Quantitative Analysis of Range Image Patches by NEB Method

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Abstract. In this paper we analyze sampled high dimensional data with the NEB method from a range image database. Select a large random sample of log-valued, high contrast, normalized, 8×8 range image patches from the Brown database. We make a density estimator and we establish 1-dimensional cell complexes from the range image patch data. We find topological properties of 8×8 range image patches, prove that there exist two types of subsets of 8×8 range image patches modelled as a circle.

Keywords-NEB; Range Image patch; Morse theory; cell complex; k-cells

1 Introduction

Computational topology becomes an important method to analyze large sets of high-dimensional data for all branches of science [1], [2], [3], [4]. In order to study multi-dimensional data, we usually make a series of simplicial complexes from data to produce a presentation. The constructed simplicial complexes usually compose of lots of simplexes, often they are hard to compute. The authors utilize the NEB method to construct cell complexes by density functions [5], they built models for nonlinear data sets, and effectively detected the homology of the data.

In this paper, we find a circle model for 8×8 range image patches with the methods in [5], and give some topological characteristic of 8×8 range image patches. The range image patches used in this paper are from the Brown database.

2 BACKGROUND

We shortly introduce three topics: CW complexes, Morse theory, and the NEB method.

1.1 CW complexes.

First, A CW complex is a type of cell complex. For k a nonnegative integer, a k -cell is the closed ball $\{y \in R^k \mid \|y\| \leq 1\}$ of dimension k . So a 0-cell is a point, a 1-cell is a line segment, a 2-cell is a disk, etc. See Figure 1 for an example and [13] for further details.

Figure 1 A stick figure represented as a CW complex containing eight 0-cells, eight 1-cells, and one 2-cell. The 0-skeleton is on the left, the 1-skeleton is in the center, and the full 2-skeleton is on the right.

1.2 Morse theory.

The following introduction to Morse theory is informal; see [13] for thorough treatment. Suppose M is a compact manifold of dimension d and Morse function $f: M \rightarrow \mathbb{R}$ is smooth with non-degenerate critical points $m_1, \dots, m_k \in M$ satisfying

$$a_0 < f(m_1) < a_1 < f(m_2) < \dots < a_{k-1} < f(m_k) < a_k$$

the index λ_i of critical point m_i is the number of linearly independent directions around m_i in which f decreases. So a minimum has index 0, a maximum has index d , and a saddle point has index between 1 and $d-1$. Let $M^a = f^{-1}((-\infty, a])$ be the sublevel set corresponding to $a \in \mathbb{R}$. Morse theory tells us that each M_{a_i} is homotopy equivalent to a CW complex with one λ_i -cell for each critical point m_i . In particular, M_{a_i} is homotopy equivalent to $M_{a_{i-1}}$ with a single λ_i -cell attached. For instance, M_{a_1} is homotopy equivalent to a point and is obtained from M_{a_0} , the empty set, by attaching a single 0-cell.

1.3 NEB

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To find saddle points in a high-dimensional space we use the nudged elastic band method (NEB) from computational chemistry. The nudged elastic band method is used to find minimum energy paths. An initial piecewise linear path (called a band) connecting two local minima of energy E is chosen, and forces move the band towards a minimum energy path [5].

An elastic band with $N+1$ images can be defined by $[U_0, U_1, \dots, U_N]$, U_0 and U_N are initial and final states. The $N-1$ middle images are modified by an optimization algorithm [10]. The total force acting on each image is defined as following:

$$F_i = F_i^S \Big|_{\parallel} - \nabla E(U_i) \Big|_{\perp} = (\|U_{i+1} - U_i\| - \|U_i - U_{i-1}\|) \tau_i - (\nabla E(U_i) - \nabla E(U_{i-1})) \tau_i \quad (1)$$

the first part $F_i^S \Big|_{\parallel}$ is called the spring force, the second part $\nabla E(U_i) \Big|_{\perp}$ is true force, and local tangent at image i . where E is the energy of the system.

The nudged elastic band method applies an optimization algorithm to shift the images depending to the force in (1) for finding the minimum energy path. For more details of NEB, please refer to papers [8], [9].

3 DATA SET OF RANGE IMAGE PATCHES

The Brown range image database by Lee and Huang is a set of 197 range images from indoor and outdoor scenes, mostly 444×1440 pixels. The database can be found at the following webpage:

<http://www.dam.brown.edu/ptg/brid/index.html>.

The operational range for the Brown scanner is typically 2–200 meters, and the distance values for the pixels are stored in units of 0.008 meters. From the Brown database we obtain a space of range image patches as in [5], see Figure2 for some examples of range image patches.

Let R be the resulting set of high-contrast, normalized, 8×8 range patches. Our data set is a random subset $X \subset R$ of size 5,000.

Figure 2 Sample range images from the Brown database

4 COMPUTING METHOD

In this part, the steps of calculation procedure are given. refer to the paper [5] for more details of the method. For $X \subset \mathbb{R}^n$ from probability density function $f: \mathbb{R}^n \rightarrow [0, \infty)$. We get superlevel sets $X^\alpha = f^{-1}([\alpha, \infty)) = \{x \in \mathbb{R}^n | f(x) \geq \alpha\}$, important topological information is given by the high dense regions. We may make CW complex models Z^α to estimate X^α .

Only the 1-dimensional skeleton of the cell complex is given. We give a differentiable function to estimate the unknown density function. then we can get the partial maximum of the density estimate in order to make 0-cells, we make original bands randomly, then discover the convergent bands through NEB [10], so we can get the 1-cells.

A. Density estimator

For a data set $X \subset \mathbb{R}^n$, let $\Phi_{x,\sigma}: \mathbb{R}^n \rightarrow [0, \infty)$ be the probability density of a normal distribution centered at $x \in X$, we apply a differentiable density estimator $g(y) = |X|^{-1} \sum_{x \in X} \Phi_{x,\sigma}(y)$ to approach the unknown density.

B. 0-cells

To find 0-cells, we randomly select an initial point $y_0 \in X$, and iteratively define a sequence $\{y_0, y_1, \dots\}$ with $y_{n+1} = m(y_n)$, where $m(y): \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the mean shift function given by the formula:

$$m(y) = \frac{\sum_{x \in X} \Phi_{x,\sigma}(y) x}{\sum_{x \in X} \Phi_{x,\sigma}(y)}$$

The sequence $\{y_n\}$ converges to a local maximum of g [13]. we use single-linkage clustering to cluster the convergent points, and choose the densest member from each cluster as a 0-cell.

C. 1-cells

For two 0-cells, there is a 1-cell between them if we find a convergent band between them by using NEB. For an initial band $[U_0, U_1, \dots, U_N]$, where U_0 and U_N are 0-cells. Using a similar formula as (1), the total force on each midterm node U_i is computed by

$$F_i = (\|U_{i+1} - U_i\| - \|U_i - U_{i-1}\|) \tau_i + c \nabla g(U_i) \Big|_{\perp} + F_{sm} \quad (2)$$

Where $c = (\sigma \sqrt{2\pi})^n \sqrt{e}$ is the gradient constant. Let $h_{\theta,\phi}(x): [0, \pi] \rightarrow [0, 1]$ be a function defined by

$$h_{\theta,\phi}(x) = \begin{cases} 0 & x \leq \theta \\ \frac{1 - \cos\left(\frac{x - \theta}{\phi - \theta} \pi\right)}{2} & \theta < x < \phi \\ 1 & x \geq \phi \end{cases}$$

Let α_i be the angle between $(U_{i+1} - U_i)$ and $(U_i - U_{i-1})$, the smoothing force F_{sm} is computed by $h_{\theta,\phi}(\alpha_i)(U_{i+1} - 2U_i + U_{i-1})$, here $\theta = \pi/6$ and $\phi = \pi/2$.

5 EXPERIMENTAL RESULTS

Now we analyze subsets of X_8 with two types subsets of X_n : (1) $\overline{X}_8(50000)$ of X_n with 50000; (2) $\overline{X}_n(k, p)$ with $k=200, 300$ and $p=30$. Table 1 is the detailed data of the subsets.

TABLE I. DATA SET INFORMATION

	$\overline{X}_8(50000)$	$\overline{X}_8(200, 30)$	$\overline{X}_8(300, 30)$
size of data set	50000	15000	15000
dimension n	64	64	64
standard deviation σ	0.36	0.32	0.34

For $\overline{X}_8(50000)$, with the deviation $\sigma=0.36$, we obtain four 0-cells with densities 97.15, 97.83, 105.7 and 150.8 respectively, and four 1-cells with densities 84.85, 88.99, 94.48 and 99.0, these cells make a loop. Therefore, for $\alpha = 84.85$, the Z^α is a circle (Figure 3).

Figure 3 $\overline{X}_8(50000)$ and the circle $Z^{84.85}$, projected to a plane.

For $\overline{X}_8(200, 30)$, we take the value of standard deviation $\sigma = 0.32$, we get four 0-cells having densities of 407.5, 254.8, 223.1 and 192.8, and four 1-cells having densities of 198.7, 182.9, 146.0 and 126.8, all these cells make a circle too (Figure 4).

Figure 4 $\overline{X}_8(200, 30)$ and the circle $Z^{126.8}$, projected to a plane.

For $\overline{X}_8(300, 30)$, we take the value of standard deviation $\sigma = 0.34$, we get four 0-cells having densities of 194.9, 225.4, 256.8 and 412.3, and four 1-cells having densities of 128.0, 146.7, 185.6 and 201.1, all these cells make a circle too (Figure 5).

Figure 5 $\overline{X}_8(300, 30)$ and the circle $Z^{128.0}$, projected to a plane.

6 CONCLUSIONS

We exploit the NEB technique to analyze 8×8 Range image patches, the results experimentally show that the spaces of 8×8 range image patches have different subsets modelled as a circle. We find four quarter-circular 1-cells forming a loop.

We test the approach on range image data sets and find compact complexes revealing important nonlinear patterns. But It's difficult to create higher dimensional cells with this method yet.

Acknowledgment

Project supported by the National Natural Science Foundation of China (Grant No.61471409).

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