cdots & x_2^k \\ 1 & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad (7)$$

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graph LR
    Signal --> Segmentation
    Segmentation --> Rife
    Rife --> Primary_fitting[Primary fitting]
    Rife --> Secondary_fitting[Secondary fitting]
    Primary_fitting --> First_order_coefficient[First-order coefficient]
    Secondary_fitting --> Second_order_coefficient[Second-order coefficient]
    First_order_coefficient --> Dynamic_parameters[Dynamic parameters]
    Second_order_coefficient --> Dynamic_parameters
    Dynamic_parameters --> Compensation
    Compensation --> PLL
    PLL --> Frequency_Phase[Frequency/Phase]
    subgraph Coarse_estimation [Coarse estimation]
        Segmentation
        Rife
        Primary_fitting
        Secondary_fitting
        First_order_coefficient
        Second_order_coefficient
    end
    subgraph Fine_estimation [Fine estimation]
        Dynamic_parameters
        Compensation
        PLL
    end
  
```

4 Simulation experiment

4.1 High dynamic model and simulation parameters

The high dynamic model of JPL is adopted in this paper. JPL proposed that the high dynamics is a relative concept, and considered that when the speed and acceleration value of the target is relatively large in reality, it is the high dynamic situation^[4], and the high dynamic model given by it is shown in figure 3.

Figure.3. High dynamic acceleration model of JPL

In Figure 3, the acceleration corresponds to the first order change rate of the Doppler frequency, and the jerk rate corresponds to the two order change rate of the Doppler frequency. In practice, although there may be three or even higher order information, but the value is generally very small, therefore, only two order change rate is considered.

According to the JPL model, the Doppler frequency is set as follows:

$$f_d(t) = \begin{cases} -7500t + f_{d1} & 0 \leq t < 3 \\ 15000t^2 - 7500t + f_{d2} & 3 \leq t < 4 \\ 7500t + f_{d3} & 4 \leq t < 8 \end{cases} \quad (8)$$

f_{d1} , f_{d2} and f_{d3} represent the initial values of Doppler frequency at the beginning of three periods respectively. The received signal is a standard sinusoidal signal, which is expressed as follows:

$$s(t) = A \cos((f_c + f_d(t))t + \Phi) + N(t) \quad (9)$$

In formula (9), A is signal amplitude, f_c is carrier frequency, Φ is initial phase, $N(t)$ is additive Gauss white noise.

The simulation is divided into three stages according to the Doppler frequency change and parameter settings are shown in Table 1.

Table 1. Simulation parameter settings.

Parameter	Value	Parameter	Value
f_{d1}	500Hz	f_c	14MHz
f_{d2}	500Hz	f_s	56MHz

Φ	0	Segment time of Rife	0.01s
C/N_0	50	Integral time of loop	0.001s

4.2 Simulation results

4.2.1 Stage one

When PLL is used for tracking directly, $B_L = 55\text{Hz}$. The tracking results are shown in Figure 4.

Figure.4. PLL direct tracking results

After fitting the tracking results, the Doppler frequency acceleration is -7.486 KHz/s, and the Doppler frequency initial value is 466.9 Hz.

The fitting results of estimated frequency from Rife of 300 segment signals are shown in Figure 5.

Figure.5. Frequency estimation results of Rife

The Doppler frequency acceleration estimation value is -7.469 KHz/s, and the Doppler frequency initial estimation value is 503.9 Hz. After the signal is compensated, the PLL tracking results are shown in Figure 6. At this time, $B_L = 5\text{Hz}$.

After fitting the tracking results, the Doppler frequency acceleration is -30.9 Hz/s, and the Doppler frequency initial value is -3.88 Hz.

Figure.6. PLL tracking results after compensating

4.2.2 Stage two

When PLL is used for tracking directly, $B_L = 60\text{Hz}$. The tracking results are shown in Figure 7.

Figure.7. PLL direct tracking results

After second order polynomial fitting the tracking results, the Doppler frequency jerk is 14.987KHz/s^2 , the Doppler frequency acceleration is -7.437KHz/s , and the Doppler frequency initial value is 459.7Hz .

The fitting results of estimated frequency from Rife of 100 segment signals are shown in Figure 8.

Figure.8. Frequency estimation results of Rife

The Doppler frequency jerk estimation value is 14.966KHz/s^2 , acceleration estimation value is -7.407KHz/s , and the initial estimation value is 499.7Hz . After the signal is compensated, the PLL tracking results are shown in Figure 9. At this time, $B_L = 30\text{Hz}$.

Figure.9. PLL tracking results after compensating

After fitting the tracking results, the Doppler frequency jerk is 33.91Hz/s^2 , acceleration is -92.7Hz/s , and initial value is 0.18Hz .

4.2.3 Stage three

The processing method of stage three is the same as stage one.

It can be seen from the above simulation results that the tracking accuracy is improved obviously and the steady phase error is smaller when there exists a jerk.

4.3 Comparison of estimation accuracy of FLL and Rife for dynamic characteristic parameters

Compared with the FLL-PLL frequency tracking method, the difference of the method proposed in this paper is that the open-loop frequency estimation (Rife algorithm) and the fitting method are used to eliminate most of the dynamic characteristics of the signal instead of FLL. Therefore, the final frequency tracking precision determined by FLL and Rife algorithms. The Doppler dynamic parameter estimation errors of FLL and Rife algorithms under different SNR are shown in Table 2 (1000 Monte Carlo simulations):

It can be seen from table 2 that when the SNR is low, the estimation accuracy of Rife algorithm is much higher than that of FLL. With the increase of SNR, the estimation accuracy of the two algorithms is improved significantly and tends to be stable. When the SNR is high, the dynamic parameter estimation performance of Rife algorithm has little difference with that of FLL.

Table 2. Doppler dynamic parameter estimation errors of FLL and Rife ($B_L=10\text{Hz}$)

C/N_0 (dB/Hz)	Algorithm	Initial Value(Hz)	Acceleration (Hz/s)	Jerk (Hz/s ²)

50	FLL	93	311	244
	Rife	5	83	65
55	FLL	41	140	104
	Rife	4	83	61
60	FLL	24	71	46
	Rife	4	83	61
65	FLL	24	69	46
	Rife	4	83	61
70	FLL	23	66	45
	Rife	4	83	61

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5 Conclusion

This paper proposes an approved frequency tracking method in connected element interferometry. The estimation and compensation of Doppler dynamic parameters overcome the limitation of the dynamic stress on the traditional PLL tracking capability. Simulation results show that the accuracy of the new method is much higher than that of the traditional PLL method. At the same time, the dynamic parameter estimation performance of Rife algorithm is better than that of FLL under low SNR.

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